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TRACE

Orbit Determination Program
Version D

SEPTEMBER 1966

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Electronics Division
AEROSPACE CORPORATION

61

AEROSPACE CORPORATION

Prepared for COMMANDER SPACE SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
Los Angeles, California

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El Segundo Technical Operations
AEROSPACE CORPORATION
El Segundo, California

September 1966

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COMMANDER SPACE SYSTEMS DIVISION
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FOREWORD

This report is published by the Aerospace Corporation, El Segundo, California, under Air Force Contract No. AF 04(695)-669. The report was authored by Charles C. Tonies and other members of the technical staff of Aerospace Corporation as part of a continuing support effort extended by the Computation and Data Processing Center to the Satellite Systems Division.

This report describes a computer program which was developed during the period 1 November 1964 through 1 March 1966. The report was submitted on 9 September 1966 to Captain Michael A. Ikezawa, SSTDG, for review and approval.

The computer program (TRACE) has evolved over the past five years at Aerospace Corporation from an original version designed by M. M. Bennett, R. J. Mercer, D. D. Morrison, L. C. Sachnoff, and C. C. Tonies. Subsequent analysis and programming contributions have been made by D. A. Adams, C. C. Christensen, D. E. Groves, R. L. Hale, K. W. Hubbard, A. R. Jacobsen, J. D. Ostlie, A. J. Skulich and, recently, G. Buechler, P. A. Thompson, and D. C. Walker. Much of this material has been collected and revised, with their help, for publication at this time. This TRACE-D document, like the TRACE-D program, is the result of a joint effort.

The contributions of Messrs. Bennett, Hubbard and Mercer require special mention. They have been especially active in TRACE related activities and major parts of this document are due to their efforts. Finally, the editorial assistance provided by Mr. C. R. Feller is gratefully acknowledged; his many hours of diligent and conscientious attention were invaluable.

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Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Michael A. Ikezawa

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ABSTRACT

The TRACE-D computer program is designed for use on the IBM 7094 machine as the principal tool in design and analysis studies of all aspects of orbital operations. Its principal characteristics are completeness of the equations of motion, a comprehensive set of differential correction parameters, and the ability to simultaneously process observations of several satellites taken by a number of different types of sensors. The report includes objectives, equations, program structure, and complete instructions for input data preparation and program operation.

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CONTENTS

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CONTENTS (cont.)

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CONTENTS (cont.)

CONTENTS (cont.)

TABLES

FIGURES

FIGURES (cont.)

FIGURES (cont.)

FIGURES (cont.)

SECTION 1

INTRODUCTION

1.1 OBJECTIVES

The principal value of this document will lie in its usefulness as a technical reference manual and usage guide for people who work with the TRACE-D program. Sections 3 (equations), 4 (program structure), and 5 (usage) are devoted entirely to that purpose. Some theoretical background is given in Section 2 in the interest of technical completeness. The material in Section 1 is presented as an introduction to both the TRACE-D program and the principal problem of concern to those who use TRACE-D.

The basic objective of the TRACE-D Orbit Determination Program (written for the IBM 7094) is to meet the needs of members of the technical staff at Aerospace Corporation for a multipurpose, flexible computational tool for application to problems in orbit analysis. That statement of objective, however, does not fully characterize the program itself. A more interesting and useful statement necessarily includes some description of the specific problems to which the program is addressed, a discussion of program characteristics, and some explanation of how the two are related.

Sections 1.2 and 1.3 therefore include some material related to the nature of the orbit determination problem and to the manner in which TRACE-D is applied. Specific characteristics of the program are given in the context of the problem description. This may be a disadvantage to the reader who wants to know specifically what the program does or does not contain, but subsequent sections, specially Sections 4 and 5, will be useful for this. It is assumed that anyone who is interested in TRACE-D is also interested in the orbit determination problem and in some of the physical considerations and the analytic technique typically associated with the use of TRACE-D at Aerospace Corporation.

Section 1.4 is devoted to a few remarks about future orbit determination programs at Aerospace Corporation.

1.2 GENERAL PROGRAM DESIGN

The idealized design objective for the Aerospace Corporation orbit determination program would be for a nearly automatic computational tool which would provide answers to a wide range of problems relating to orbit and tracking system design, space vehicle performance, and force model analysis. In designing the actual TRACE-D program, however, considerations of automatic operation were relinquished in favor of flexibility. TRACE-D is not a "real time" program. It is used by analysts in a research or investigation environment, and its features are implemented to augment the analytical ability and imagination of its users.

The force model--i.e., the list of accelerations which appear in the equations of motion--is intended to be complete for a near-earth satellite (actually up to six satellites may be treated simultaneously for orbit determination purposes). Since these include atmospheric drag and vehicle originated accelerations and since the entire spectrum of near-earth orbits--all inclinations and eccentricities, and all altitudes up to 100,000 miles--must be considered, numerical integration and the Cowell trajectory formulation are employed. A rather extensive list of observation types is available for input to the orbit determination function or for output in the data generation mode.

1.3 MAJOR PROGRAM FUNCTIONS

The program has four major modes of operation: ephemeris generation or trajectory, orbit determination, data generation, and residuals analysis. These are all related in some direct way to the orbit determination problem, although all the functions are used extensively in solving a variety of associated problems. The characteristics of these four functions and a few specific applications examples are given in Section 1.3.1 through 1.3.4.

1.3.1 Ephemeris Generation (Trajectory) Mode

Basic to all TRACE-D functions is generation of a time history of the positions of the orbiting object in inertial space. For example, an ephemeris is necessary to calculate the "computed" or estimated observations that are required

in the orbit determination, data generation, and residuals analysis modes. Hence, every TRACE-D run involves either calculation of an ephemeris for one or more satellites or use of previously generated ephemeris information which has been stored on magnetic tape. In addition, the inertial ephemeris (trajectory) of a satellite, as well as associated quantities such as the ground track and altitude history, often are of interest in themselves.

Given the appropriate input data, the object of the trajectory or ephemeris generation function is to generate the foregoing types of information. The motion of an orbiting object is simulated by numerical integration of the applicable differential equations of motion. Specifically, the equations of motion are represented in the Cowell formulation, wherein the total acceleration vector is expressed as three components in a cartesian coordinate system. The resulting three second-order nonlinear differential equations are integrated directly by using a predictor/corrector technique with time as the independent variable.

The TRACE-D equations of motion include earth geopotential (gravity) effects in the form of a spherical harmonic expansion with provision for zonal harmonic terms J_2 through J_{10} and all tesseral and sectorial terms through $J_{6,6}$. Effects due to other bodies in the solar system (Sun, Moon, Venus, Mars, and Jupiter) are computed from inverse-square-law formulas, and positions of the other bodies are obtained from tabulated coordinates stored on magnetic tape. Acceleration due to atmospheric drag is assumed to be directly proportional to the square of the velocity relative to the air. Atmospheric density is obtained from one of three different model atmospheres that are incorporated in the program.

Instantaneous changes in the inertial velocity vector may be applied at specified times to simulate maneuvers such as orbit adjust or vehicle separation. Also, an included low-thrust acceleration term may be used to simulate thrust tailoff in cases involving large engines, long-term constant thrust, or, in some instances, leaking tanks or valves.

Although the terms of the equations of motion (i. e. , the force model) are programmed into TRACE-D, the model which is used on any particular run is to a great extent optional, and the program user may specify terms to be included in calculating the accelerations by means of indicators supplied at the time of program execution. In keeping with TRACE-D programming philosophy of avoiding built-in constants of any kind, all coefficients and arguments in the equations of motion are loaded at execution time and may be changed at the discretion of the user.

The TRACE-D trajectory or ephemeris generation mode may be employed for a variety of specific analyses. The comparative listing of associated input and output quantities given in Table 1-1 suggests the range of potential application. Complete instructions for preparation of trajectory mode input data and a sample of a typical trajectory mode output listing are given in Section 5.

1. 3. 2 Orbit Determination Mode

Solving the orbit determination problem is the principal objective of the TRACE-D program, consequently not only is it the most important program feature, but all other major program functions are related to it in some way.

Stated in simplest form, the orbit determination problem consists of extracting information from observations of a satellite in orbit. The observational data typically are gathered by a network of tracking stations on the surface of the earth. The information to be extracted from these data nearly always includes the orbital elements and may include other parameters as well. In the case of TRACE-D, the information extraction process takes the form of a generalized least-squares differential correction procedure.

Because the problems encountered in orbit determination are numerous and varied, comprehensive discussion of all related TRACE-D applications is not possible within the scope of this document. Rather, it is convenient to illustrate the features and characteristics of the TRACE-D orbit determination

Table 1-1. Input/Output Quantities Associated with TRACE-D Trajectory Mode

Input Data		Output Data
1. Earth-model constants	Items Printed Out at Regular Time Intervals	1. Satellite inertial position and velocity in rectangular and spherical coordinates
2. Atmosphere-model constants		2. Magnitudes of geocentric-radius and inertial-velocity vectors
3. Solar system constants		3. Latitude and longitude of sub-vehicle point
4. Units conversion factors		4. Altitude above earth
5. Numerical integration-control constants		5.* Differences between two trajectories in rectangular, spherical, classical element, and orbit-plane coordinates
6. Epoch time		6.* Time difference between corresponding points of two trajectories
7. Position and velocity or orbital elements at epoch time		7.* Magnitudes of distance and velocity difference vectors for two trajectories
8. Satellite ballistic coefficient		8.** Partial derivatives of trajectory position with respect to differential-equation parameters
9. Time, magnitude, and direction of velocity increments		9.** Differences between changes in position and velocity produced by perturbing parameters and changes predicted by corresponding calculated partial derivatives
10. Interval, amplitude, and decay rate of low thrust	Items Printed Out at Special Time Points	10. At time when vehicle crosses ascending node: Output Items 1 through 4 above Classical orbital elements Mean and true anomaly Nodal regression rate Rate of advance of the line of apsides Apogee and perigee radius and altitude Keplerian, anomalistic, and nodal periods Revolution number Nodal period, Nodal period decay, Nodal regression (this set obtained by simple differencing rather than by formula)
11. Table of time points where output is to occur		11. At the time of the event: Orbit adjust magnitude and direction Magnitude of low thrust at start and stop times and at ascending node times
12. Parameter selection indicators		12. At the time when the flight path angle passes through ninety degrees (roughly at apogee and perigee): Output Items 1 through 4 above
13. Tape-unit numbers for trajectory differencing		13. At the time when the vehicle reaches local maximum or minimum altitude (when $\dot{h} = 0$): Output Items 1 through 4 above
14. Latitudes and longitudes where special print-outs are to occur		
15. Minor option indicators		

*Associated with a trajectory differencing.

**Refer to the partial derivatives of trajectory position with respect to selected parameters.

function by outlining the procedure that would be involved in obtaining the solution to a hypothetical but typical problem. For example, a satellite moving for two days in a roughly circular polar orbit (i.e., 90-deg orbital inclination) at an approximately 100-n mi altitude will execute approximately sixteen revolutions around the earth and will come into view of a typical five-station tracking network approximately forty times. Each pass over a tracking station lasts a maximum of about seven minutes, during which time station radar equipment might record range, azimuth, and elevation measurements at 4-sec intervals. A typical TRACE-D application would be to reconstruct the time history of the satellite's position in an earth-centered coordinate system over the entire 2-day period by fitting such a set of tracking observations.

It is important to note that the latter requirement implies the necessity to simulate satellite translational motion over the periods when the vehicle is out of tracking-network view as well as during its passes over the various stations. The TRACE-D program simulates orbital motion by using a model consisting of the differential equations of motion as described in Section 1.3.1. Because these equations define position as a function of time, it is possible to simulate the motion of a satellite for any time period of interest if initial conditions for the equations are available. Determination of applicable initial conditions from observations of a satellite is part of the TRACE-D orbit determination function.

If the equations of motion for a near-earth satellite, together with associated physical constants, were completely known and if the positions of the tracking stations were exactly identified, then determination of initial conditions, either in terms of initial position and velocity or of any equivalent set of six orbital elements, would constitute a complete solution to a problem such as the previously noted reconstruction of a 2-day period. However, the equations

and their constants can in fact be specified only to a certain degree of accuracy because of such error sources as the following:

a. Incomplete knowledge of the earth's gravitational field

b. Dynamic fluctuations in atmospheric density

c. Extraneous impulses or thrusts originated by a satellite itself

d. Uncertainties in the geometrical shape of the earth and in locations of tracking stations on its surface

e. Computing errors accumulated in the solution of the equations of motion

It thus is apparent that determination either of initial position and velocity or of orbital elements is not sufficient to completely define the motion of a satellite at subsequent times, even if their exact values are obtainable.

Although the foregoing factors will always impose a certain degree of inaccuracy on the TRACE-D computational model, its further improvement and refinement remains a continuing objective.

The following sections indicate the manner in which the model problem interacts with orbit determination investigations in general.

1.3.2.1 TRACE-D Applications

One approach that may be adopted by a TRACE-D user when dealing with a problem such as that outlined above is to accept the environment model as programmed and use some "current-best-estimate" set of values for the associated physical constants. The processing then involves determination of initial conditions for the differential equations of motion and possibly of some physical constants directly associated with the satellite system itself.

The seven quantities most frequently determined in TRACE-D applications are the six components of the position and velocity vectors at the beginning of the time interval spanned by the observations and the satellite ballistic coefficient, which is a scale factor associated with acceleration due to atmospheric drag on

a vehicle. Because several thousand observations would have been accumulated even with the limited amount of tracking assumed for the sample situation, a least squares approach to the resulting overdetermined system is advantageous. Since it is a safe assumption that overdetermined systems would always arise in connection with TRACE-D applications even though the number of quantities to be determined may be greater than seven and the number of observations fewer than in the above typical example, the least-squares differential correction method has been adopted for TRACE-D purposes. Throughout this document the quantities so determined are conventionally referred to as parameters. This process is illustrated schematically in Figure 1-1, wherein it should be noted that a feedback to the beginning of the process, implying an iterative procedure, is indicated.

Nearly every TRACE-D application to orbit determination problems requires iteration because of the usual reason that linear techniques are used to approximate a process which inherently is nonlinear. The iteration process ordinarily is continued until the residuals (i. e., the differences between the actual observations and the computed observations) are minimized in the weighted least-squares sense. A "one-batch" processing approach is used, wherein all observations to be fit are processed (i. e., residuals and corresponding partial derivatives are computed) before corrections to the parameters are computed.

Available TRACE-D parameters are itemized in Section 1. 3. 2. 2 together with characteristics of the other major features of the orbit determination function.

1. 3. 2. 2 Specific Features of the Orbit Determination Function

The following listings of specific features and options of the TRACE-D orbit determination function are given for convenient reference as well as to characterize more clearly the environment of the orbit determination problem. Complete explanation of TRACE-D program operation is given in Section 5.

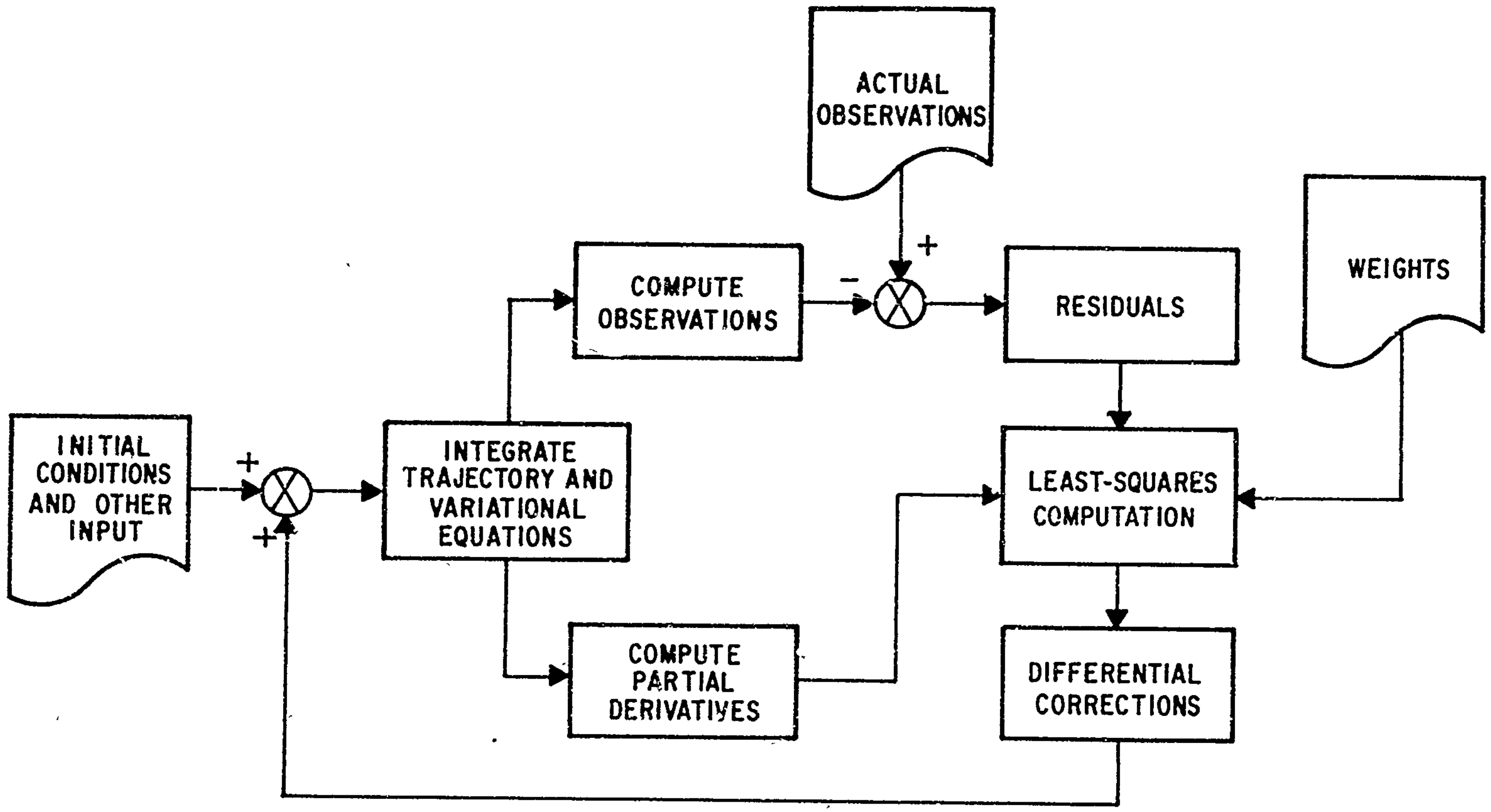


Figure 1-1. Schematic Diagram of Least-Squares Differential Correction Process

1.3.2.2.1 The Force Model

The satellite is assumed to move in an environment in which the following physical forces may be acting:

a. Gravitational attraction of the earth (zonal harmonics through J_{10} and tesseral harmonics through $J_{6,6}$ are included.)

b. Atmospheric drag (three density models are incorporated in the Trace-D program.)

c. Gravitational attraction of other bodies in the solar system

d. Vehicle-originated forces (instantaneous changes in velocity (kicks) and a low-thrust effect of exponential form are available.)

Inclusion of any of these types of forces or of any subset is completely under the control of the program user through input options.

1.3.2.2.2 Parameter List

TRACE-D is programmed to determine by differential correction the following quantities:

a. Initial position and velocity components in either spherical, rectangular, or classical element form for up to six independent orbital arcs

b. Reciprocals of ballistic coefficients (C_DA/W) for up to six different satellites

c. Up to six velocity increments (kicks) for one satellite

d. Amplitude and time constant for an exponentially decaying low thrust

e. Zonal harmonic coefficients J_2 through J_{10} and all tesseral harmonics $J_{2,1}$ through $J_{6,6}$

f. Constant biases on all types of observations

g. Scale factors for range and range-rate observations

h. Time biases (i.e., biases in reported times at which observations were made)

i. Latitudes, longitudes, and altitudes above sea level of up to 100 observing stations

On any one computer run any set of the foregoing quantities may be selected as parameters to be determined by means of differential correction, subject to a maximum limit of sixty items from items a through e above and a maximum of one hundred items total.

1.3.2.2.3 Observation Types

The following types of observations are accepted by TRACE-D:

a. Slant range
b. Local azimuth
c. Local elevation
d. Topocentric right ascension
e. Topocentric declination
f. Topocentric hour angle
g. Geocentric right ascension
h. Geocentric declination
i. Horizon sensor outputs
j. Altitude above the earth
k. Earth-fixed geocentric rectangular coordinates
l. Range rate
m. Differences in range and range rate between two stations

1.3.2.2.4 Residuals Editor

Each time an iteration is carried out, the residuals are subject to an editing criterion. In general terms, the criterion is determined by the root-mean square (RMS) value for all residuals on the previous iteration which carried the appropriate tracking station and observation-type identifiers. For example, the RMS value of the residuals for range data from Station A is used to edit the range residuals for Station A on the subsequent iteration. An important by product of this process is the printed RMS of the residuals by station and by observation type, or even by pass and observation type, at the end of each iteration.

1.3.2.2.5 Weights

In general, observations are given a priori weights by station and observation type. The twofold purpose of the weights is to normalize units so that different types of observations may be mixed in the least-squares process and to adjust their relative influence in the process of fitting observations of different accuracies. An important example of use of weights is zero weighting. If observations from a certain source are given zero weights they will have no influence in determining the values of the parameters, but corresponding residuals will appear and the RMS summary of residuals will be given. Non-zero weights may be thought of as divisors--the smaller the number, the greater the influence of the associated observations.

1.3.2.2.6 The Correlation Matrix

A by-product of the least-squares process is the inverse normal matrix. If certain assumptions are made regarding the statistical properties of the observation errors and the linearity of certain partial derivatives, this matrix may be interpreted as a variance/covariance matrix for the parameters.

A correlation matrix is computed from the inverse normal matrix by the program. Premised by the same assumptions, elements of this matrix may be interpreted as a measure of the correlation among parameter estimates.

1.3.2.3 TRACE-D As An Analytical Tool

Additional insight into the use of TRACE-D as a tool for analysis of satellite motion may be gained by further consideration of the 2-day arc-fitting problem discussed above. Assuming the previously described seven-parameter fit approach were adopted as a typical first step, the program would be put into execution and the least-squares differential correction process would be continued to convergence, which in practice is obtained when no significant further reduction in the numerical value of the RMS of all weighted residuals is obtainable. It is important to realize that corrections to the parameters

do not become arbitrarily small near convergence in accordance with theoretical prediction, but that the very best that can be expected in reduction of correction size is to approach the level of round-off error. Often it is not possible to obtain even this degree of resolution because of contributions from other error sources such as those previously itemized in Section 1. 3. 2, and because the parameters are correlated--correction to one may be partially equivalent to correction to another.

The interaction of convergence behavior and a typical model error may be illustrated by supposing that convergence occurs somewhat more slowly than experience would indicate for this type of fit, requiring perhaps six iterations (no more than three would normally be involved with correct model selection and all data consistent), and that convergence resolution also was poor; i. e., the magnitudes of the corrections with respect to the parameters remain large when the residuals RMS value has apparently reached its lowest value. Using output from the final iteration for construction of corresponding residual patterns, plots of range data residuals versus time, for example, might be of the general form shown in Figure 1-2, where each graph represents one pass (approximately four minutes) over some radar station and obviously exhibits a systematic pattern in addition to the expected random noise.

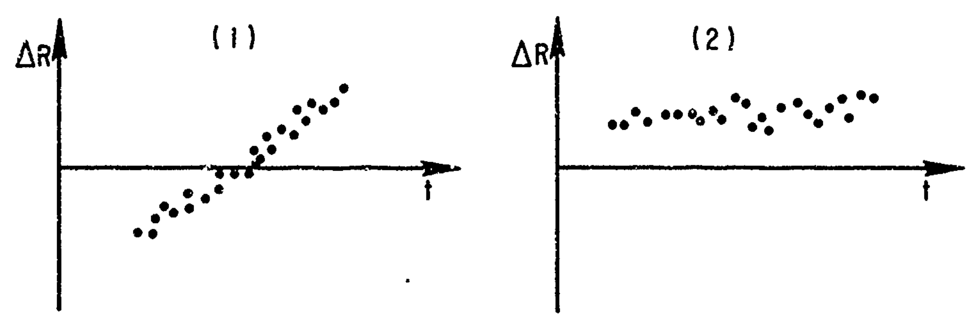


Figure 1-2. Typical Plots of Range Data Residuals

It should be noted that in this case the magnitudes of residuals for all other types of observations would be found to be large compared to the known quality of the data, and also that all the patterns of residuals versus time would be systematic.

Although the values of the initial conditions and drag parameters given by the last iteration of the run could be accepted as the solution at this point, the fact that something is obviously not right with the fit would indicate that an ephemeris derived from those parameters could not be expected to be very accurate. Consequently, the first step that might be applied by experienced analysts in this situation would be to determine whether some bad data have been included in the fit, since a batch of observations that are completely inconsistent with all others present will prevent acceptable convergence. Ordinarily a check of the listing of residuals produced on the last iteration of the run will serve to answer this question since residuals for observations that are grossly inaccurate will stand out by virtue of magnitude provided that the majority of the data are not grossly inaccurate. If a group of offending observations is detected, it may of course be deleted, which is the usual procedure if the bad data constitute a small fraction of the total set and if the reason for the poor quality is either known or not of interest.

In cases where a significant amount of data appears to be in error, additional investigation is usually required. The listing of residuals will often reveal correlation between large residuals and a station designation, which would occur when a station is reporting especially noisy or biased observations, when the station clock is inaccurate, or when the specified station location is incorrect. Careful examination of plotted residuals patterns on a pass-by-pass basis often supplies a clue as to the nature of the trouble, as in the case of the two residuals plots shown in Figure 1-2, which suggest a time bias and a range bias, respectively. However, it is not possible to conclude immediately that such biases are indeed present in the data because patterns such as the two shown may also be explained in terms of other error sources.

At this point the job of the analyst thus becomes one of accumulating evidence with respect to various hypotheses concerning the characteristics of his observation set. If the accumulated evidence supports the hypothesis that one station did indeed have a bias in the range system, for instance, a range-bias parameter for that station may be selected on the next run. Usually such a parameter is added to the list of parameters previously adopted, which in this case would make the run an eight-parameter fit. Similar discussion also is applicable to the other bias and scale-factor parameters previously itemized in Section 1.3.2.2.2, a number of which might be selected for a single run. TRACE-D will adjust these factors by differential correction and automatically apply them to corresponding observations on each iteration.

If inspection of the residuals listing suggests a station location error, some combination of latitude, longitude, and altitude parameters for that station is selected on a subsequent run and the location of the station is shifted by the program on each iteration as it attempts to minimize the residuals. The ability to determine the location of a tracking site from satellite observations has proved to be very useful, especially in cases where the site is remotely located and ground surveys are difficult to undertake.

If no strong evidence of systematic errors in the observations or station locations is apparent, the manner in which the programmed force model may differ from the real environment experienced by the satellite must be evaluated.

The necessarily inaccurate nature of the programmed models, stemming from incomplete present knowledge of real world forces, is manifested by systematic residuals patterns remaining at convergence in most of the current cases of TRACE-D orbit determination applications involving orbital arcs of more than a few revolutions. Thus it is not expected that residuals would be reduced to the level of random noise by a seven-parameter fit to observations over a 2-day arc. It therefore would be necessary for an analyst to decide by inspecting the magnitudes of the final residuals whether or

not they may be attributed to the estimated deficiencies in his model. If so, the choice would be either to accept the parameter solution or to proceed to refine the model itself by differential correction.

If residuals remain above the noise level, further significant information may still be extracted from the observation set, at least in theory. (In fact, if residuals were reduced to noise level, further information might even be extracted by means of statistical inference.) In an alternative sense, the matter may be visualized as a circumstance wherein, if the program is given additional degrees of freedom or parameters that it can adjust, the residuals will be reduced and the values of the additional parameters--model constants in this case--presumably will be improved. For example, the latter general motivation has led to inclusion of all programmed geopotential harmonic coefficients among available TRACE-D parameters.

It should be pointed out again, however, that the program user is not usually free to select simultaneously all the parameters which he may wish to determine. Parameters generally are correlated with one another; i.e., an adjustment to the value of one will result in changes in the residuals nearly identical to changes caused by an adjustment to another. The program therefore has no means by which to discriminate between parameters in computing differential corrections, and poor convergence behavior results. Given observations associated with only one orbital arc, for example, it is possible to estimate only a very limited number of geopotential parameters.

Another significant error source derives from the atmospheric density models programmed into TRACE-D, one of which is selected by the user to account for effects of atmospheric drag. The error arises because the programmed density models do not truly represent actual environment at 100-n mi altitude for instance. In this case the modeling problem is generally more difficult than that associated with the gravity field because the physical processes are dynamic, complicated, and partly unknown, and the

indices used to characterize the process are difficult to measure. Nevertheless, experienced analysts will be aware of the magnitudes of residuals that reasonably may be attributed to atmospheric uncertainties. Such judgment will be facilitated to some degree by the availability of the values of the above-noted indices, since they will serve to indicate whether or not the real atmosphere was undergoing any major perturbation such as a magnetic storm during the period when the satellite of interest was in orbit.

Use of the drag parameter, allowing the program to differentially correct the inverse ballistic coefficient, is equivalent to finding the best average density for observation-fitting purposes. Later versions of the TRACE-D program will also include additional atmosphere model parameters whose differential correction may help to reduce further the effects of uncertainties in atmospheric density. However, to the present time, the drag parameter technique has proved effective even for such demanding applications as using data from satellite observations for studying the geopotential field at 100-n mi altitudes.

One further major aspect of the model problem may be illustrated by supposing that the residuals printout reveals no obvious questions associated with the observations themselves even though the residuals magnitudes are much larger than can be attributed to uncertainties in the gravity and atmosphere models. If such a condition were to occur in combination with the poor convergence assumed for the original seven-parameter fit, the next logical hypothesis would be that the motion of the satellite was influenced by some phenomenon that was not included in the chosen model. An obvious candidate would be some force originated by the vehicle system itself, for instance thrusting by an on-board system.

An intentional maneuver involving on-orbit thrusting would not be considered in this connection because it would have been simulated as part of the

computed trajectory in the original fit. However, this does not preclude the unknown force assumed to be operating on the real satellite from also being a thrust, inasmuch as an unanticipated expulsion of material from any satellite-vehicle assemblage might result in application of a net impulse to the center of gravity of the satellite. If the satellite were injected into orbit by an engine that remained attached to the orbiting payload, the payload might be subjected to continued low-level thrusting by the engine following its shutdown at time of injection, for example, because of imperfect propellant valves. Another possibility would be development of a small leak in some pressurized tank during injection, since the cumulative effect of fuel escaping through a pinhole-sized leak in a fuel tank would easily be detectable in the motion of a satellite as measured by ground tracking systems.

One method for testing the extraneous-thrust hypothesis would be to use the time bias parameters available in TRACE-D, originally included in the program to search for and remove biases in reported observation times (i. e., station clock errors) but currently used much more extensively to aid in identifying model discrepancies. Under this procedure, observations from each pass over each station are uniquely identified. A fit permitting derivation of a time bias parameter for each pass is then made. Since the principal measureable effect of a model error usually becomes apparent as discrepancies in time of arrival at given points along an orbit, the time bias parameters offer a way for the program to reduce the residuals artificially by finding an adjustment to the reported observation times for each pass (one constant time bias for all observations from a given pass) and automatically applying this adjustment to the observation times. A plot of the time bias values versus revolution number for the present hypothetical case might appear as shown in Figure 1-3. In this connection it must be remembered that the time bias values are not actual errors in the observations, but are empirical factors introduced by the program for the purpose of fitting observations to the model used.

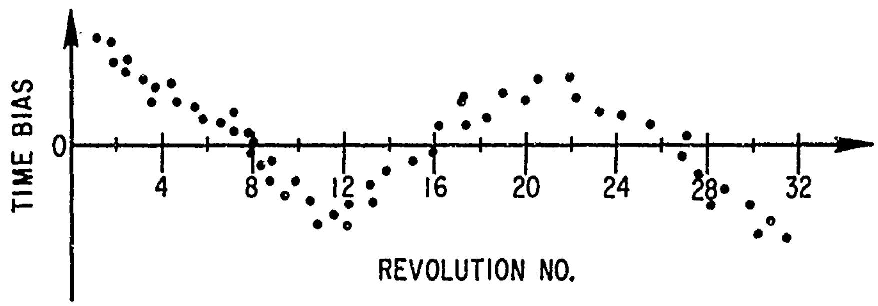


Figure 1-3. Plot of Time Bias Values Versus Revolution Number for Fit of Revolutions 1 Through 32

The plot of Figure 1-3 reflects two distinct segments in that Revolutions 1 through 11 show a monotonically decreasing pattern whereas Revolutions 12 through 32 show a concave arc. Since it is known from experience that the curved arc shape is produced when the program is forced to fit with the wrong drag factor and since drag was a parameter in the original fit, it must be true that a single value of drag would not fit both segments and that the plot in fact appears to represent two different orbits.

The next step would be to fit one of the segments at a time. Revolutions 12 through 32 would be the logical choice since a seven-parameter fit over that interval would converge quickly, final corrections would be small, and residuals would be acceptable. If the hypotheses were correct and the solution from this run were used to solve for time biases as before, a plot would then take the form shown in Figure 1-4.

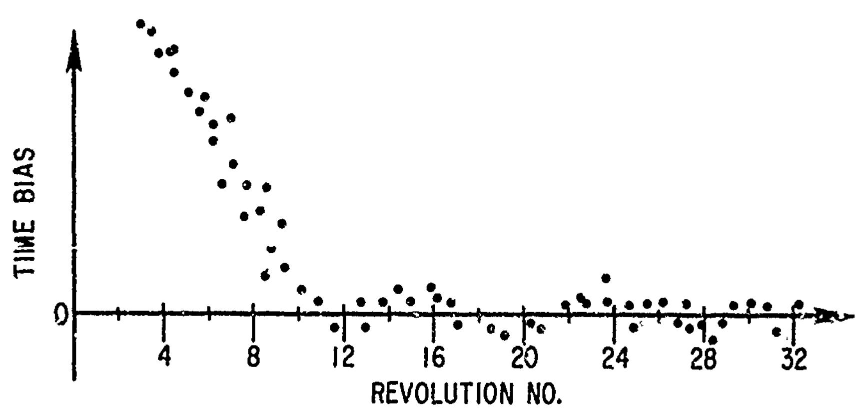


Figure 1-4. Plot of Time Bias Values Versus Revolution Number for Fit of Revolutions 12 Through 32

On the other hand, an attempt to fit Revolutions 1 through 11 with seven parameters would meet with little success since, again, convergence would be slow and it would not be possible to reduce the residuals to a desirable level.

Analysis and some intuitive deduction lead to the conclusion that the trouble on Revolutions 1 through 11 must be a continuous low thrust, probably caused by a leak that stopped somewhere on Revolutions No. 11 or 12. Since TRACE-D incorporates a low-thrust acceleration option with two associated parameters, a subsequent fit over all 32 revolutions, with the low thrust added to the model for the interval through Revolution 11, would yield the desired solution to the fitting problem.

Once the initial-condition, drag, and thrust parameters have been determined, they may be input to the TRACE-D trajectory function to produce a complete time history of position together with associated quantities such as the ground track and altitude above the earth, as described in Section 1. 3. 1.

1. 3. 3 Data Generation

As noted in Section 1. 3. 1, the TRACE-D trajectory, or ephemeris generation, function is a means for computing the path of an orbiting vehicle in inertial (space-fixed) coordinates as well as the associated ground track and other related quantities. However, no information directly related to the tracking problem is produced in the trajectory mode. The object of the data generation function therefore is to generate various forms of simulated observation measurements from a given definition of the ephemeris of a vehicle and the locations of observing stations on the earth's surface.

The most frequent application of the TRACE-D data generation feature is associated with visibility considerations. When and how long each of the tracking stations would see a particular satellite might be determined for a certain given set of burnout conditions for a launch from a particular launch

site. If associated measurements, for example, of range, azimuth, and elevation readings as functions of time also were needed, input of the appropriate indicators would produce data at whatever time frequencies were desired.

Such simulated data may either be punched on cards in appropriate format for input to the TRACE-D orbit determination function or be written on magnetic tape for subsequent input to an orbit determination run. These options are oriented toward the typical application of generating simulated data to be fit with TRACE-D in connection with orbit determination feasibility studies and error analyses. Experiments in convergence behavior, for example, may be conducted by producing simulated observation sets with various bias, noise, and distribution characteristics and then observing the influence of these factors upon convergence speed and resolution in the orbit determination mode. Fitting data with a force model different from the one used to generate the data is another typical activity providing insight into the real-world problem of fitting actual observations with a programmed force model that is always less than perfect.

Required input to the data generation function includes force model definition, epoch conditions, tracking station locations, and specifications defining the types and frequency of the data to be generated for each station. Items to be output are selected by the user through input indicators. Among available output items are all observation types accepted by the TRACE-D orbit determination function, uncertainties in certain observations as determined by uncertainties in orbit elements, indication of mutual visibility between two tracking stations, satellite ground track coordinates, and aspect angles associated with orientation of on-board antennas. A detailed list of data generation output quantities is shown in Table 5-5.

Special features of the data generation mode include:

a. A minimum-output option which allows computation and printing only of quantities associated with rise (initial visibility) time, maximum elevation angle incidence, and set (loss-of-visibility) time

b. An option to write generated observations on magnetic tape in card image format

c. Capability to compute satellite attitude in terms of pitch, roll, and yaw angles referenced to the local vertical and in-track directions

d. Optional addition of random noise and biases to generated observation data

1.3.4 Residuals Analysis

One of the end products of a TRACE-D orbit determination run is a group of residuals (differences between the actual observations and the values corresponding to the derived ephemeris). Because errors in both the observations and the program model are significant in typical TRACE-D orbit determination applications, residuals remaining after a final iteration are an important source of information for program users, as previously suggested in Section 1.3.2.

Formation of residual vectors for certain types of residuals and rotation of these vectors into an orbit-plane coordinate system, where the vector components may be more easily associated with the direction of satellite motion, is a device that often facilitates analysis of data system malfunctions and model effects. The residuals analysis function accomplishes these operations and also provides statistics such as the RMS of residuals in terms of the rotated vector components.

Residuals analysis input data and usage are essentially the same as for the orbit determination mode except for the following special operating features:

a. In normal mode the residuals analysis link computes observations from ephemeris information supplied via tape. These observations are then subtracted from the

actual observations to form the residuals. Alternatively, the "computed" observations may be supplied directly to provide a means for differencing either sets of actual observations or possibly sets of observations generated by other computer programs.

b. An edit and punch option permits a set of observations to be subjected to a residuals-editing criterion, after which a revised deck of observation cards is punched. The new deck includes only those measurements which passed the editing test.

c. Ephemeris points may be treated in the same way as observations in that they may be differenced and the resulting difference vectors resolved into orbit-plane coordinates. For example, ephemeris differences for two orbit reconstructions obtained by using the same observations but different models may be generated. This feature is frequently used at Aerospace Corporation to compare ephemeris tapes generated by orbit determination programs other than TRACE-D.

1.4 THE FUTURE OF TRACE-D

Among the most significant modifications and additions to the basic or reference version of the TRACE-D program that are now in preparation are the following:

a. An additional atmosphere model (identified as the Jacchia-Nicolet model at Aerospace Corporation) which computes density values based on actual measured values of magnetic and radiation flux indices. Seventeen constant coefficients of the model are available as parameters.

b. Additional geopotential terms which will provide a means for investigating resonance phenomena (i. e., resonance associated with orbital period and rate of earth rotation)

c. An option to sort residuals by station and observation type before printing

d. Interpolation for impact time and location

e. Availability of fifty orbit-adjust events (not parameters) in addition to the six events that are available as parameters in the reference version

f. A lunar orbiter mode

g. Many minor additional output items and format improvements

TRACE-D will become obsolete at the end of 1966 and will no longer be used at Aerospace Corporation. Documentation describing all program modifications will be published at that time which, in conjunction with this document, will completely define the terminal TRACE-D program.

A new program designated TRACE-66, to be operated in the Chippewa Operating System FORTRAN programming language, is being written at Aerospace Corporation for the CDC 6600 computer. The first operating version of TRACE-66 is expected to be in use at Aerospace Corporation in the Fall of 1966 and a full production version, together with supporting documentation, is expected to be available early in 1967. The first reference manual for the TRACE-66 program will be published at that time.

SECTION 2

THEORY

2.1 INTRODUCTION

2.1.1 General

Section 2 develops some mathematical statements of the various TRACE-D applications to orbit determination problems previously identified in Section 1. Section 2 emphasizes theoretical aspects and functional relations; Section 3 presents the specific equations and methods used in the TRACE-D program.

The applications treated in the following analysis are considered in order of increasing scope, but not in uniform depth. Thus, less familiar topics are emphasized, but more usual problems such as the numerical solution of differential equations are largely ignored.

2.1.2 Derivative with Respect to a Vector

The indication of a derivative with respect to a vector is a very convenient notational device for representing partial derivative matrices and chain rule differentiation. In this connection, the following conventions are observed throughout this document:

a. The term "vector" denotes a column vector. A row vector is described as such or denoted as a transposed vector (for example, $(x, y, z) = X^T$).

b. The derivative of a vector with respect to a scalar is a vector.

c. The derivative of a scalar with respect to a vector is a row vector.

d. The derivative of a vector with respect to a vector is a matrix (for example, if F is a vector function of a vector variable X, then $\partial F/\partial X$ is the matrix of partial derivatives whose i-j^{th} element is $\partial F_i/\partial X_j$).

The neatness of this approach is exemplified by assuming X(t) is the vector $[x_1(t), x_2(t), \cdot \cdot \cdot , x_n(t)]^T$ and y is a scalar function of X, or $y = f[x_1(t), x_2(t), \cdot \cdot \cdot, x_n(t)] = f[X(t)]$. Differentiating then leads to

$$\frac{dy}{dt} = \frac{\partial f}{\partial x_1} \frac{dx_1}{dt} + \frac{\partial f}{\partial x_2} \frac{dx_2}{dt} + \cdot \cdot \cdot + \frac{\partial f}{\partial x_n} \frac{dx_n}{dt} = \frac{\partial f}{\partial X} \frac{dX}{dt} \tag{2-1}$$

where $\partial f/\partial X$ is by convention a row vector, dX/dt is a column vector, and their juxtaposition indicates the scalar product.

2.2 THE TRAJECTORY AND ITS PARTIAL DERIVATIVES

The trajectory of a space vehicle is described by the differential equation of motion

$$\ddot{X} = - \frac{\mu X}{r^3} + F \tag{2-2}$$

with the initial values

$$X(t_o) = X_o \quad , \quad \dot{X}(t_o) = X_o$$

where

X = 3-vector of rectangular components of position (x, y, z) in an inertial coordinate system

$r = |X| = (x^2 + y^2 + z^2)^{1/2}$

μ = product (GM) of the Newtonian gravitational constant and mass of the earth

F = acceleration vector resulting from perturbing forces; i. e., all forces other than the inverse square central force due to gravitation.

One application of TRACE-D is the solving of the foregoing differential equation. The solution $X(t)$, $\dot{X}(t)$ is generated numerically at time points $t = t_j$ $(j = 0, 1, 2, \ldots)$ and defined at $t \neq t_j$ by an interpolation formula.

More sophisticated TRACE-D applications additionally require the sensitivity, as expressed by partial derivatives, of the trajectory to its initial conditions and other parameters.

It is obvious that $\ddot{X}$ is a function of μ, which is an example of a "differential-equation parameter." Other parameters of this nature, such as ballistic or oblateness coefficients, also may appear in F. The solution depends on the initial conditions X_o and $\dot{X}_o$, which in turn may be computed from "initial-condition parameters." Letting vectors of these types of parameters be represented by β and α respectively, the functional relations associated with Eq. (2-2) may be written

$$\ddot{X} = -\frac{\mu X}{r^3} + F(X, \dot{X}, \beta, t) \tag{2-3a}$$

or

$$\ddot{X} = \ddot{X}(X, \dot{X}, \beta, t) \tag{2-3b}$$

in conjunction with the initial conditions

$$X(t_o) = X_o(\alpha) \quad , \quad \dot{X}(t_o) = \dot{X}_o(\alpha)$$

It should be noted that F and $\ddot{X}$ will be functions of $\dot{X}$ whenever drag forces are present.

The dependence of the solution upon applicable parameters may be indicated by

$$X(t) = X(\alpha, \beta, t_o, t) \quad , \quad \dot{X}(t) = \dot{X}(\alpha, \beta, t_o, t)$$

Thus, the solution also can be given by the integral equations

$$\dot{X}(\alpha, \beta, t_o, t) = \dot{X}_o(\alpha) + \int_{t_o}^{t} \ddot{X}[X(\alpha, \beta, t_o, t''), \dot{X}(\alpha, \beta, t_o, t''), \beta, t'']dt''$$

and

$$X(\alpha, \beta, t_o, t) = X_o(\alpha) + \int_{t_o}^{t} \dot{X}(\alpha, \beta, t_o, t')dt'$$

$$= X_o(\alpha) + (t - t_o)\dot{X}_o(\alpha)$$

$$+ \int_{t_o}^{t}\int_{t_o}^{t'} \ddot{X}[X(\alpha, \beta, t_o, t''), \dot{X}(\alpha, \beta, t_o, t''), \beta, t'']dt''\,dt'$$

The double integral form may be reduced to a single integral form (see Ref. 2 for intermediate steps):

$$X(t) = X_o(\alpha) + (t - t_o)\dot{X}_o(\alpha)$$

$$+ \int_{t_o}^{t} (t - t'')\ddot{X}[X(\alpha, \beta, t_o, t''), \dot{X}(\alpha, \beta, t_o, t''), \beta, t'']dt'' \qquad (2\text{-}4)$$

Although they are generally unsuitable for computations, the integral expressions are of value for showing the involved functional relations explicitly.

It is now possible to show how the partial derivatives $\partial X/\partial\alpha$, $\partial X/\partial\beta$, and $\partial X/\partial t_o$, which measure to first order the sensitivity of solutions to variations in the trajectory parameters α, β, and t_o, are obtained. In addition to their extensive use in numerous TRACE-D applications, the physical interpretations of these partial derivatives often are of independent interest.

Differentiating Eqs. (2-3a) and (2-3b) with respect to α, interchanging orders of differentiation, and using the notation X_α for $\partial X/\partial\alpha$ leads to

$$\ddot{X}_\alpha = \left[\frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) + \frac{\partial F}{\partial X}\right]X_\alpha + \frac{\partial F}{\partial \dot{X}}\dot{X}_\alpha \tag{2-5}$$

with initial conditions

$$X_\alpha(t_o) = \frac{\partial X_o}{\partial \alpha} \quad , \quad \dot{X}_\alpha(t_o) = \frac{\partial \dot{X}_o}{\partial \alpha}$$

Equation (2-5) is called a "variational equation," and is a second-order linear vector differential equation whose solution is the vector of partial derivatives $X_\alpha = \partial X/\partial\alpha$ of the components of position with respect to the initial condition parameter α. In the course of solving Eq. (2-5), $\dot{X}_\alpha = \partial\dot{X}/\partial\alpha$ will also be obtained. A similar equation can be derived for each initial condition parameter.

Equation (2-5) also may be obtained by differentiating the integral, Eq. (2-4), with respect to α to give

$$X_\alpha = X_\alpha(t_o) + (t - t_o)\dot{X}_\alpha(t_o) + \int_{t_o}^{t} (t - t'')\left(\frac{\partial \ddot{X}}{\partial X}\frac{\partial X}{\partial \alpha} + \frac{\partial \ddot{X}}{\partial \dot{X}}\frac{\partial \dot{X}}{\partial \alpha}\right)dt'' \tag{2-6}$$

It should be noted that Eq. (2-6) corresponds to Eq. (2-5) in exactly the same way that Eq. (2-4) corresponds to Eq. (2-3).

The variational equations for initial time t_o are of the same form

$$\ddot{X}_{t_o} = \left[\frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) + \frac{\partial F}{\partial X}\right]X_{t_o} + \frac{\partial F}{\partial \dot{X}}\dot{X}_{t_o}$$

However, they are associated with different initial conditions

$$X_{t_o}(t_o) = -\dot{X}_o(a) \quad , \quad \dot{X}_{t_o}(t_o) = -\ddot{X}_o(a)$$

These expressions may be derived by differentiating with respect to t_o integral Eq. (2-4), which best shows the dependence upon t_o.

The variational equations for the differential equation parameter β are

$$\ddot{X}_\beta = \left[\frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) + \frac{\partial F}{\partial X}\right] X_\beta + \frac{\partial F}{\partial \dot{X}} \dot{X}_\beta + \frac{\partial F}{\partial \beta} \qquad \text{(2-7a)}$$

and

$$X_\beta(t_o) = \dot{X}_\beta(t_o) = 0 \qquad \text{(2-7b)}$$

As a source of partial derivatives, variational equations yield more accurate results than typical analytic derivatives, which usually assume two-body motion. Also, the partials can be generated more rapidly than difference quotient approximations because the terms $[\partial(-\mu X/r^3)/\partial X + \partial F/\partial X]$ and $\partial F/\partial \dot{X}$ of Eq. (2-5) are identical in all the variational equations, and only the nonhomogeneous term $\partial F/\partial \beta$ of Eq. (2-7a) varies with a particular parameter.

A further advantage of the variational equations is that they permit use of the difference quotient technique as a checking device since lack of substantial agreement between the partial derivative estimates obtained by the two methods would indicate the presence of error. Although this test should not be considered fool proof, it is valuable and should not be overlooked.

2.3 BASIC ORBIT DETERMINATION PROBLEM

The basic orbit determination problem, as discussed in Section 1, is to find values for a set of observational-model parameters such that the differences between the actual measured observations and corresponding values computed

from the model will be minimized in a generalized least-squares (GLS) sense. With respect to TRACE-D, GLS refers to a fitting process which allows weighting matrices to contain some off-diagonal elements as described in Section 2.5.2.

The TRACE-D program model includes the trajectory of the vehicle with associated initial-condition and differential-equation parameters, locations of observing stations, and certain systematic errors in the observation sensing equipment. The full list of parameters available in TRACE-D is given in Section 1. Relative significance may be assigned to observations of various types and quality by means of a weighting matrix. This weighting matrix is also used to a limited extent in processing correlated observations (See Section 2.5 2 for discussion of TRACE-D correlated data option).

The basic orbit determination problem thus may be restated: Given a set of n observations of orbiting objects, an appropriate weighting matrix, and a model from which corresponding observations may be computed, determine values of the model parameters to minimize the expression[1]

$$||O_m - O_c(P)||_W^2 = [O_m - O_c(P)]^T W [O_m - O_c(P)] \qquad (2\text{-}8)$$

where

O_m = vector of measured observations

O_c = vector of corresponding computed observations

P = vector of model parameters

W = observation weighting matrix

The particular significance of the minimization process depends on the nature of the weighting matrix. In TRACE-D this process usually is simple weighted least squares (WLS) if W is diagonal, or generalized least squares (GLS)

[1]The notation $|| \ ||_W$ will be used in subsequent discussions to indicate a quadratic form in which the weighting matrix W appears.

otherwise. However, under certain circumstances the process may be minimum variance. Detailed explanation of the weighting-matrix options and their significance is given in conjunction with the statistical aspects of the orbit determination problem (Section 2. 5).

An approximate solution P_o of Eq. (2-8) is nearly always available, since trajectory initial conditions may be estimated from design information or preliminary orbit determination methods. Also, the "current best" values for force model parameters and station locations usually represent an excellent first approximation. Thus, expanding $O_c(P)$ in a Taylor series about P_o to first order, the quantity to be minimized, Eq. (2-8), becomes

$$||O_m - O_c(P)||_W^2 = ||O_m - O_c(P_o) - A \cdot \Delta P||_W^2 \tag{2-9}$$

where

$$A = \frac{\partial O_c}{\partial P} = \text{matrix of partial derivatives evaluated at } P = P_o$$

For the case where the parameters are quantities appearing in the equations of motion (including initial conditions), the partial derivatives are computed from the chain rule

$$\frac{\partial O_c}{\partial P} = \frac{\partial O_c}{\partial X}\frac{\partial X}{\partial P} + \frac{\partial O_c}{\partial \dot{X}}\frac{\partial \dot{X}}{\partial P} \tag{2-10}$$

where

$$\frac{\partial X}{\partial P}, \ \frac{\partial \dot{X}}{\partial P} = \text{matrices of solutions of the variational equations}$$

The matrices $\partial O_c/\partial X$ and $\partial O_c/\partial \dot{X}$ and those rows of $\partial O_c/\partial P$ that correspond to parameters not appearing in the equations of motion (station locations and observation-error terms) are computed directly from geometrical relations.

The differences $O_{mc}(P_o) = O_m - O_c(P_o)$ between the observations and the corresponding quantities computed from the assumed values P_o are called residuals. These result from the presence of random observational errors, inadequacies in the form of the model, incorrect values for the model parameters and computational errors due to roundoff and truncation effects. Since the original nonlinear GLS problem has been replaced by the approximate linear problem of finding a correction vector ΔP such that $||O_{mc}(P_o) - A \cdot \Delta P||_W^2$ is minimized, the expression $P = P_o + \Delta P$ is in general only an approximate solution, and an iterative process is required. Then $||O_{mc}(P_o)||_W$ measures the degree to which an orbit computed from the current values P_o of the parameters fits the observations and the expression $||O_{mc}^p||_W = ||O_{mc}(P_o) - A \cdot \Delta P||_W$ (superscript p means "predicted") then is an approximation, based upon the linearity assumption, to the value of $||O_{mc}||_W$ that would be obtained by replacing P_o with $P_o + \Delta P$. In a well-behaved iteration the observed $||O_{mc}||_W$ should follow the predicted $||O_{mc}^p||_W$, the relative agreement between the two factors being a measure of convergence of the process.

The correction vector ΔP may be shown to be the solution of the linear system $(A^T W A)\Delta P = A^T W O_{mc}$ in various ways. Two proofs of this are the following:

Proof 1:

If

$$f(\Delta P) = ||A \cdot \Delta P - O_{mc}||_W^2 = (A \cdot \Delta P - O_{mc})^T W (A \cdot \Delta P - O_{mc})$$

and if $f(\Delta P)$ is differentiated with respect to ΔP, one obtains

$$\frac{\partial f}{\partial(\Delta P)} = 2\left(A^T W A \cdot \Delta P - A^T W O_{mc}\right)^T$$

The latter expression must be zero if ΔP minimizes $f(\Delta P)$.

Proof 2:

If

$$A^T W A \cdot \Delta P = A^T W O_{mc}$$

then, for any $\Delta P' \neq \Delta P$, one obtains

$$\begin{aligned} f(\Delta P') &= ||A \cdot \Delta P - O_{mc} + A(\Delta P' - \Delta P)||_W^2 \\ &= ||A \cdot \Delta P - O_{mc}||_W^2 + 2[A \cdot \Delta P - O_{mc}]^T W [A(\Delta P' - \Delta P)] \\ &\quad + ||A(\Delta P' - \Delta P)||_W^2 \\ &= f(\Delta P) + 2\left(A^T W A \cdot \Delta P - A^T W O_{mc}\right)^T (\Delta P' - \Delta P) \\ &\quad + ||A(\Delta P' - \Delta P)||_W^2 \\ &= f(\Delta P) + ||A(\Delta P' - \Delta P)||_W^2 \end{aligned}$$

or

$$f(\Delta P') > f(\Delta P)$$

From Proof 2 it is evident that ΔP minimizes $F(\Delta P)$.

2.4 CONSTRAINED AND BOUNDED LEAST-SQUARES SOLUTIONS

Two distinct types of restrictions upon the solution of the generalized least-squares (GLS) problem may be necessary or desirable, in that constraints among the parameters may be a part of the physical problem, and bounds upon the magnitude of the corrections may facilitate computation.

2.4.1 Constraints

An example of a physical constraint which could be imposed upon parameters would be precise knowledge of the relative locations of two nearby observing stations. If the actual locations of such a pair of stations were among the parameters P in a differential correction, it would be important to constrain the ΔP corrections in such a manner that the relative locations of the stations

were preserved. In TRACE-D this is accomplished by introducing linear constraints in the form

$$\Delta P = B \cdot \Delta P' + C \tag{2-11}$$

where

B = rectangular matrix

$\Delta P'$ = reduced set of independent parameters

and by solving the GLS problem in terms of $\Delta P'$ and using Eq. (2-11) to obtain the constrained correction ΔP. Solving the GLS problem in terms of $\Delta P'$ requires minimizing $||A \cdot \Delta P - O_{mc}||^2_W$ subject to the constraint Eq. (2-11), which is equivalent to minimizing $||A \cdot (B \cdot \Delta P' + C) - O_{mc}||^2_W = ||(AB) \cdot \Delta P' - (O_{mc} + AC)||^2_W$. This required minimum is obtained as the solution of the linear system

$$(AB)^T W (AB) \Delta P' = (AB)^T W (O_{mc} + AC) \tag{2-12}$$

2.4.2 Bounds

Under conditions fairly common in the solving of the GLS problem, which for example might involve inadequacies in the observational model or a poor initial approximation P_o, the observed $||O_{mc}||_W$ may fail to follow the predicted $||O^P_{mc}||_W$, or may even diverge. In the presence of such manifestations of nonlinearity, it may become necessary to introduce a side condition bounding the magnitude of the correction vector ΔP at each iteration to assure eventual convergence. In this connection, if the reciprocals of the bounds are collectively referred to as the diagonal matrix G, the problem becomes one of minimizing the quantity $||A \cdot \Delta P - O_{mc}||^2_W$, subject to the bounding condition $||G \cdot \Delta P||^2 < 1$. Thus, if $||G \cdot \Delta P||^2 = \sum_i (\Delta p_i / g_i)^2 \leq 1$, then one obtains $|\Delta p_i| \leq g_i$ for each component, where g_i are the individual diagonal elements of G, (i.e. $G_{ii} = g_i^{-1}$).

The simple geometrical interpretation for a two-parameter example is illustrated in Figure 2-1, wherein the problem is to find a minimum of the surface $f(\Delta P) = ||A \cdot \Delta P - O_{mc}||_W^2$ over all ΔP within the ellipse defined by g_1 and g_2. An elliptic rather than a circular region is used to account for the range of magnitudes of the various parameters.

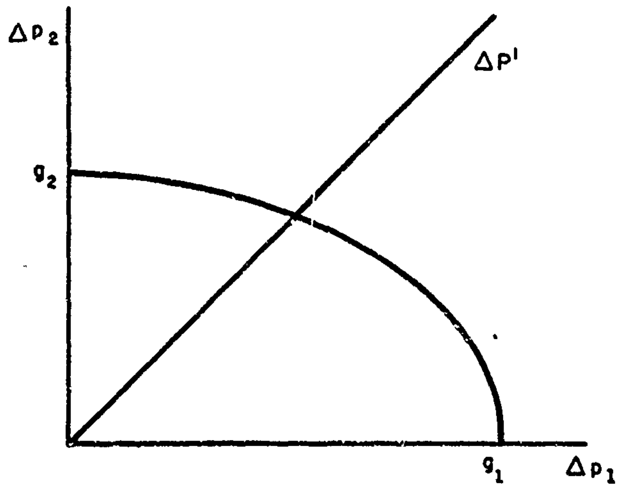


Figure 2-1. Two-Parameter Constraint Ellipse

If the unconstrained solution does not occur within the ellipse $||G \cdot \Delta P||^2 = 1$, a new function $F(\Delta P) = f(\Delta P) + z||G \cdot \Delta P||^2$ is minimized, whose minimum point $\Delta P'(z)$ is found as the solution of

$$(A^T WA + zG^T G)\Delta P' = A^T WO_{mc} \tag{2-13}$$

As z increases, minimization of F requires smaller and smaller values of $\Delta P'(z)$. More precisely, it will be shown that $||G \cdot \Delta P'(z)||$ is a decreasing function of z and that, in particular, by a search and interpolation procedure a value z' of z can be found such that $||G \cdot \Delta P'(z')||^2 = 1$. Since minimizing $F = f + z'||G \cdot \Delta P'(z')|| = f + z'$ actually is equivalent to minimizing f, because these terms differ only by the constant z', the point $\Delta P'(z')$ which minimizes

$f(\Delta P)$ along the bounding ellipse is identified. It also will be shown that $f[\Delta P'(z)]$ is an increasing function of z. Thus, any interior point of the ellipse corresponds to larger values of z and of f, and therefore the constrained minimum point is on the boundary and is the solution $\Delta P'(z')$ of $(A^TWA + z'G^TG)\Delta P = A^TWO_{mc}$ for which $||G\cdot\Delta P'(z')||^2 = 1$. Throughout the following proofs of the above, primes have been dropped for convenience.

The monotonic decreasing character of $||G\cdot\Delta P||$ as a function of z may be demonstrated by differentiating

$$(A^TWA + zG^TG)\Delta P(z) = A^TWO_{mc} \tag{2-14}$$

with respect to z to obtain

$$(A^TWA + zG^TG)\frac{d}{dz}(\Delta P) + (G^TG)\Delta P = 0 \tag{2-15}$$

or

$$\frac{d}{dz}(\Delta P) = -(A^TWA + zG^TG)^{-1}(G^TG)\Delta P \tag{2-16}$$

Differentiating $d||G\cdot\Delta P||^2/dz$ and substituting Eq. (2-16) then leads to

$$\begin{aligned}\frac{d}{dz}||G\cdot\Delta P||^2 &= 2(\Delta P^T)(G^TG)\left(\frac{d}{dz}\,\Delta P\right)\\ &= -2\Delta P^T(G^TG)(A^TWA + zG^TG)^{-1}(G^TG)\Delta P \end{aligned} \tag{2-17}$$

Since $(A^TA + zG^TG)^{-1}$ is positive definite for positive z, the following is true

$$\frac{d}{dz}||G\cdot\Delta P||^2 < 0 \tag{2-18}$$

whenever $(G^TG)\cdot\Delta P \neq 0$.

The monotonic increasing character of $f[\Delta P(z)]$ may similarly be established by showing that $df/dz > 0$. Differentiating $f[\Delta P(z)]$ then leads to

$$\frac{df}{dz} = \frac{\partial f}{\partial(\Delta P)} \frac{d(\Delta P)}{dz}$$

$$= \left[2\left(A^TWA\Delta P - A^TWO_{mc}\right)\right]^T\left[-(A^TWA + zG^TG)^{-1}(G^TG)\Delta P\right]$$

$$= -2\left[(A^TWA + zG^TG)\Delta P - A^TWO_{mc} - z(G^TG)\Delta P\right]^T\left[(A^TWA + zG^TG)^{-1}(G^TG)\Delta P\right] \tag{2-19}$$

However, since by Eq. (2-14) $(A^TWA + zG^TG)\Delta P(z) = A^TWO_{mc}$, differentiating leads to

$$\frac{df}{dz} = +2z\Delta P^T(G^TG)(A^TWA + zG^TG)^{-1}(G^TG)\Delta P \tag{2-20}$$

which is positive whenever $(G^TG)\Delta P \neq 0$.

2.4.3 Solution of the Linear System

Solution of the linear system $(A^TWA + zG^TG)\Delta P = A^TWO_{mc}$ and inversion of the coefficient matrix $C = A^TWA + zG^TG$ is accomplished by a special method related to a procedure known classically as the square root method (Ref. 3). This method is finite, or noniterative, is applicable only to symmetric matrices, and is based on the fact that a symmetric matrix can be decomposed as a product of the form $C = LDL^T$, where L is a lower triangular matrix with (-1) as diagonal elements and D is a diagonal matrix. In such a representation, det (L) = ±1 and det (D) = det (C). Therefore, L^{-1} exists and in fact is also a lower triangular matrix with (-1) on the diagonal, and D has no zero elements if C is nonsingular. Two equivalent forms consequently are

(1) $L^{-1}C(L^T)^{-1} = L^{-1}C(L^{-1})^T = D$

(2) $C^{-1} = (L^{-1})^TD^{-1}L^{-1}$

or

$$(1') \quad SCS^T = D$$

$$(2') \quad C^{-1} = S^T D^{-1} S$$

where $S = L^{-1}$.

It thus is apparent that the inversion of C and the solution $\Delta P' = (C^{-1})A^T WO_{mc}$ require matrices S and D such that $SCS^T = D$.

A bordering technique used to find S and D assumes at the k^{th} stage that the k^{th}-order principal minors of S and D have been found. The $(k + 1)^{st}$-order minors require the vector V and the scalar b such that

$$\begin{pmatrix} S_k & 0 \\ V^T & -1 \end{pmatrix} \begin{pmatrix} C_k & d \\ d^T & a \end{pmatrix} \begin{pmatrix} S_k^T & V \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} D_k & 0 \\ 0 & b \end{pmatrix} \qquad (2\text{-}21)$$

where

$$C_k = k^{th}\text{-order minor of } C$$

$$\begin{pmatrix} C_k & d \\ d^T & a \end{pmatrix} = (k+1)^{st}\text{-order minor of } C$$

It may easily be verified that the required V and b are

$$V = S_k^T D_k^{-1} S_k d$$

and (2-22)

$$b = a - V^T d$$

2.5 THE STATISTICS OF ORBIT DETERMINATION

In the process of orbit determination by the method of weighted least squares (WLS), no assumptions regarding the statistics of the observational errors are necessary. Although no statistical conclusions can be drawn from obtained results, the justification for the method is that solutions gained by minimizing residuals in the least squares sense have proved to be very useful. It should be noted that all previous TRACE- program versions employed WLS exclusively.

On the other hand, if the two frequently adopted assumptions that (1) the observational errors ϵ_i are random with mean zero and known covariance matrix Σ and (2) that the weighting matrix $W = \Sigma^{-1}$, then the inverse normal matrix is the linear approximation of the variance-covariance matrix (often abbreviated "covariance matrix") of the parameters being determined. Inasmuch as this matrix depends only on the partial derivatives of observations with respect to the parameters and on the weighting matrix, it may be generated and used for statistical analysis of tracking networks and spacecraft systems in the absence of actual or simulated observations. Details pertinent to the variance-covariance matrix, as well as the relation of WLS orbit determinations to minimum-variance and maximum-likelihood criteria and the particular parameter estimation methods available in the TRACE-D program are outlined in the following sections.

2.5.1 The Variance-Covariance Matrix

If it is assumed that the vector of measured observations O_m is the true value $O_c(P_t)$ (subscript t denotes "true") plus a random error ϵ, the linear approximation to $O_c(P_t)$ is

$$O_c(P_t) = O_c(P_o) + A \cdot \Delta P \qquad (2\text{-}23)$$

and the residual vector is

$$O_{mc} = O_m - O_c(P_o) = A \cdot \Delta P_t + \epsilon \qquad (2\text{-}24)$$

where

$O_c(P_o)$ = vector of computed quantities

P_o = estimate of the true parameter vector P_t

$\Delta P_t = P_t - P_o$

A = matrix of partial derivatives $\partial O_c / \partial P_o$

ϵ = vector of unbiased observational errors

As noted previously, the GLS problem is minimizing $f(\Delta P) = \|O_{mc} - A \cdot \Delta P\|^2_W$, for which the solution is $\Delta P' = (A^T WA)^{-1} A^T WO_{mc}$.

It can readily be shown for the linear problem that the vector $\Delta P'$ is an unbiased estimate of the true value ΔP_t and that the expected value of $\Delta P'$ is the true value ΔP_t, even though $\Delta P'$ is a random quantity since it depends upon the residuals and therefore upon the observational errors. Thus, with

$$
\begin{aligned}
\Delta P' &= (A^T WA)^{-1} A^T WO_{mc} \\
&= (A^T WA)^{-1} A^T W(A \cdot \Delta P_t + \epsilon) \\
&= \Delta P_t + (A^T WA)^{-1} A^T W\epsilon
\end{aligned}
$$

then

$$
E(\Delta P') = \Delta P_t + E[(A^T WA)^{-1} A^T W\epsilon] = \Delta P_t \qquad (2\text{-}25)
$$

The latter expression reflects appeal to the linearity of $E(\cdot)$ and the assumption that $E(\epsilon) = 0$. It should be noted that the vector $\delta P' = \Delta P' - \Delta P_t$ would be the deviation, due to random errors, of the solution $\Delta P'$ from the true value ΔP_t. This vector has previously been shown to have the expected value zero.

The expected values of the square of the deviations (the product of two components of the deviations) are summarized in the quantity $E(\delta P' \delta P'^T)$, which by definition is the covariance matrix $C(P')$ of the (estimated) parameters. If the symmetry of the matrices W and A^TWA is used, the expression may be written

$$E(\delta P' \delta P'^T) = E[(A^TWA)^{-1}A^TW(\epsilon\epsilon^T)WA(A^TWA)^{-1}] \tag{2-26}$$

However, since $E(\epsilon\epsilon^T) = \Sigma$, the covariance matrix of the observational errors, Eq. (2-26) becomes

$$C(P') = (A^TWA)^{-1}A^TW\Sigma WA(A^TWA)^{-1} \tag{2-27}$$

Equation (2-27) is the general form of the covariance matrix for a GLS estimate of the parameters. However, use of the second assumption, $W = \Sigma^{-1}$, results in the important simplification

$$C(P') = (A^TWA)^{-1} = (A^T\Sigma^{-1}A)^{-1} \tag{2-28}$$

This latter expression represents the basic covariance matrix calculated in TRACE-D.

2.5.2 Minimum-Variance and Maximum-Likelihood Estimates

In most instances of orbit determination from observations, least-squares methods (WLS or GLS) not only require no statistical justification, but in fact none exists. These methods are intended simply to produce fits and predictions of acceptable quality. Conversely, statistical conclusions are sought in other applications such as systems analysis and design. These are commonly based on minimum-variance (MV, or "Markov") or on maximum-likelihood (ML) estimations. This section describes in general terms the assumptions governing MV and ML techniques and their relation to TRACE-D program procedure.

The MV estimate of ΔP is the linear unbiased estimate which minimizes the diagonal terms (variances) of the variance-covariance matrix of parameters P (Ref. 4). The applicable formulas are

$$\Delta P_{MV} = (A^T \Sigma^{-1} A)^{-1} A^T \Sigma^{-1} O_{mc} \tag{2-29}$$

and

$$C(P_{MV}) = (A^T \Sigma^{-1} A)^{-1} \tag{2-30}$$

When Σ^{-1} is used as the weighting matrix W, the GLS estimate and the covariance matrix in TRACE-D also are MV.

The WLS procedure, which has been used in prior TRACE-program versions, also is available in TRACE-D and in fact remains the most frequently utilized weighting option. However, one alternative weighting option that has been added to TRACE-D, and whose inclusion in the program represents a step in the direction of a generalized MV capability, is the possibility of supplying non-diagonal weighting submatrices up to 3 X 3 in dimension. In this connection, all observations associated with each weighting matrix must be of the same set number (see Section 5. 1. 3. 4), and their observation times must be the same. Although this feature is rather restrictive in that it precludes treatment of cases involving known time correlations in the observations, it permits one important class of observations to be handled correctly, which previously was not possible.

For example, if sets of raw observations of some arbitrary type are subjected to a data-reduction process which finally produces a series of three-component (x, y, z) vectors representing estimates of the position of a satellite at various times, the data-reduction process typically also will yield a 3 X 3 covariance matrix for the estimated errors in (x, y, z) that is derived from estimates of errors in the original raw observations. In the fitting of the

(x, y, z) points with the TRACE program, the desirability of weighting these points with the inverse of the covariance matrix for each (x, y, z) set has been realized for the first time in TRACE-D.

Since the criterion for MV is that $W = \Sigma^{-1}$, which is assumed to be the case for data such as those noted above, the solution obtained by fitting the data with TRACE-D may said to be an MV estimate.

Nothing has so far been assumed about the actual form of the distribution of the random errors. If a specific probability density function is assumed, then it is possible to seek the estimate that maximizes the probability or likelihood that the actual observations would occur, given the estimated values of the parameters. In the case of a joint normal (or gaussian) distribution of observational errors with covariance matrix Σ, the ML estimate reduces to MV. In practice the gaussian assumption is always made (Ref. 5).

The foregoing may be summarized: If the weighting matrix applied in the least-squares process is the inverse of the estimated covariance matrix for the observational errors, the GLS estimate obtained by TRACE-D is also MV; if the errors are further assumed to be normally distributed, the estimate is also ML.

SECTION 3

EQUATIONS

3.1 COORDINATE SYSTEMS

Several coordinate systems are employer in TRACE-D. Basic to all is the earth-centered inertial system, which has as its reference plane and direction the true equator at epoch and the mean equinox at 0 hour GMT of the date of epoch. This description follows from the manner in which input quantities are treated, in which connection the longitude reference is α_g (Ref. 6), or the right ascension of the Greenwich meridian from the mean equinox of date at 0^h GMT (α_g is also known as Greenwich mean sidereal time), and the reference plane for initial position and velocity parameters is the equator.[2]

Three types of coordinates within the foregoing system that are useful for TRACE-D program purposes are described in Section 3.1.1. A station-dependent coordinate system that has been introduced to facilitate radar computations and an orbit-plane coordinate system that is used for analysis of residuals are described in Sections 3.1.2 and 3.1.3. The station-centered inertial and the vehicle-centered coordinate systems discussed in Section 3.1.4 and 3.1.5 are introduced for convenience in the computation of partial derivatives and vehicle-centered quantities, respectively.

[2]TRACE-A and TRACE-D are identical in this regard. The corresponding statement in Section 3 of Ref. 1 is incorrect.

3.1.1 Earth-Centered Inertial System

The basic earth-centered coordinate system is shown in Figure 3-1,

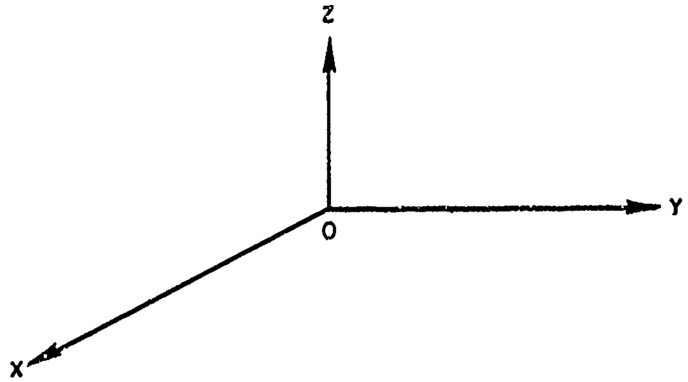


Figure 3-1. Earth-Centered Coordinate System

where

0 = center of mass of the earth

X = vector from 0 in the equatorial plane directed to the vernal equinox at t_g, (0 hour GMT of epoch date)

Y = vector from 0 perpendicular to X in such direction that (X, Y, Z) is a right-handed system

Z = vector from 0 perpendicular to the equatorial plane and directed north

The position and velocity of a body at a point P within this reference frame may be expressed in rectangular or spherical coordinates or in terms of the classical elements of its orbit.

3.1.1.1 Rectangular Coordinates

Within rectangular coordinates, representation of a point P may be expressed by

$$P = P(x, y, z, \dot{x}, \dot{y}, \dot{z})$$

where x, y, z are, respectively, the components of position of the point in the X, Y, Z directions as defined in Figure 3-1 and $\dot{x}$, $\dot{y}$, $\dot{z}$ are the components of its velocity in those directions.

3.1.1.2 Spherical Coordinates

In the spherical coordinate system shown in Figure 3-2, position and velocity

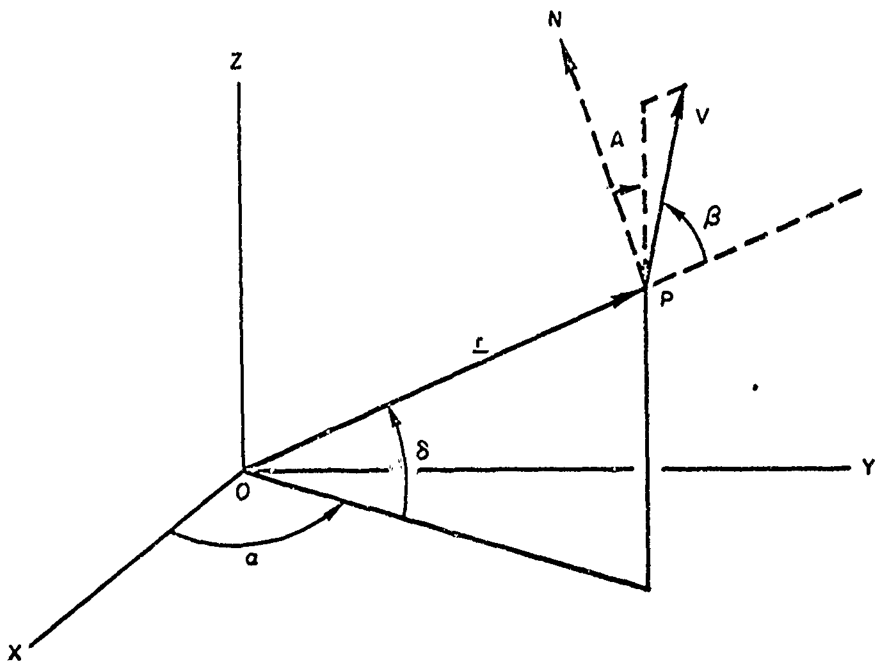


Figure 3-2. Spherical Coordinate System

at a point may be written

$$P = P(\alpha, \ \delta, \ \beta, \ A, \ r, \ v)$$

where

V = a vector equal in magnitude and direction to the velocity of the vehicle at P

α = right ascension measured from X-axis, positive eastward

δ = geocentric latitude ($-\pi/2 \le \delta \le \pi/2$)

β = angle between V and the geocentric vertical at P ($0 \le \beta \le \pi$)

A = azimuth of V from true north, measured eastward on a plane normal to geocentric vertical

r = magnitude of $\overline{OP}$

v = magnitude of V

3.1.1.3 Orbital Elements

In Figure 3-3, P is a point on the osculating conic described by the parameters a, e, i, Ω, and ω. The position of P on this conic is determined by τ and by a value for the current time. Within this reference framework,

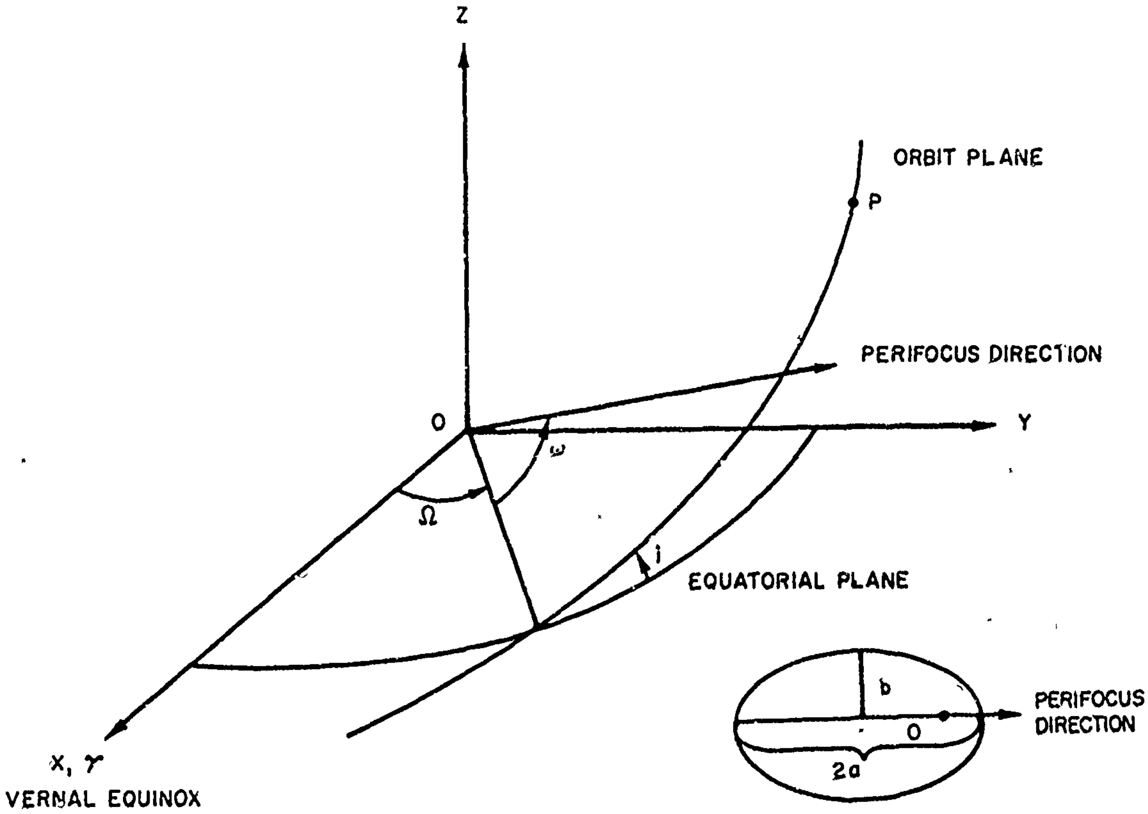


Figure 3-3. Orbital Elements

the expression for position is given by

$$P = P(a, e, i, \Omega, \omega, \tau)$$

where

a, b = semi-major and semi-minor axes

e = eccentricity $= \sqrt{a^2 - b^2}/a$

i = inclination of orbit plane

Ω = right ascension of ascending node

ω = angle between direction of perigee and the line of nodes

τ = time in minutes from t_g of last previous perigee passage (t_g = 0 hours GMT of day of epoch)

3.1.2 Station-Dependent Coordinate System

A station-dependent coordinate system intended to facilitate radar computations is shown in Figure 3-4,

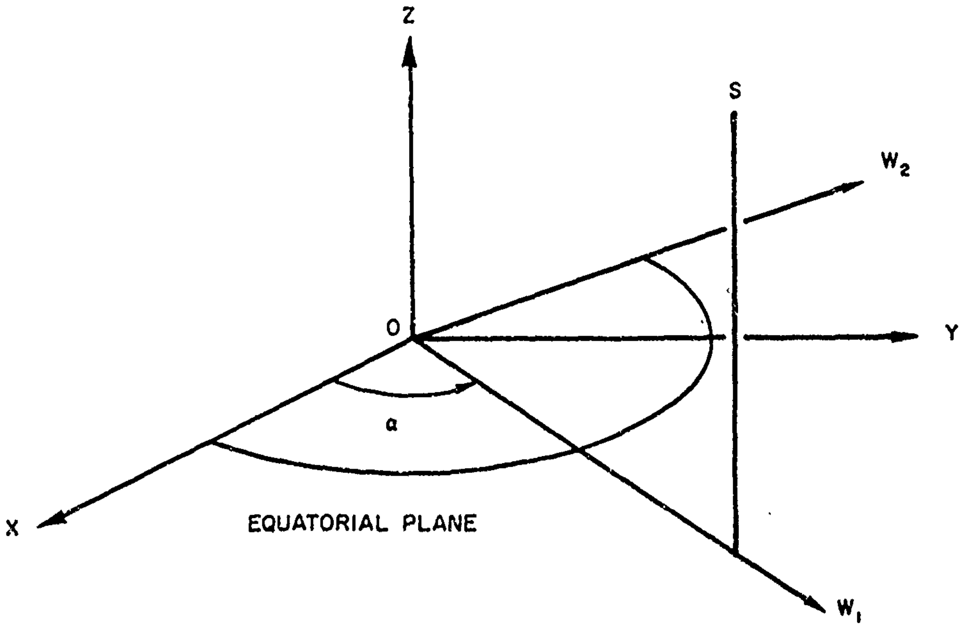


Figure 3-4. Station-Dependent Coordinate System (W_1, W_2, Z)

where

S = station location at some time t

$\alpha = \ell + a_g + \omega_e(t - t_g)$

ℓ = geographic longitude of station

a_g = right ascension of Greenwich at time t_g

ω_e = rate of rotation of the earth

W_1, W_2 = axes X and Y rotated through angle α.

3.1.3 Orbit-Plane Coordinate System

In Figure 3-5 the basic coordinate system is the earth-centered inertial reference frame described in Section 3.1. However, in this instance the

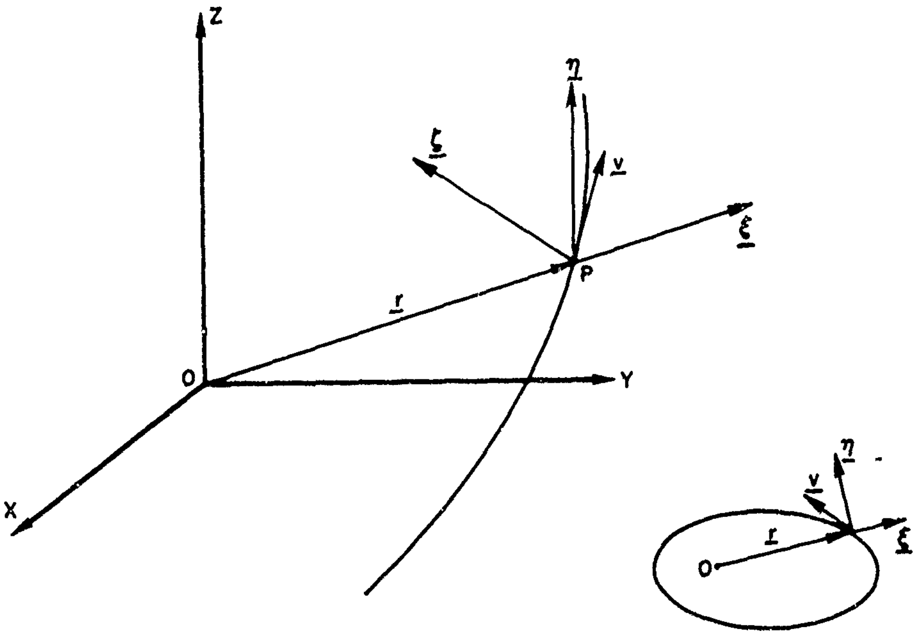


Figure 3-5. Orbit-Plane Coordinate System

point P, which usually is the position of a reference vehicle, represents the origin of the orbit-plane coordinate system defined by the vectors $\underline{\xi}$, $\underline{\eta}$, $\underline{\zeta}$, which are in the radial, in-track, and cross-track directions, respectively. In the smaller sketch it should be noted that the $\underline{\xi}$ axis is an extension of the geocentric radius vector. The $\underline{\eta}$ axis is both normal to the $\underline{\xi}$ axis and positive in the same general direction as the inertial velocity vector $\underline{v}$. All lie in the instantaneous orbit plane. The $\underline{\zeta}$ axis is directed out of the page, or normal to the orbit plane, thus forming a right-handed orthogonal system.

The position and velocity of an alternate point P_1 relative to the point P are then given by

$$P_1 = P_1(\xi, \eta, \zeta, \dot{\xi}, \dot{\eta}, \dot{\zeta})$$

3.1.4 Station-Centered Inertial Coordinate System

The station-centered inertial coordinate system used for partials computation is shown in Figure 3-6,

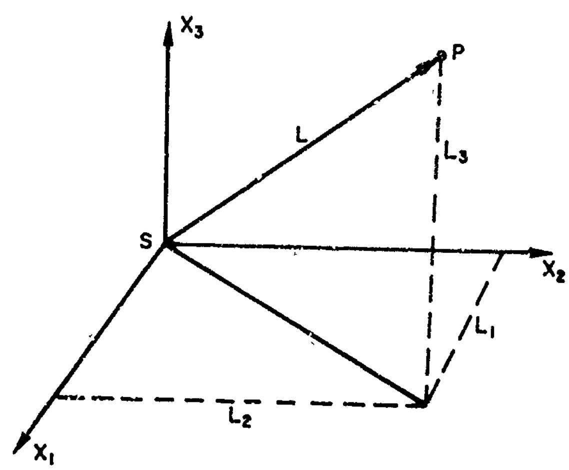


Figure 3-6. Station-Centered Inertial Coordinate System

where

$$S = \text{station location at some time } t = (x_s, y_s, z_s)$$

$$\alpha = \ell + \alpha_g + \omega_e(t - t_g)$$

$$\left.\begin{aligned} x_s &= w_1^s \cos \alpha \\ y_s &= w_1^s \sin \alpha \\ z_s &= w_3^s \end{aligned}\right\} = \text{station position at time } t \text{ in ECI system (see Section 3.4.1 for definition of } w_1^s \text{ and } w_3^s\text{)}$$

$$L_1 = x - x_s$$

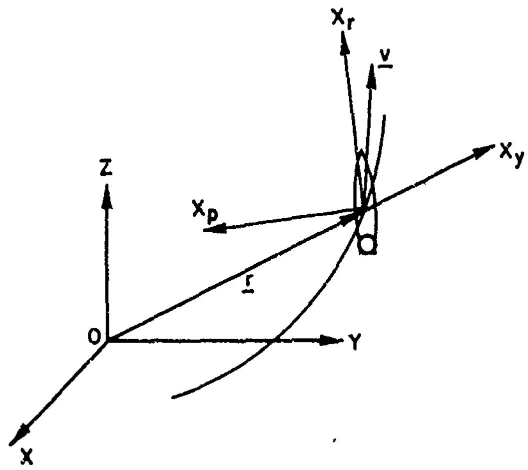
$$L_2 = y - y_s$$

$$L_3 = z - z_s$$

and where vectors X_1, X_2, and X_3 are parallel to axes X, Y, Z, respectively.

3.1.5 Vehicle Coordinate System

A coordinate system convenient for computing vehicle-fixed effects is shown in Figure 3-7,



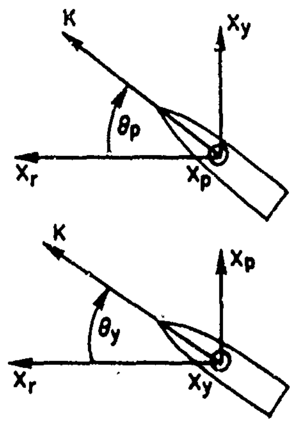


Figure 3-7. Vehicle Coordinate System

where

X = vehicle position vector in basic ECI system

$\dot{X}$ = vehicle velocity vector in basic ECI system

$$\left.\begin{aligned} X_y &= X/|X| = \text{yaw axis} \\ X_p &= \frac{X \times \dot{X}}{|X \times \dot{X}|} = \text{pitch axis} \\ X_r &= X_p \times X_y = \text{roll axis} \end{aligned}\right\} \quad \text{unit vectors}$$

K = magnitude of instantaneous velocity adjustment (kick) vector

θ_p = direction of deflection for positive pitch-angle adjustment

θ_y = direction of deflection for positive yaw-angle adjustment

X and $\dot{X}$ are identical to the usual symbols $\underline{r}$ and $\underline{v}$, and the vectors X_y, X_p, X_r are in the radial, in-track, and cross-track directions, respectively.

3.2 INITIAL CONDITIONS AND COORDINATE TRANSFORMATIONS

The parameters of an orbit may be given within any of the coordinate categories described in Section 3.1.1. The primary trajectory computations are accomplished in earth-centered rectangular coordinates. Supplemental outputs include spherical coordinates and orbital elements. Formulas for necessary transformations are given in Sections 3.2.1 through 3.2.4, wherein the date chosen to determine the X-axis is t_g (0 hour GMT of epoch date). The time t_o at which the parameters are specified is referenced to t_g, i.e., the program operates with time in units of minutes from t_g.

3.2.1 Spherical to Rectangular Coordinates

$$x = r \cos \delta \cos \alpha$$

$$y = r \cos \delta \sin \alpha$$

$$z = r \sin \delta$$

$$\dot{x} = v[\cos \alpha(-\cos A \sin \beta \sin \delta + \cos \beta \cos \delta) - \sin A \sin \beta \sin \alpha]$$

$$\dot{y} = v[\sin \alpha(-\cos A \sin \beta \sin \delta + \cos \beta \cos \delta) + \sin A \sin \beta \cos \alpha]$$

$$\dot{z} = v[\cos A \cos \delta \sin \beta + \cos \beta \sin \delta]$$

If longitude (ℓ) is input instead of α in the foregoing expressions, α is computed as in Section 3.1.2. In this case, $t - t_g = t_o$.

3.2.2 Rectangular to Spherical Coordinates

$$\alpha = \tan^{-1}(y/x)$$

$$\delta = \tan^{-1}\left(z/\sqrt{x^2 + y^2}\right)$$

$$\beta = \cos^{-1}[(x\dot{x} + y\dot{y} + z\dot{z})/rv]$$

$$A = \tan^{-1}\left[\frac{r(x\dot{y} - y\dot{x})}{y(y\dot{z} - z\dot{y}) - x(z\dot{x} - x\dot{z})}\right]$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

3.2.3 Orbital Elements to Rectangular Coördinates

$$x = x_\omega P_x + y_\omega Q_x$$

$$y = x_\omega P_y + y_\omega Q_y$$

$$z = x_\omega P_z + y_\omega Q_z$$

$$\dot{x} = \dot{x}_\omega P_x + \dot{y}_\omega Q_x, \text{ etc}$$

where, if the conic is an ellipse,

$$P_x = \cos\Omega \cos\omega - \sin\Omega \sin\omega \cos i$$

$$P_y = \sin\Omega \cos\omega + \cos\Omega \sin\omega \cos i$$

$$P_z = \sin\omega \sin i$$

$$Q_x = -\cos\Omega \sin\omega - \sin\Omega \cos\omega \cos i$$

$$Q_y = -\sin\Omega \sin\omega + \cos\Omega \cos\omega \cos i$$

$$Q_z = \cos\omega \sin i$$

$p = a(1 - e^2)$ = semi-latus rectum

μ = gravitational constant

$n = \sqrt{|\mu/a^3|}$ = mean motion

$M = n(t - \tau)$ = mean anomaly

E = solution of $(M = E - e \sin E)$ = eccentric anomaly

$$r_\omega = a(1 - e\cos E)$$

$$x_\omega = a(\cos E - e)$$

$$y_\omega = \sqrt{|ap|}\sin E$$

$$\dot{x}_\omega = -\frac{\sqrt{|\mu a|}}{r_\omega}\sin E$$

$$\dot{y}_\omega = \frac{\sqrt{\mu p}}{r_\omega}\cos E$$

If the orbital conic is a hyperbola ($e > 1$), E is the solution of $M = e \sinh E - E$. Also, sin E and cos E above must be replaced by sinh E and cosh E.

3.2.4 Rectangular Coordinates to Orbital Elements

$$a = \left(\frac{2}{r} - \frac{v^2}{\mu}\right)^{-1}$$

$$e = \sqrt{(e \cos E)^2 + (e \sin E)^2} \qquad \text{(for elliptic orbits)}$$

$$e = \sqrt{(e \cosh E)^2 - (e \sinh E)^2} \qquad \text{(for hyperbolic orbits)}$$

$$i = \tan^{-1}\left(\frac{\sqrt{P_z^2 + Q_z^2}}{P_x Q_y - P_y Q_x}\right)$$

$$\Omega = \tan^{-1}\left(\frac{P_y Q_z - P_z Q_y}{P_x Q_z - P_z Q_x}\right)$$

$$\omega = \tan^{-1}\left(\frac{P_z}{Q_z}\right)$$

$$\tau = t - \frac{M}{n}$$

where

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$$

$$\left.\begin{aligned} e \cos E &= 1 - \frac{r}{a} \\ e \sin E &= \frac{x\dot{x} + y\dot{y} + z\dot{z}}{\sqrt{|a|\mu}} \end{aligned}\right\} \qquad \text{(for elliptic orbits)}$$

$$\left.\begin{aligned} e \cosh E &= 1 - \frac{r}{a} \\ e \sinh E &= \frac{x\dot{x} + y\dot{y} + z\dot{z}}{\sqrt{|a|\mu}} \end{aligned}\right\} \quad \text{(for hyperbolic orbits)}$$

$$p = \frac{r^2 v^2 - (x\dot{x} + y\dot{y} + z\dot{z})^2}{\mu}$$

$$D = \frac{x\dot{x} + y\dot{y} + z\dot{z}}{e\mu}$$

$$\dot{D} = \frac{e \cos E}{er}$$

$$H = \frac{1}{e\sqrt{\mu p}}(r - p)$$

$$\dot{H} = \frac{1}{e\sqrt{\mu p}} \; \frac{x\dot{x} + y\dot{y} + z\dot{z}}{r}$$

$$P_x = \dot{D}x - D\dot{x}$$

$$P_y = \dot{D}y - D\dot{y}$$

$$P_z = \dot{D}z - D\dot{z}$$

$$Q_x = \dot{H}x - H\dot{x}, \text{ etc}$$

$$n = \sqrt{\left|\frac{\mu}{a^3}\right|}$$

$$M = E - e \sin E \quad \text{(for eliiptic orbits)}$$

$$M = e \sinh E - E \text{ (for hyperbolic orbits)}$$

$$E = \tan^{-1} \frac{e \sin E}{e \cos E}$$

3.3 OBSERVATION DATA

Although in previous TRACE program models all observations were transformed to R, A, E, and $\dot{R}$, all observations used in the TRACE-D orbit-determination process are retained in the same coordinate system in which they are input. In general, input is in units of feet, seconds, and degrees, and computations are carried out in earth radii, minutes, and radians. The details of units conversion have been omitted from this section.

It should be noted that the coordinate systems shown in Section 3.3 also are pertinent to the discussion of partials presented in Section 3.4.

3.3.1 Station Coordinates

Station coordinate system relationships are shown in Figure 3-8,

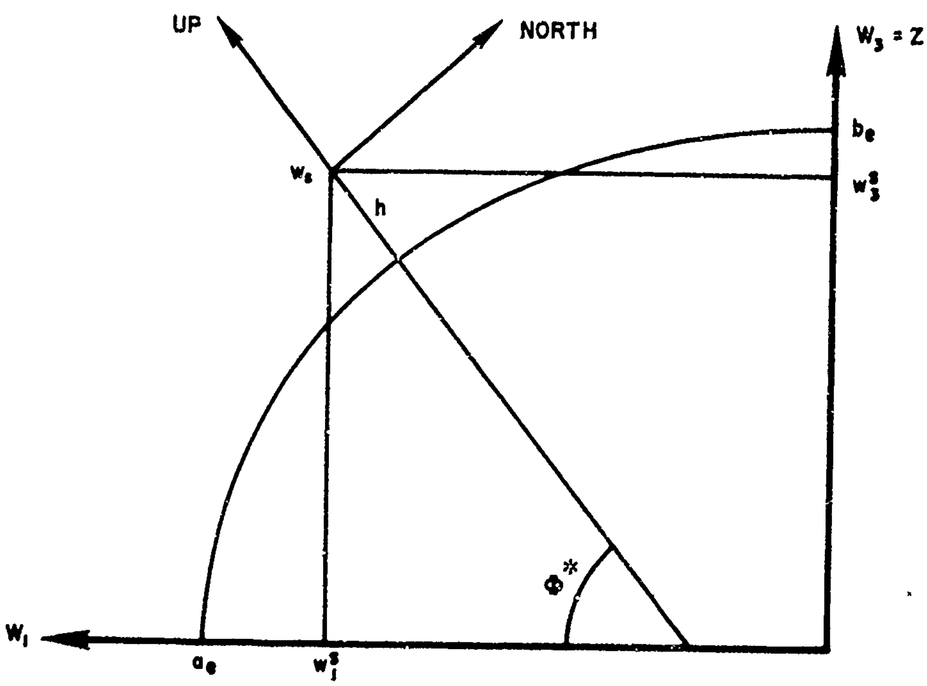


Figure 3-8. Station Coordinates

where

Φ^* = geodetic latitude

a_e = semi-major axis of the earth (equatorial radius)

b_e = semi-minor axis of the earth (polar radius)

h = station altitude with respect to reference ellipsoid

W^s = station position in W-coordinate system = $(w_1^s, 0, w_3^s)$

3.3.2 Azimuth and Elevation Angles

Azimuth and elevation angle coordinate system relationships are shown in Figure 3-9,

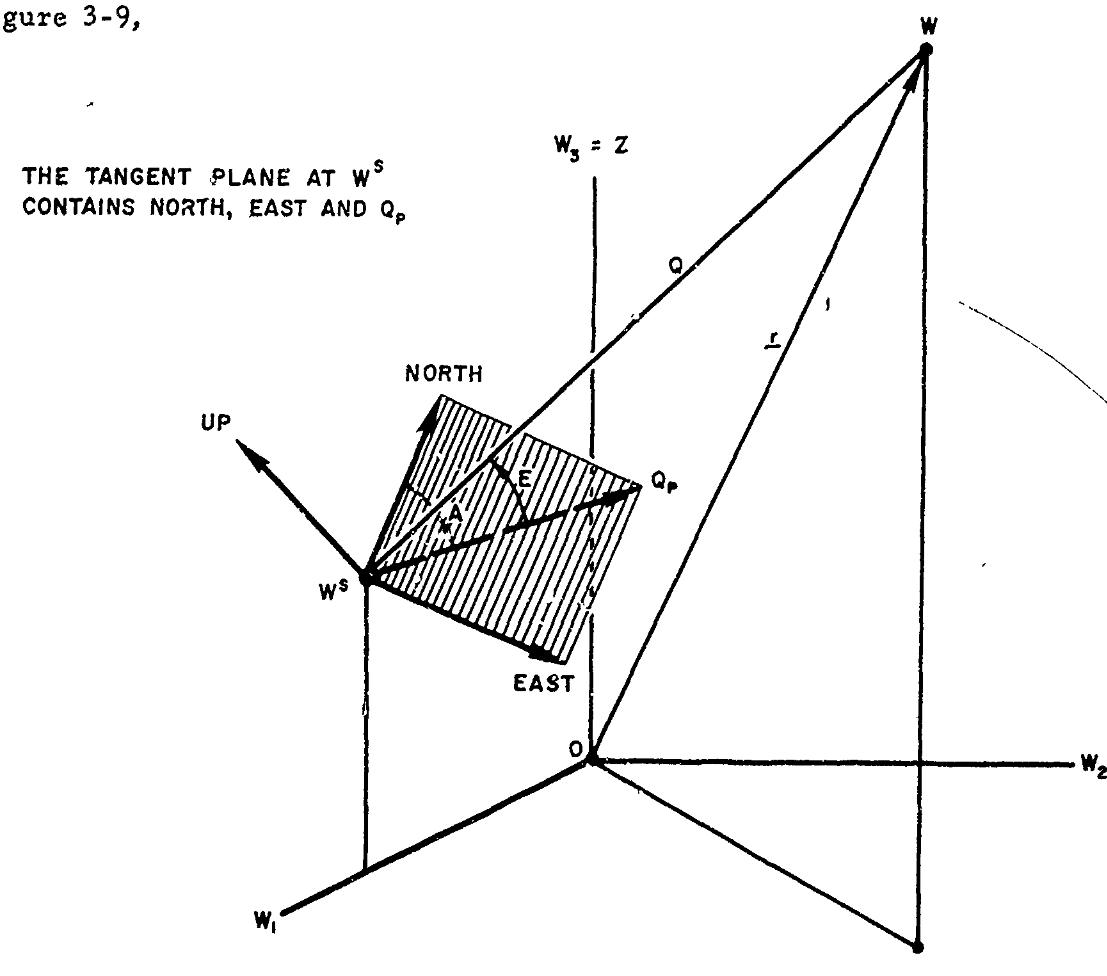


Figure 3-9. Azimuth and Elevation in Station Coordinate System

where

W = vehicle position

W^S = station position

Q_p = projection of $Q = W - W^S$ onto tangent plane at W^S

$R = |Q|$ = slant range

$\underline{r}$ = geocentric radius vector

3.3.3 Topocentric Angular Measurement

Coordinate system relationships of topocentric angular measurements are shown in Figure 3-10,

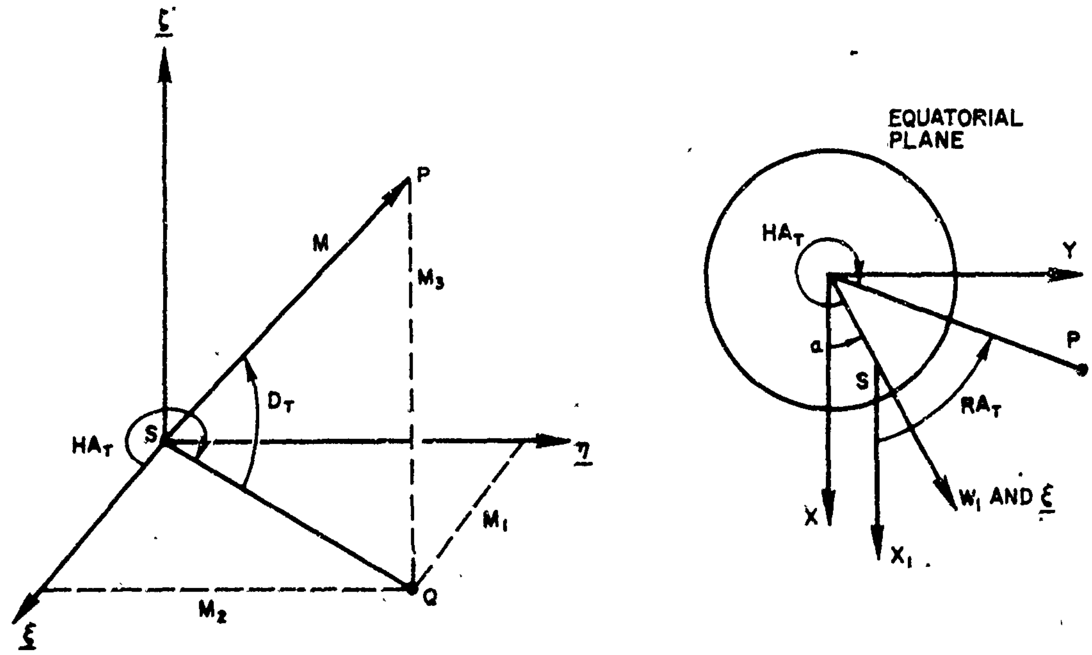


Figure 3-10. Right Ascension, Declination, and Hour Angle in Station-Centered Coordinate System

where

$\underline{\xi}$, $\underline{\eta}$, $\underline{\zeta}$, are parallel to W_1, W_2, W_3, respectively

SQ = projection of M onto ξ, η plane

HA_T = topocentric hour angle (measured westward in the ξ, η plane)

RA_T = topocentric right ascension = $\alpha - HA_T$

D_T = topocentric declination

$R = |M|$

α = right ascension of station

3.3.4 Interferometer Data

In Figure 3-11, S, S_P, and S_Q are a network of stations reporting range and

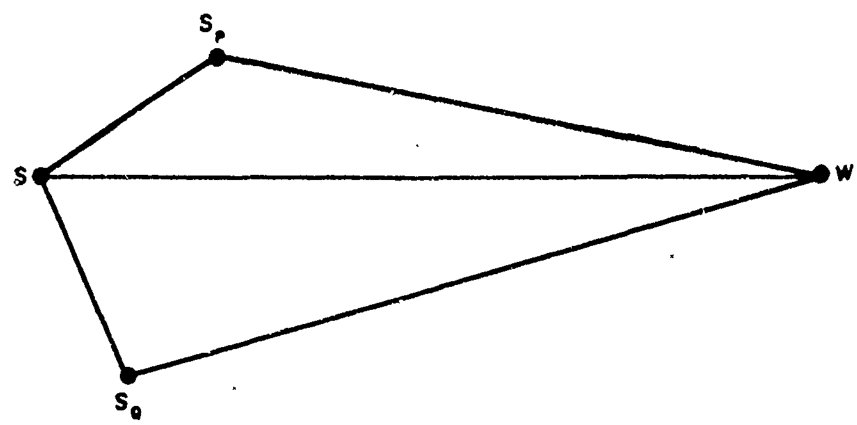


Figure 3-11. Station Network for Interferometer Data

range-rate differences. Letting $R = |W - S|$, $R_P = |W - S_P|$, and $R_Q = |W - S_Q|$, then

$$P = R - R_P$$
$$Q = R - R_Q$$
$$\dot{P} = \dot{R} - \dot{R}_P$$
$$\dot{Q} = \dot{R} - \dot{R}_Q$$

3.3.5 Horizon-Sensor Angles

Horizon-sensor angles related to an earth-centered inertial coordinate system are shown in Figure 3-12,

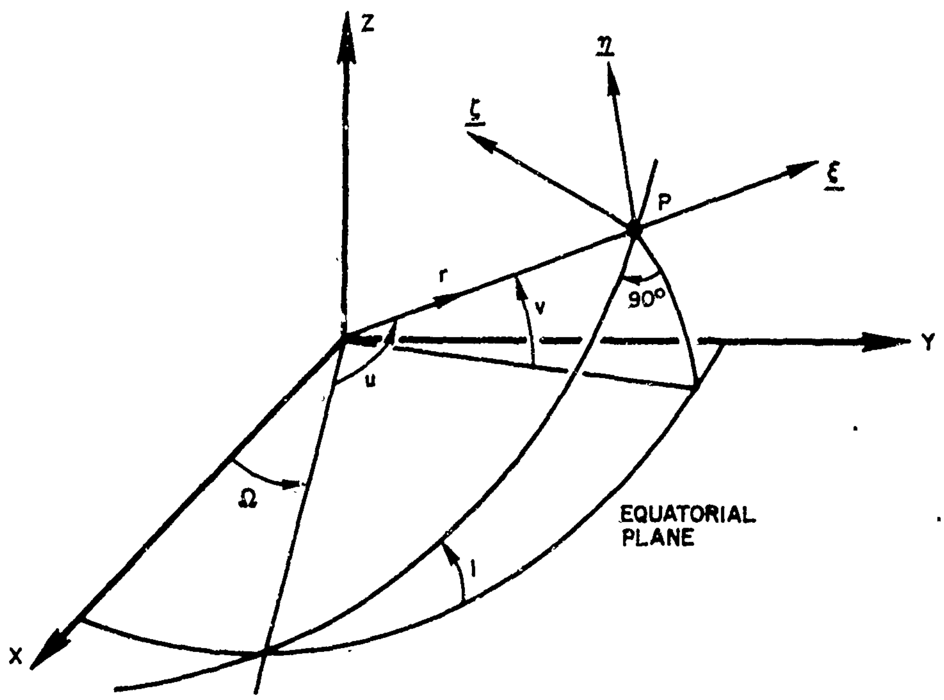


Figure 3-12. Horizon-Sensor Angles u, v in Earth-Centered Inertial Coordinate System

where, assuming $\underline{\zeta}$, $\underline{\eta}$, and $\underline{\xi}$ are unit vectors,

$$N_1 = N_2 / |N_2|$$

$$N_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \underline{\zeta}$$

$$u = \sin^{-1} |N_1 \times \underline{\xi}|$$

$$u = \cos^{-1} (N_1 \cdot \underline{\xi})$$

$$v = \tan^{-1} (\sin u \tan i)$$

3.3.6 Other Observation Measurements

3.3.6.1 Geocentric Right Ascension and Declination

Observed values of these angles, which can be input to the orbit-determination process, are identical to α and δ as previously defined in Section 3.1.1.2.

3.3.6.2 Height Measurements

Height observations are assumed to be measured along the geocentric radius vector above the oblate earth (see Section 3.5.9 for specific equation).

3.3.6.3 Rectangular Geocentric Data

$\hat{x}$, $\hat{y}$, $\hat{z}$ data points are measured in a coordinate system with origin at the center of the earth, the $\hat{z}$ axis along the spin axis, and the $\hat{x}$ axis along the Greenwich meridian.

3.3.6.4 Range Rate and Doppler

Equations for $\dot{R}$ and Δf data are given in following Sections 3.4.5 and 3.4.7.

3.4 PARTIAL DERIVATIVES OF OBSERVATION DATA

In tracking and data studies it is necessary to compute partial derivatives of observation data with respect to the parameters of initial conditions, differential equations, station locations, observation biases, and observation scale factors. For the purposes of this section, it will be assumed that the integrated position of the vehicle in earth-centered inertial rectangular coordinates and the partial derivatives of these coordinates with respect to the first two types of parameters are known.

The following notation is applicable to the derivations presented in this section:

p_i $(i = 1, 2, \ldots, n)$	=	the ordered list of initial conditions and differential equation parameters for which partials are to be computed
$\frac{\partial x}{\partial p_i}, \frac{\partial y}{\partial p_i}, \ldots, \frac{\partial \dot{z}}{\partial p_i}$	=	partial derivatives of $x, y, \ldots, \dot{z}$ with respect to the p_i
ℓ	=	longitude of station
Φ^*	=	geodetic latitude of station
h	=	height of station
α	=	$\alpha_g + \omega_e(t - t_g) + \ell$ (Ref. Sec. 3.1.1.2)
$w_j, \dot{w}_j$ $(j = 1, 2, 3)$	=	position and velocity of vehicle in the station-dependent W system
w_1^s, w_3^s	=	position of station in station-dependent W system
ϵ	=	ellipticity of reference ellipsoid
a_e	=	semi-major axis of the earth
$b_e = a_e(1 - \epsilon)$	=	semi-minor axis of the earth
K_R	=	range scale factor
K_D	=	range-rate scale factor

3.4.1 Position and Velocity in the W System and Associated Preliminary Computations

$$w_1 = x \cos \alpha + y \sin \alpha$$

$$w_2 = -x \sin \alpha + y \cos \alpha$$

$$w_3 = z$$

$$\dot{w}_1 = (\dot{x} + \omega_e y) \cos \alpha + (\dot{y} - \omega_e x) \sin \alpha$$

$$\dot{w}_2 = -(\dot{x} + \omega_e y) \sin \alpha + (\dot{y} - \omega_e x) \cos \alpha$$

$$\dot{w}_3 = \dot{z}$$

The following derivation develops quantities that are necessary for transforming the partial derivatives of the earth-centered inertial (ECI) rectangular coordinates to the station-dependent W system. By differentiating the foregoing six expressions it becomes apparent that a simple substitution of $\partial w_j / \partial p_i$ for w_j and $\partial \dot{w}_j / \partial p_i$ for $\dot{w}_j$, where $j = 1, 2, 3$, and of $\partial x / \partial p_i$, $\partial y / \partial p_i$, . . . , $\partial \dot{z} / \partial p_i$ for x, y, . . . , $\dot{z}$ yields

$$\frac{\partial w_1}{\partial p_i} = \frac{\partial x}{\partial p_i} \cos \alpha + \frac{\partial y}{\partial p_i} \sin \alpha$$

$$\frac{\partial w_2}{\partial p_i} = \frac{-\partial x}{\partial p_i} \sin \alpha + \frac{\partial y}{\partial p_i} \cos \alpha$$

etc . . . .

Also, differentiating with respect to ℓ, since $\partial\alpha/\partial\ell = 1$,

$$\frac{\partial w_1}{\partial \ell} = -x \sin \alpha + y \cos \alpha = w_2$$

$$\frac{\partial w_2}{\partial \ell} = -x \cos \alpha - y \sin a = -w_1$$

$$\frac{\partial \dot{w}_1}{\partial \ell} = \dot{w}_2$$

$$\frac{\partial \dot{w}_2}{\partial \ell} = -\dot{w}_1$$

To find the station position in the W system, the expressions

$$w_1^s = (a_e A_s + h) \cos \Phi^*$$

$$w_3^s = (b_e B_s + h) \sin \Phi^*$$

are used, where

$$A_s = \left(\cos^2 \Phi^* + \frac{b_e^2}{a_e^2} \sin^2 \Phi^* \right)^{-1/2}$$

$$B_s = \left(\sin^2 \Phi^* + \frac{a_e^2}{b_e^2} \cos^2 \Phi^* \right)^{-1/2}$$

Differentiating the expressions for w_1^s and w_3^s with respect to Φ^* and h, then,

$$\frac{\partial w_1^s}{\partial \Phi^*} = -w_3^s - a_e(2\epsilon - \epsilon^2)A_s B_s^2 \sin^3 \Phi^*$$

$$\frac{\partial w_3^s}{\partial \Phi^*} = w_1^s - a_e(2\epsilon - \epsilon^2)A_s^3 \cos^3 \Phi^*$$

$$\frac{\partial w_1^s}{\partial h} = \cos \Phi^*$$

$$\frac{\partial w_3^s}{\partial h} = \sin \Phi^*$$

It is convenient to introduce the three intermediate vectors Q, U, and V and the quantity R_1, where

$Q = W - W^s$ = vehicle position relative to the station

with

$$q_1 = w_1 - w_1^s$$
$$q_2 = w_2$$
$$q_3 = w_3 - w_3^s$$

$U = Q/R$ = a unit vector in the direction of Q, wherein $R = |Q| = \sqrt{q_1^2 + q_2^2 + q_3^2}$,

with

$$u_1 = q_1/R$$
$$u_2 = q_2/R$$
$$u_3 = q_3/R$$

V = vector U referred to the East/North/Up system

with

$$v_1 = u_2$$

$$v_2 = -u_1 \sin \Phi^* + u_3 \cos \Phi^*$$

$$v_3 = u_1 \cos \Phi^* + u_3 \sin \Phi^*$$

$$R_1 = vR$$

with

$$v = \sqrt{v_1^2 + v_2^2}$$

Then,

$$v_3 = \sin E$$

$$v = \cos E$$

$$\frac{v_2}{v} = \cos A$$

$$\frac{v_1}{v} = \sin A$$

The expression for $\dot{v}$ that is needed to determine $\partial A/\partial t$ and $\partial E/\partial t$ may be obtained by computing $\dot{U} = 1/R[\dot{W} - U\dot{\lambda}] = 1/R[\dot{W} - (U \cdot \dot{W})U]$ whereby

$$\dot{v}_1 = \dot{u}_2$$

$$\dot{v}_2 = -\dot{u}_1 \sin \Phi^* + \dot{u}_3 \cos \Phi^*$$

$$\dot{v} = \frac{v_1 \dot{v}_1 + v_2 \dot{v}_2}{v}$$

3.4.2 Range Partials

Differentiating $R = \sqrt{q_1^2 + q_2^2 + q_3^2}$ results in

$$\frac{\partial R}{\partial p_i} = U \cdot \frac{\partial Q}{\partial p_i} = u_1 \frac{\partial q_1}{\partial p_i} + u_2 \frac{\partial q_2}{\partial p_i} + u_3 \frac{\partial q_3}{\partial p_i} \cdot$$

$$\frac{\partial R}{\partial \Phi^*} = u_1 w_3^s - u_3 w_1^s$$

$$\frac{\partial R}{\partial \ell} = u_1 w_2 - u_2 w_1$$

$$\frac{\partial R}{\partial h} = -u_1 \cos \Phi^* - u_3 \sin \Phi^*$$

$$\frac{\partial R}{\partial R_{bias}} = 1 .$$

$$\frac{\partial R}{\partial K_R} = R$$

$$\frac{\partial R}{\partial t} = \dot{R} = (U \cdot \dot{W})$$

3.4.3 Azimuth Partials

Differentiating $A = \tan^{-1}(v_1/v_2)$.

$$\frac{\partial A}{\partial p_i} = \frac{1}{R_1} \left[\frac{\partial w_2}{\partial p_i} \cos A - \left(- \frac{\partial w_1}{\partial p_i} \sin \Phi^* + \frac{\partial w_3}{\partial p_i} \cos \Phi^* \right) \sin A \right]$$

$$\frac{\partial A}{\partial \Phi^*} = \frac{\sin A}{R_1}(w_1 \cos \Phi^* + w_3 \sin \Phi^*)$$

$$\frac{\partial A}{\partial \ell} = \frac{-w_1 \cos A + w_2 \sin \Phi^* \sin A}{R_1}$$

$$\frac{\partial A}{\partial h} = 0$$

$$\frac{\partial A}{\partial A_{bias}} = 1$$

$$\frac{\partial A}{\partial t} = \frac{1}{v^2}(v_2 \dot{v}_1 - v_1 \dot{v}_2)$$

3.4.4 Elevation Partials

Differentiating $E = \sin^{-1} v_3 = \cos^{-1} v$,

$$\frac{\partial E}{\partial p_i} = \frac{1}{R_1}\left(\frac{\partial w_1}{\partial p_i} \cos \Phi^* + \frac{\partial w_3}{\partial p_i} \sin \Phi^* - \frac{\partial R}{\partial p_i} \sin E\right)$$

$$\frac{\partial E}{\partial \Phi^*} = \frac{1}{R_1}\left(w_3 \cos \Phi^* - w_1 \sin \Phi^* - \frac{\partial R}{\partial \Phi^*} \sin E\right)$$

$$\frac{\partial E}{\partial \ell} = \frac{1}{R_1}\left(w_2 \cos \Phi^* - \frac{\partial R}{\partial \ell} \sin E\right)$$

$$\frac{\partial E}{\partial h} = \frac{-1}{R_1}\left(1 + \frac{\partial R}{\partial h} \sin E\right)$$

$$\frac{\partial E}{\partial E_{bias}} = 1$$

$$\frac{\partial E}{\partial t} = \frac{\dot{u}_1 \cos \Phi^* + \dot{u}_3 \sin \Phi^*}{\cos E}$$

3.4.5 Range Rate Partials

Differentiating $\dot{R} = (U \cdot \dot{W})$,

$$\frac{\partial \dot{R}}{\partial p_i} = \left(\frac{\partial W}{\partial p_i} \cdot \dot{U}\right) + \left(U \cdot \frac{\partial \dot{W}}{\partial p_i}\right)$$

$$\frac{\partial \dot{R}}{\partial \Phi^*} = w_3^s \dot{u}_1 - w_1^s \dot{u}_3$$

$$\frac{\partial \dot{R}}{\partial \ell} = (w_2 \dot{u}_1 - w_1 \dot{u}_2) + (\dot{w}_2 u_1 - \dot{w}_1 u_2)$$

$$\frac{\partial \dot{R}}{\partial h} = - \dot{u}_1 \cos \Phi^* - \dot{u}_3 \sin \Phi^*$$

$$\frac{\partial \dot{R}}{\partial \dot{R}_{bias}} = 1$$

$$\frac{\partial \dot{R}}{\partial K_D} = \dot{R}$$

$$\frac{\partial \dot{R}}{\partial t} = \ddot{R} = (\dot{U} \cdot \dot{W}) + (U \cdot \ddot{W})$$

where

$$\ddot{W} = -\left(\omega_e^2 J + \frac{\mu}{|X|^3} I\right) W + 2L\dot{X}$$

$$J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$L\dot{X} = \begin{pmatrix} -\omega_e \dot{x} \sin \alpha + \omega_e \dot{y} \cos \alpha \\ -\omega_e \dot{x} \cos \alpha - \omega_e \dot{y} \sin \alpha \\ 0 \end{pmatrix}$$

$$|X| = \left(\sum_{i=1}^{3} w_i^2\right)^{1/2}$$

3.4.6 P, Q, $\dot{P}$, $\dot{Q}$, Partials

These partial derivatives are obtained by differencing the R, $\dot{R}$ partials, using the appropriate station locations.

3.4.7 Doppler (Δf) Partials

Differentiating $\Delta f = -K_D \dot{R}(1 - \dot{R}/c)$ with respect to $\dot{R}$,

$$\frac{\partial(\Delta f)}{\partial \dot{R}} = -K_D\left(1 - \frac{2\dot{R}}{c}\right)$$

Then,

$$\frac{\partial(\Delta f)}{\partial p_i} = \frac{\partial(\Delta f)}{\partial \dot{R}} \cdot \frac{\partial \dot{R}}{\partial p_i}$$

$$\frac{\partial(\Delta f)}{\partial \Phi^*} = \frac{\partial(\Delta f)}{\partial \dot{R}} \cdot \frac{\partial \dot{R}}{\partial \Phi^*}$$

$$\frac{\partial(\Delta f)}{\partial \ell} = \frac{\partial(\Delta f)}{\partial \dot{R}} \cdot \frac{\partial \dot{R}}{\partial \ell}$$

$$\frac{\partial(\Delta f)}{\partial h} = \frac{\partial(\Delta f)}{\partial \dot{R}} \cdot \frac{\partial \dot{R}}{\partial h}$$

$$\frac{\partial(\Delta f)}{\partial(\Delta f)_{bias}} = 1$$

$$\frac{\partial(\Delta f)}{\partial t} = \frac{\partial(\Delta f)}{\partial \dot{R}} \cdot \frac{\partial \dot{R}}{\partial t}$$

$$\frac{\partial(\Delta f)}{\partial K_D} = -\dot{R}\left(1 - \frac{\dot{R}}{c}\right)$$

3.4.8 Topocentric Right Ascension Partials

Differentiating $RA_T = \tan^{-1} L_2/L_1 = \tan^{-1}(y - y_s)/(x - x_s)$,

$$\frac{\partial RA_T}{\partial p_i} = \frac{\partial RA_T}{\partial x} \frac{\partial x}{\partial p_i} + \frac{\partial RA_T}{\partial y} \frac{\partial y}{\partial p_i}$$

Also,

$$\frac{\partial RA_T}{\partial x} = \frac{-L_2}{L_1^2 + L_2^2}$$

$$\frac{\partial RA_T}{\partial y} = -\frac{L_1}{L_2}\frac{\partial RA_T}{\partial x}$$

$$\frac{\partial RA_T}{\partial \Phi^*} = \frac{(y \cos a - x \sin a)\dfrac{\partial W_1^s}{\partial \Phi^*}}{L_1^2 + L_2^2}$$

$$\frac{\partial RA_T}{\partial \ell} = \frac{-(x_s L_1 + y_s L_2)}{L_1^2 + L_2^2}$$

$$\frac{\partial RA_T}{\partial h} = \frac{(y \cos a - x \sin a)\cos \Phi^*}{L_1^2 + L_2^2}$$

$$\frac{\partial RA_T}{\partial RA_{T(bias)}} = 1$$

$$\frac{\partial RA_T}{\partial t} = \frac{L_1(\dot{y} - \omega_e x_s) - L_2(\dot{x} + \omega_e y_s)}{L_1^2 + L_2^2}$$

3.4.9 Topocentric Declination Partials

Differentiating $D_T = \sin^{-1} L_3/R = \sin^{-1}(z - z_s)/\left[(x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2\right]^{1/2}$,

$$\frac{\partial D_T}{\partial p_i} = \frac{\partial D_T}{\partial x}\frac{\partial x}{\partial p_i} + \frac{\partial D_T}{\partial y}\frac{\partial y}{\partial p_i} + \frac{\partial D_T}{\partial p_i} + \frac{\partial D_T}{\partial z}\frac{\partial z}{\partial p_i}$$

Also,

$$\frac{\partial D_T}{\partial x} = \frac{-L_1 L_3}{R^2\left(L_1^2 + L_2^2\right)^{1/2}}$$

$$\frac{\partial D_T}{\partial y} = \frac{L_2}{L_1}\frac{\partial D_T}{\partial x}$$

$$\frac{\partial D_T}{\partial z} = \frac{\left(L_1^2 + L_2^2\right)^{1/2}}{R^2}$$

$$\frac{\partial D_T}{\partial \Phi^*} = \frac{\left(w_1 - w_1^s\right)L_3 \frac{\partial w_1^s}{\partial \Phi^*} + \left(L_1^2 + L_2^2\right)\frac{\partial w_3^s}{\partial \Phi^*}}{R^2\left(L_1^2 + L_2^2\right)^{1/2}}$$

$$\frac{\partial D_T}{\partial \ell} = \frac{L_3\left(x_s y - y_s x\right)}{R^2\left(L_1^2 + L_2^2\right)^{1/2}}$$

$$\frac{\partial D_T}{\partial h} = \frac{1}{R^2\left(L_1^2 + L_2^2\right)^{1/2}}\left\{-R^2 \sin \Phi^* + L_3\left[\left(w_1 - w_1^s\right)\cos \Phi^* + \left(w_3 - w_3^s\right)\sin \Phi^*\right]\right\}$$

$$\frac{\partial D_T}{\partial D_{T(\mathrm{bias})}} = 1$$

$$\frac{\partial D_T}{\partial t} = \frac{-L_3[L_1\dot{x} + L_2\dot{y} + (xy_x - yx_s)\omega_e] + \dot{z}\left(L_1^2 + L_2^2\right)}{R^2\left(L_1^2 + L_2^2\right)^{1/2}}$$

3.4.10 Topocentric Hour Angle Partials

Differentiating $HA_T = \alpha - RA_T$,

$$\frac{\partial HA_T}{\partial p_i} = -\frac{\partial RA_T}{\partial p_i}$$

Also,

$$\frac{\partial HA_T}{\partial \Phi^*} = -\frac{\partial RA_T}{\partial p_i}$$

$$\frac{\partial HA_T}{\partial \ell} = 1 - \frac{\partial RA_T}{\partial \ell}$$

$$\frac{\partial HA_T}{\partial h} = -\frac{\partial RA_T}{\partial h}$$

$$\frac{\partial HA_T}{\partial HA_{T(\mathrm{bias})}} = 1$$

$$\frac{\partial HA_T}{\partial t} = \omega_e - \frac{\partial RA_T}{\partial t}$$

3.4.11 Geocentric Right Ascension Partials

Differentiating $\alpha = \tan^{-1}(y/x)$,

$$\frac{\partial \alpha}{\partial p_i} = \frac{\partial \alpha}{\partial x}\,\frac{\partial x}{\partial p_i} + \frac{\partial \alpha}{\partial y}\,\frac{\partial y}{\partial p_i}$$

Also,

$$\frac{\partial a}{\partial x} = \frac{-y}{x^2 + y^2}$$

$$\frac{\partial \alpha}{\partial y} = \frac{x}{x^2 + y^2}$$

$$\frac{\partial \alpha}{\partial \alpha_{(bias)}} = 1$$

$$\frac{\partial \alpha}{\partial t} = \frac{x\dot{y} - y\dot{x}}{x^2 + y^2}$$

3.4.12 Geocentric Declination Partials

Differentiating $\delta = \tan^{-1}\left(z/\sqrt{x^2 + y^2}\right)$,

$$\frac{\partial \delta}{\partial p_i} = \frac{\partial \delta}{\partial x}\,\frac{\partial x}{\partial p_i} + \frac{\partial \delta}{\partial y}\,\frac{\partial y}{\partial p_i} + \frac{\partial \delta}{\partial z}\,\frac{\partial z}{\partial p_i}$$

where

$$\frac{\partial \delta}{\partial x} = \frac{-xz}{(x^2 + y^2)^{1/2}(x^2 + y^2 + z^2)}$$

$$\frac{\partial \delta}{\partial y} = \frac{-yz}{(x^2 + y^2)^{1/2}(x^2 + y^2 + z^2)}$$

$$\frac{\partial \delta}{\partial z} = \frac{(x^2 + y^2)^{1/2}}{x^2 + y^2 + z^2}$$

Also,

$$\frac{\partial \delta}{\partial \delta_{(bias)}} = 1$$

$$\frac{\partial \delta}{\partial t} = \frac{z(x\dot{x} + y\dot{y}) + \dot{z}(x^2 + y^2)}{(x^2 + y^2)^{1/2}(x^2 + y^2 + z^2)}$$

3.4.13 Horizon Scanner Angle (u) Partials

Differentiating $u = \sin^{-1}(N_1 \times \xi)$

$$\frac{\partial u}{\partial p_i} = \frac{\partial u}{\partial X}\frac{\partial X}{\partial p_i}$$

Also,

$$\frac{\partial u}{\partial X} = \frac{\underline{\eta}^T}{R}$$

$$\frac{\partial u}{\partial u_{(bias)}} = 1$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial X}\dot{X}$$

3.4.14 Horizon Scanner Angle (v) Partials

Differentiating $v = \tan^{-1}(\sin u \tan i)$,

$$\frac{\partial v}{\partial p_i} = \frac{\partial v}{\partial X}\frac{\partial X}{\partial p_i}$$

Also,

$$\frac{\partial v}{\partial X} = \frac{\zeta^T}{R}$$

$$\frac{\partial v}{\partial v_{(bias)}} = 1$$

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial X}\dot{X}$$

3.4.15 Height (h) Partials

$$\frac{\partial h}{\partial p_i} = \frac{\partial h}{\partial X}\frac{\partial X}{\partial p_i}$$

Also,

$$\frac{\partial h}{\partial X} = \left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}, \frac{\partial h}{\partial z}\right)$$

where

$$\frac{\partial h}{\partial x} = \frac{x}{r}\left\{1 - \frac{a_e\,\epsilon(2 - 3\epsilon + \epsilon^2)\,z^2}{[r^2 - (2\epsilon - \epsilon^2)(x^2 + y^2)]^{3/2}}\right\}$$

$$\frac{\partial h}{\partial y} = \frac{y}{r}\left\{1 - \frac{a_e\ \epsilon(2 - 3\epsilon + \epsilon^2)\ z^2}{[r^2 - (2\epsilon - \epsilon^2)(x^2 + y^2)]^{3/2}}\right\}$$

$$\frac{\partial h}{\partial z} = \frac{z}{r}\left\{1 - \frac{a_e\ \epsilon(2 - 3\epsilon + \epsilon^2)(x^2 + y^2)}{[r^2 - (2\epsilon - \epsilon^2)(x^2 + y^2)]^{3/2}}\right\}$$

$$\frac{\partial h}{\partial h_{(bias)}} = 1$$

$$\frac{\partial h}{\partial t} = \frac{\partial h}{\partial X}\dot{X}$$

3.4.16 Cartesian Earth-Fixed ($\hat{x}$, $\hat{y}$, $\hat{z}$) Partials

Differentiating $\hat{x} = x \cos a + y \sin a$,

$$\frac{\partial \hat{x}}{\partial p_i} = \cos a \frac{\partial x}{\partial p_i} + \sin a \frac{\partial y}{\partial p_i}$$

$$\frac{\partial \hat{x}}{\partial t} = \dot{x} \cos a + \dot{y} \sin a + \omega_e(y \cos a - x \sin a)$$

Differentiating $\hat{y} = y \cos a - x \sin a$,

$$\frac{\partial \hat{y}}{\partial p_i} = - \sin a \frac{\partial x}{\partial p_i} + \cos a \frac{\partial y}{\partial p_i}$$

$$\frac{\partial \hat{y}}{\partial t} = - \dot{x} \sin a + \dot{y} \cos a - \omega_e(x \cos a + y \sin a)$$

Also, since $\hat{z} = z$,

$$\frac{\partial \hat{z}}{\partial p_i} = \frac{\partial z}{\partial p_i}$$

$$\frac{\partial \hat{z}}{\partial p_t} = \dot{z}$$

3.4.17 $\dot{A}$, $\dot{E}$ Partials

The TRACE-D program does not accept $\dot{A}$ and $\dot{E}$ as observations, but can calculate these quantities for the benefit of tracking stations and can compute the variance of the quantities $\dot{A}$, $\dot{E}$, $\dot{R}$, A, E, and R, given a covariance matrix for the trajectory parameters. The partial derivatives of $\dot{A}$ and $\dot{E}$, which are required for calculation of these variances, may be written as

$$\frac{\partial \dot{A}}{\partial p_i} = \frac{1}{Rv} \Bigg\{ \cos A \left[\frac{\partial \dot{w}_2}{\partial p_i} + \dot{A} \left(\frac{\partial w_1}{\partial p_i} \sin \Phi^* - \frac{\partial w_3}{\partial p_i} \cos \Phi^* \right) \right] \Bigg\}$$

$$+ \sin A \left[\left(\frac{\partial \dot{w}_1}{\partial p_i} \sin \Phi^* - \frac{\partial \dot{w}_3}{\partial p_i} \cos \Phi^* \right) - \dot{A} \frac{\partial w_2}{\partial p_i} \right] - (\dot{R}v + R\dot{v}) \frac{\partial A}{\partial p_i}$$

and

$$\frac{\partial \dot{E}}{\partial p_i} = \frac{1}{Rv} \left[\frac{\partial \dot{w}_1}{\partial p_i} \cos \Phi^* + \frac{\partial \dot{w}_3}{\partial p_i} \sin \Phi^* - \frac{\partial \dot{R}}{\partial p_i} \sin E - \dot{E} \frac{\partial R}{\partial p_i} \cos E \right.$$

$$\left. - (\dot{R}v + R\dot{v}) \frac{\partial E}{\partial p_i} \right]$$

3.5 DATA GENERATION CALCULATIONS

The formulas for computing data for the data generation function are a subset of the formulas presented in Sections 3.1 through 3.4, except as otherwise noted in Sections 3.5.1 through 3.5.12.

3.5.1 Rise-Set Prediction

The expression

$$\underline{r} \cdot R_s - rR_s \cos\left(\frac{\pi}{2} - E_m - \sin^{-1} \frac{R_s \cos E_m}{r}\right) = 0$$

where

$\underline{r}$ = vehicle position vector

R_s = station position vector

E = elevation angle

E_m = input minimum elevation or input maximum elevation, whichever applicable

holds when the elevation E is equal to E_m in a two-body model. This expression is positive when $E > E_m$ and negative when $E < E_m$. Preliminary values of rise-set times are generated by converting the above expression to a function of eccentric anomaly, θ, stepping from θ_o to $\theta_o + 2\pi$, and noting the times of the appropriate sign changes.

3.5.2 Rise and Set Times

The actual rise-set times may be computed from the integrated trajectory by use of the expressions

$$t_{(\text{rise or set})} = t_n + \Delta t$$

$$\Delta t = -\left[\frac{v_3 - \sin(E_m)}{\dot{u}_1 \cos \Phi^* + \dot{u}_3 \sin \Phi^*}\right]$$

where

t_n = current time

v_3, $\dot{u}_i$ = values as defined in Section 3.4.1

3.5.3 Elevation Angle Refraction Correction

The computed elevation angles (E) are corrected for atmospheric refraction effects by means of the expressions

$$E' = E + \eta_{si} \cot E \qquad (E \geq 0.1 \text{ radian})$$

$$E' = E + \frac{1}{1000} \cdot \frac{\eta_{si} \times 10^6}{12 + 1000E} - \frac{80}{6 + 1000E} \qquad (E < 0.1 \text{ radian})$$

where

E = geometric elevation angle

E' = elevation angle measured by radar (data generation output quantity)

η_{si} = appropriate refractivity index from REFR table (if $\eta_{si} = 0$, no refraction correction is applied)

3.5.4 Observations with Normally Distributed Random Noise

For observations accompanied by normally distributed random noise,

$$o = o_c + r_n$$

where

o = observation output on data generation run

o_c = nominal computed observation

r_n = added noise

and

$$r_n = n\sigma_{sj} + \beta_{sj}$$

where

σ_{sj} = appropriate sigma for Type j, Station s

β_{sj} = appropriate bias (if any)

n = a random element from a set of numbers with mean zero and unity standard deviation

3.5.5 Aspect Angles

Aspect angles Φ and θ as shown in Figure 3-13 are defined respectively as the angle between the negative yaw axis and the projection of the range vector

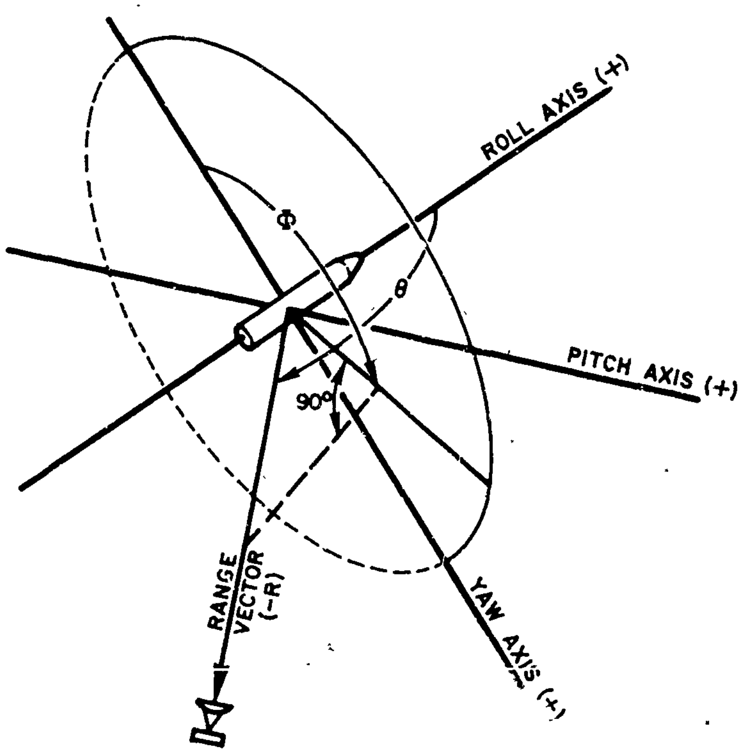


Figure 3-13. Aspect Angles with Respect to Vehicle-Centered Coordinates

in the roll plane, and as the angle between the positive roll axis and the range vector. The (+) roll direction is the in-track direction and the yaw axis is coincident with the geocentric radius vector.

To compute these angles, the vector from the vehicle to the station first is transformed from the basic coordinate system to the vehicle-centered system. It should be noted that the range vector is taken in the sense opposite to the convention adopted for the TRACE-D program for consistency with radar antenna pattern conventions (Ref. 7). Thus,

$$R_1 = A_{1,2}(-R)$$

where

$$A_{1,2} = \begin{bmatrix} -(\sin A \sin \alpha + \cos A \sin \delta \cos \alpha) & (\sin A \cos \alpha - \cos A \sin \delta \sin \alpha) & \cos A \cos \delta \\ -(\cos A \sin \alpha - \sin A \sin \delta \cos \alpha) & (\cos A \cos \alpha + \sin A \sin \delta \sin \alpha) & -\sin A \cos \delta \\ -\cos \delta \cos \alpha & -\cos \delta \sin \alpha & -\sin \delta \end{bmatrix}$$

and wherein A is the inertial azimuth angle, α is right ascension, and δ is geocentric declination of the vehicle.

The vector is then further transformed through the yaw, pitch, and roll angles which describe the instantaneous attitude of the vehicle, or

$$R_2 = A_{2,3}R_1$$

where

$$A_{2,3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_r & \sin\theta_r \\ 0 & -\sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} \cos\theta_p & 0 & -\sin\theta_p \\ 0 & 1 & 0 \\ \sin\theta_p & 0 & \cos\theta_p \end{bmatrix} \begin{bmatrix} \cos\theta_y & \sin\theta_y & 0 \\ -\sin\theta_y & \cos\theta_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and wherein θ_y, θ_p, θ_r are the instantaneous yaw, pitch, and roll angles as calculated from input attitude-control data.

If then

$$R_2 = \begin{bmatrix} R_{2x} \\ R_{2y} \\ R_{2z} \end{bmatrix}$$

the desired angles are given by

$$\cos\Phi = \frac{-R_{2z}}{\sqrt{R_{2y}^2 + R_{2z}^2}}$$

$$\sin\Phi = \frac{R_{2y}}{\sqrt{R_{2y}^2 + R_{2z}^2}}$$

$$\cos\theta = \frac{R_{2x}}{\sqrt{R_{2x}^2 + R_{2y}^2 + R_{2z}^2}}$$

3.5.6 Look Angle

The look-angle computation is similar to that for θ above, or

$$\cos \theta_L = \frac{R \cdot \underline{b}}{|R|}$$

where

R = station-to-vehicle range vector
(The components of $\underline{b}$ are the direction cosines of one of the missile axes in the basic coordinate system. The $\underline{b}$ components are completely arbitrary and independent of the aspect-angle computations.)

3.5.7 Observation Variances

The uncertainties in observations which are due to trajectory uncertainties that in turn result from uncertainties in the initial conditions and differential equation parameters can be computed by

$$C(O_t) = \left(\frac{\partial O_t}{\partial X_t}\right)\left(\frac{\partial X_t}{\partial P_o}\right) C(P_o)\left(\frac{\partial X_t}{\partial P_o}\right)^T\left(\frac{\partial O_t}{\partial X_t}\right)^T$$

where

$C(O_t)$ = covariance matrix of generated observations (diagonal)

$\frac{\partial O_t}{\partial X_t}$ = matrix of partial derivatives of observations with respect to trajectory position

$\frac{\partial X_t}{\partial P_o}$ = matrix of trajectory partial derivatives or solutions of variational equations

$C(P_o)$ = a priori covariance matrix of initial-condition and differential-equation parameters

Only variances (not covariances) are computed, and these only for the quantities R, A, E, $\dot{R}$, $\dot{A}$, $\dot{E}$.

3.5.8 Surface Range

The surface range equation is

$$R_{surf} = \cos^{-1}[\sin \delta \sin \phi + \cos \delta \cos \phi \cos(\ell - \lambda)]$$

where

$\phi = \tan^{-1}[(1 - \epsilon)^2 \tan \Phi^*]$

δ = geocentric declination of intersection of radius vector and ellipsoid

Φ^* = geodetic latitude of station

ℓ = longitude of station

λ = longitude of vehicle (east from Greenwich)

ϵ = ellipticity of reference ellipsoid

3.5.9 Height

The equation for height is

$$h = r - \frac{a_e(1 - \epsilon)}{\left[1 - (2\epsilon - \epsilon^2)\frac{x^2 + y^2}{r^2}\right]^{1/2}}$$

where

r = geocentric radius to vehicle

a_e = equatorial radius of the earth

ϵ = ellipticity of reference ellipsoid

3.5.10 Kappa (K)

Kappa, or the angle between the slant-range vector and the geocentric radius vector to a vehicle, is given by

$$K = \cos^{-1}\left[\frac{r^2 + R^2 - \left(w_1^s\right)^2 - \left(w_3^s\right)^2}{2rR}\right]$$

where

R = slant range

r = geocentric distance to vehicle

w_1^s, w_3^s = station position in W system

3.5.11 Doppler

The doppler frequency shift is computed by

$$\Delta f = - K_D \dot{R}\left(1 - \frac{\dot{R}}{c}\right)$$

where

K_D = an input constant (C(29))

c = speed of light (INTEG(40))

3.5.12 Attenuation

Attenuation is obtained from

$$A = -40 \log_{10} R$$

where

A = amplitude attenuation in decibels

R = slant range in feet

3.6 TRAJECTORY

The position and velocity components $X = (x, y, z)$ and $\dot{X} = (\dot{x}, \dot{y}, \dot{z})$ of the vehicle and their partial derivatives X_{p_i} and $\dot{X}_{p_i}$ $(i = 1, 2, 3, \ldots, n)$ with respect to the trajectory initial-condition and differential-equation parameters are functions of time defined by their differential equations and appropriate initial conditions. The equations are integrated numerically, and at each observation or print time all the quantities X, $\dot{X}$, X_{p_i} and $\dot{X}_{p_i}$ $(i = 1, 2, 3, \ldots, n)$ are obtained by interpolation in the integrated results. From these the computed radar observations and their partial derivatives and the trajectory output are obtained (see Section 3.3 and 3.4).

3.6.1 Differential Equations

The equations of motion of the vehicle follow from

$$\ddot{X} = \frac{-\mu X}{r^3} + F$$

where

μ = gravitational constant (GM) of the earth

$r = |X| = (x^2 + y^2 + z^2)^{1/2}$

$F = F_1 + F_2 + F_3 + F_4$ = perturbative acceleration due to asphericity of the earth, extra-terrestrial gravitational forces, atmospheric drag, and low thrust, respectively.

The initial conditions $X(t_o)$ and $\dot{X}(t_o)$, if not given directly, are computed from the initial spherical coordinates or elliptic elements (see Section 3.2 for applicable formulas).

The perturbative acceleration F_1 due to the asphericity of the earth is derived from the assumed potential function

$$U = \frac{\mu}{r}\left[1 - \sum_{n=2}^{n_1} J_n \left(\frac{a_e}{r}\right)^n P_n(\sin\phi) + \sum_{n=2}^{n_2} \sum_{m=1}^{n} J_{nm} \left(\frac{a_e}{r}\right)^n P_n^m(\sin\phi) \cos m(\lambda - \lambda_{nm})\right]$$

where

μ = product (GM) of the Newtonian gravitational constant and mass of the earth

r, ϕ, λ = geocentric distance, geocentric latitude, and (east) longitude of a point

a_e = mean equatorial radius of the earth

J_n, J_{nm} = numerical coefficients

P_n = Legendre polynomial of the first kind of degree $n < n_1$

P_n^m = associated Legendre function of the first kind of degree $n < n_2$ and order m

λ_{nm} = longitudes associated with the J_{nm}

In the local horizontal coordinate system, in which the coordinate axes are directed Up (along the radius vector), East, and North, the F_1 force components g_U, g_E, and g_N are given by

$$g_U = \frac{\partial U}{\partial r}$$

$$= -\frac{\mu}{r^2}\left[1 - \sum_{n=2}^{n_1} (n+1) J_n \left(\frac{a_e}{r}\right)^n P_n(\sin\phi)\right.$$

$$\left. + \sum_{n=2}^{n_2} \sum_{m=1}^{n} (n+1) J_{nm} \left(\frac{a_e}{r}\right)^n P_n^m(\sin\phi) \cos m(\lambda - \lambda_{nm})\right]$$

$$g_E = \frac{1}{r\cos\phi}\frac{\partial U}{\partial\lambda}$$

$$= -\frac{\mu}{r^2}\sum_{n=2}^{n_2}\sum_{m=1}^{n} mJ_{nm}\left(\frac{a_e}{r}\right)^n \frac{P_n^m(\sin\phi)}{\cos\phi}\sin m(\lambda - \lambda_{nm})$$

and

$$g_N = \frac{1}{r}\frac{\partial U}{\partial\phi}$$

$$= -\frac{\mu}{r^2}\left[\sum_{n=2}^{n_1} J_n\left(\frac{a_e}{r}\right)^n P_n'(\sin\phi)\cos\phi\right.$$

$$\left. - \sum_{n=2}^{n_2}\sum_{m=1}^{n} J_{nm}\left(\frac{a_e}{r}\right)^n P_n^{m\prime}(\sin\phi)\cos\phi\,\cos m(\lambda - \lambda_{nm})\right]$$

Also, the Legendre functions and their derivatives are computed from the recursion formulas

$$P_n(\sin\phi) = \frac{-(n-1)P_{n-1}(\sin\phi) + (2n-1)\sin\phi\,P_{n-1}(\sin\phi)}{n}$$

$$P_n'(\sin\phi) = \sin\phi\,P_{n-1}'(\sin\phi) + nP_{n-1}(\sin\phi)$$

$$\frac{P_n^m(\sin\phi)}{\cos\phi} = \frac{-(n+m-1)\dfrac{P_{n-2}^m(\sin\phi)}{\cos\phi} + (2n-1)\sin\phi\dfrac{P_{n-1}^m(\sin\phi)}{\cos\phi}}{n-m}$$

$$\frac{P_m^m(\sin\phi)}{\cos\phi} = 1 \cdot 3 \cdot \ldots (2m - 1)(\cos\phi)^{m-1}$$

and

$$P_n^{m'}(\sin\phi)\cos\phi = (n+1)\sin\phi \, \frac{P_n^m(\sin\phi)}{\cos\phi} - (n - m + 1)\frac{P_{n+1}^m(\sin\phi)}{\cos\phi}$$

wherein the initial values are

$$P_o(\sin\phi) = P_1'(\sin\phi) = 1$$

$$P_1(\sin\phi) = \sin\phi,$$

$$\frac{P_{m-1}^m(\sin\phi)}{\cos\phi} = 0$$

The force vector in the ECI coordinate system is then

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = \begin{bmatrix} \cos\phi\cos\alpha & -\sin\alpha & -\sin\phi\cos\alpha \\ \cos\phi\sin\alpha & \cos\alpha & -\sin\phi\sin\alpha \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} g_U \\ g_E \\ g_N \end{bmatrix}$$

where

$$\alpha = \alpha_g + \omega_e(t - t_g) = \text{right ascension}$$

The gravitational attraction of other bodies contributes the effect

$$F_2 = -\mu \sum_{j=1}^{k} m_j\left(\frac{X - X_j}{|X - X_j|^3} + \frac{X_j}{|X_j|^3}\right)$$

where

m_j = mass relative to the earth of j^{th} body

X_j = vector position of j^{th} body as obtained from the JPL/STL planetary coordinate tapes.

In connection with the JPL/STL planetary tapes (Ref. 8) it should be noted that the tabular planetary coordinates are with respect to the Mean Equator and Equinox 1950.0 coordinate system whereas TRACE-D program calculations are referred to 0 hour GMT of start day. The planetary coordinates are transferred to the TRACE-D coordinate system before F_2 is calculated (Refs. 9 and 10).

The effect of atmospheric drag is expressed by

$$F_3 = -\rho\left(\frac{V_A}{2}\right)\left(\frac{C_D A}{W}\right)\dot{X}_A$$

where

ρ = density at height h above the oblate earth

with

$$h = r - \frac{a_e(1-\epsilon)}{\left[1-(2\epsilon-\epsilon^2)\frac{x^2+y^2}{r^2}\right]^{1/2}}$$

$\frac{C_D A}{W}$ = drag coefficient (reciprocal of "ballistic coefficient")

$\dot{X}_A$ = vehicle velocity vector relative to rotating atmosphere

with

$$\dot{x}_A = \dot{x} + \omega_a y$$

$$\dot{y}_A = \dot{y} - \omega_a x \qquad \text{(where } \omega_a \text{ is the rotation rate of the atmosphere)}$$

$$\dot{z}_A = \dot{z}$$

$$V_A = |\dot{X}_A|$$

The atmospheric density is computed from an atmosphere model or certain combinations of models (Refs. 11 and 12).

The low-thrust term F_4, which simulates only a special form of acceleration such as might be due to the influence of residual gases, may be expressed as

$$F_4 = T_1 e^{-T_2(t-t_s)} \frac{\dot{X}}{|\dot{X}|} \qquad (\text{for } t_s \leq t \leq t_f)$$

or

$$F_4 = 0 \qquad (\text{otherwise})$$

where

T_1 = initial magnitude of acceleration

T_2 = time constant for thrust decay

t_s, t_f = start and finish times

$\dot{X}$ = inertial velocity vector

It should be recognized that arbitrary velocity impulses, or "kicks," also may be inserted into a trajectory profile. These velocity increments are described by the time of application t_K of the magnitude K of the kick, and by the deflections θ_p and θ_y in pitch and yaw, respectively, of the direction of the kick from the in-track vector. θ_p and θ_y are referenced to the in-track vector in accordance with the coordinate conventions previously specified in Figure 3-7.

The vector velocity increment then is

$$\Delta \underline{v} = K(\sin \theta_p X_y + \cos \theta_p \cos \theta_y X_r + \cos \theta_p \sin \theta_y X_p)$$

where

$$X_y = \text{yaw axis} = \frac{X}{|X|} = \text{unit vector in radial direction}$$

$$X_p = \text{pitch axis} = \frac{X \times \dot{X}}{|X \times \dot{X}|} = \text{unit vector normal to orbit plane}$$

$$X_r = \text{roll axis} = X_p \times X_y = \text{unit vector in in-track direction}$$

3.6.2 Trajectory Partial Derivatives

The partial derivatives of vehicle position and velocity with respect to trajectory parameters either can be approximated analytically or can be obtained by a simultaneous numerical integration of the variational equations.

3.6.2.1 Variational Equations

The variational equation for an initial-condition parameter α is

$$\ddot{X}_\alpha = \left[\frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) + \frac{\partial F}{\partial X}\right] X_\alpha + \frac{\partial F}{\partial \dot{X}} \dot{X}_\alpha$$

with initial conditions $X_\alpha(t_o) = (\partial X/\partial \alpha)t_o$, $\dot{X}_\alpha(t_o) = (\partial \dot{X}/\partial \alpha)t_o$. Also, for any differential-equation parameter β, except the specific case where the parameter is μ (i.e., GM) (see Section 3.6.2.3), the variational equation is

$$\ddot{X}_\beta = \left[\frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) + \frac{\partial F}{\partial X}\right] X_\beta + \frac{\partial F}{\partial \dot{X}} \dot{X}_\beta + \frac{\partial F}{\partial \beta}$$

with initial conditions $X_\beta(t_o) = \dot{X}_\beta(t_o) = 0$.

In the foregoing expressions for $\ddot{X}_\alpha$ and $\ddot{X}_\beta$, $X_\alpha = \partial X/\partial\alpha$, $\dot{X}_\alpha = \partial\dot{X}/\partial\alpha$, and X_β, $\dot{X}_\beta$ and $\partial F/\partial\beta$ are all 3-component vectors. The contents of the square brackets and $\partial F/\partial\dot{X}$ are 3×3 matrices. The system is solved for each parameter, and all the numerical integrations are carried out simultaneously.

The matrix in square brackets is calculated as the sum V + T, where

$$V = \frac{\partial}{\partial X}\left(-\frac{uX}{r^3}\right) + \frac{\partial F_1}{\partial X}$$

$$T = \frac{\partial F_3}{\partial X}$$

represent the dependence of the gravitational and drag accelerations, respectively, upon the position of the vehicle. The other-body term $\partial F_2/\partial X$ is ignored (see Appendices B and C for derivations of the V and T matrices). The matrix $\partial F/\partial\dot{X}$ then is

$$\frac{\partial F_3}{\partial \dot{X}} + \frac{\partial F_4}{\partial \dot{X}}$$

where

$$\frac{\partial F_3}{\partial \dot{X}} = -\frac{1}{2}\rho V_A \frac{C_D A}{W}\left(\frac{\dot{X}_A X_A^T}{V_A^2} + I\right)$$

$$\frac{\partial F_4}{\partial \dot{X}} = \left(T_1 e^{-T_2(t-t_s)}\right) \frac{\partial}{\partial \dot{X}}\left(\frac{\dot{X}}{|\dot{X}|}\right) = \frac{T_1 e^{-T_2(t-t_s)}}{|\dot{X}|}\left[I - \frac{\dot{X}\dot{X}^T}{|\dot{X}|^2}\right]$$

It should be noted that for the two initial-condition parameters α (right ascension) and Ω (right ascension of the ascending node), analytic solutions for the variational equations are available when the forces upon the vehicle are symmetric about the polar axis. The solutions for α,

$$\frac{\partial x}{\partial \alpha} = -y \qquad \frac{\partial \dot{x}}{\partial \alpha} = -\dot{y}$$

$$\frac{\partial y}{\partial \alpha} = x \qquad \frac{\partial \dot{y}}{\partial \alpha} = \dot{x}$$

$$\frac{\partial z}{\partial \alpha} = 0 \qquad \frac{\partial \dot{z}}{\partial \alpha} = 0$$

and similarly those for Ω are employed in TRACE-D. The additional terms arising from the tesseral harmonics in the geopotential and longitudinal density variations are ignored (see Appendix D for derivation of foregoing equations).

3.6.2.2 Variational-Equation Initial Conditions

Paragraphs 3.6.2.2.1 through 3.6.2.2.3 define initial conditions $X_\alpha(t_o)$ and $\dot{X}_\alpha(t_o)$ for the variational equations for the parameters of initial position and velocity in rectangular and spherical coordinates and in elliptic orbital elements.

3.6.2.2.1 Rectangular Coordinates

$$\left(\frac{\partial X}{\partial X}\right)_{t_o} = \left(\frac{\partial \dot{X}}{\partial \dot{X}}\right)_{t_o} = I \qquad \text{(the } 3 \times 3 \text{ identity matrix)}$$

$$\left(\frac{\partial X}{\partial \dot{X}}\right)_{t_o} = \left(\frac{\partial \dot{X}}{\partial X}\right)_{t_o} = 0$$

3.6.2.2.2 Spherical Coordinates

α (Right ascension)

(Initial conditions are not required for this parameter due to the fact that an analytic solution of the variational equation for α is employed.)

δ (Declination)

$$\frac{\partial x}{\partial \delta} = -r \sin \delta \cos \alpha$$

$$\frac{\partial y}{\partial \delta} = -r \sin \delta \sin \alpha$$

$$\frac{\partial z}{\partial \delta} = r \cos \delta$$

$$\frac{\partial \dot{x}}{\partial \delta} = -\dot{z} \cos \alpha$$

$$\frac{\partial \dot{y}}{\partial \delta} = -\dot{z} \sin \alpha$$

$$\frac{\partial \dot{z}}{\partial \delta} = v (\cos \beta \cos \delta - \cos A \sin \beta \sin \delta)$$

β (Flight path angle)

$$\frac{\partial x}{\partial \beta} = \frac{\partial y}{\partial \beta} = \frac{\partial z}{\partial \beta} = 0$$

$$\frac{\partial \dot{x}}{\partial \beta} = -v [(\sin \beta \cos \delta + \cos A \cos \beta \sin \delta) \cos \alpha + \sin A \cos \beta \sin \alpha]$$

$$\frac{\partial \dot{y}}{\partial \beta} = -v\left[(\sin\beta\cos\delta + \cos A\cos\beta\sin\delta)\sin\alpha - \sin A\cos\beta\cos\alpha\right]$$

$$\frac{\partial \dot{z}}{\partial \beta} = v(\cos A\cos\beta\cos\delta - \sin\beta\sin\delta)$$

A (Azimuth)

$$\frac{\partial x}{\partial A} = \frac{\partial y}{\partial A} = \frac{\partial z}{\partial A} = 0$$

$$\frac{\partial \dot{x}}{\partial A} = v(\sin A\sin\delta\cos\alpha - \cos A\sin\alpha)\sin\beta$$

$$\frac{\partial \dot{y}}{\partial A} = v(\sin A\sin\delta\sin\alpha + \cos A\cos\alpha)\sin\beta$$

$$\frac{\partial \dot{z}}{\partial A} = -v(\sin A\cos\delta\sin\beta)$$

r (Magnitude of radial vector)

$$\frac{\partial x}{\partial r} = \frac{x}{r}$$

$$\frac{\partial y}{\partial r} = \frac{y}{r}$$

$$\frac{\partial z}{\partial r} = \frac{z}{r}$$

$$\frac{\partial \dot{x}}{\partial r} = \frac{\partial \dot{y}}{\partial r} = \frac{\partial \dot{z}}{\partial r} = 0$$

v (Velocity)

$$\frac{\partial x}{\partial v} = \frac{\partial y}{\partial v} = \frac{\partial z}{\partial v} = 0$$

$$\frac{\partial \dot{x}}{\partial v} = \frac{\dot{x}}{v}$$

$$\frac{\partial \dot{y}}{\partial v} = \frac{\dot{y}}{v}$$

$$\frac{\partial \dot{z}}{\partial v} = \frac{\dot{z}}{v}$$

3.6.2.2.3 Elliptic Orbital Elements

The equations for the partial derivatives of position and velocity components with respect to the elliptic orbital elements are used both to compute initial conditions at time t_o for the variational equations for the elliptic-element parameters and to estimate analytically the trajectory partial derivatives.

a (Semi-major axis)

$$X_a = \frac{1}{a}\left(X - \frac{3M}{2n}\dot{X}\right)$$

$$\dot{X}_a = \frac{1}{a}\left(\dot{X} - \frac{3M}{2n}\ddot{X} - \frac{3}{2}\dot{X}\right)$$

$$\ddot{X} = -\frac{\mu X}{r^3}$$

e (Eccentricity)

$$X_e = -\left[a + \frac{y_\omega^2}{r(1 - e^2)}\right]P + \frac{x_\omega y_\omega}{r(1 - e^2)}Q$$

$$\dot{X}_e = -\frac{1}{r(1-e^2)^{1/2}}\left[\frac{y_\omega}{(1-e^2)^{1/2}}\dot{y}_\omega + n\left(\frac{a}{r}\right)^2 x_\omega y_\omega\right]P$$

$$+\frac{1}{r(1-e^2)^{1/2}}\left[\frac{y_\omega}{(1-e^2)^{1/2}}\dot{x}_\omega + n\left(\frac{a}{r}\right)^2 x_\omega^2\right]Q$$

i (Inclination)

$$X_i = \frac{z}{\left(P_z^2 + Q_z^2\right)^{1/2}} W$$

$$\dot{X}_i = \frac{\dot{z}}{\left(P_z^2 + Q_z^2\right)^{1/2}} W$$

where

$$W = P \times Q$$

Ω (Longitude of ascending node)

(Initial conditions are not required for this parameter due to the fact that an analytic solution of the variational equation for Ω is employed.)

ω (Argument of perigee)

$$X_\omega = -y_\omega P + x_\omega Q$$

$$\dot{X}_\omega = -\dot{y}_\omega P + \dot{x}_\omega Q$$

τ (Time of perigee passage)

$$X_\tau = -\dot{X}$$

$$\dot{X}_\tau = \frac{\mu X}{r^3}$$

Initial conditions for the variational equation for epoch time are $X_{t_o}(t_o) = -\dot{X}(t_o)$ and $\dot{X}_{t_o}(t_o) = -\ddot{X}(t_o)$.

The following "delayed initial conditions", corresponding to the time of a kick, must be applied in the applicable variational equations for the the partial derivatives of trajectory position and velocity with respect to kick parameters.

K (Kick magnitude)

$$X_K = 0$$

$$\dot{X}_K = \sin\theta_p X_y + \cos\theta_p \cos\theta_y X_r + \cos\theta_p \sin\theta_y X_p$$

θ_p (Pitch deflection)

$$X_{\theta_p} = 0$$

$$\dot{X}_{\theta_p} = K(\cos\theta_p X_y - \sin\theta_p \cos\theta_y X_r - \sin\theta_p \sin\theta_y X_p)$$

θ_y (Yaw deflection)

$$X_{\theta_y} = 0$$

$$\dot{X}_{\theta_y} = K(-\cos\theta_p \sin\theta_y X_r + \cos\theta_p \cos\theta_y X_p)$$

3.6.2.3 Differential Equation Parameter Non-homogeneous Terms

The non-homogeneous terms $\frac{\partial F}{\partial \beta}$ for the differential-equation parameter variational equations are:

$C_D A/W$ (Drag coefficient)

$$\frac{\partial F}{\partial\left(\frac{C_D A}{W}\right)} = F_3\left(\frac{C_D A}{W}\right)^{-1}$$

μ (Gravitational constant)

$$\frac{\partial \ddot{X}}{\partial \mu} = \frac{F_1 + F_2}{\mu} - \frac{X}{r^3}$$

J_i, J_{ik}, λ_{ik} (Oblateness parameters)

Denoting the perturbative force components in the Up/East/North system (see Section 3.6.1) by

$$g_U = -\frac{\mu}{r^2}\left[\sum_{n=2}^{n_1} A_n + \sum_{n=2}^{n_2} \sum_{m=1}^{n} B_{nm} \cos m (\lambda - \lambda_{nm})\right]$$

$$g_E = -\frac{\mu}{r^2}\left[\sum_{n=2}^{n_2} \sum_{m=1}^{n} C_{nm} \sin m (\lambda - \lambda_{nm})\right]$$

$$g_N = -\frac{\mu}{r^2}\left[\sum_{n=2}^{n_1} D_n + \sum_{n=2}^{n_2}\sum_{m=1}^{n} E_{nm}\cos m\,(\lambda - \lambda_{nm})\right]$$

then

$$\frac{\partial g_U}{\partial J_i} = \frac{-\mu}{r^2}\;\frac{A_i}{J_i}$$

$$\frac{\partial g_E}{\partial J_i} = 0$$

$$\frac{\partial g_N}{\partial J_i} = \frac{-\mu}{r^2}\;\frac{D_i}{J_i}$$

$$\frac{\partial g_U}{\partial J_{ik}} = \frac{-\mu}{r^2}\;\frac{B_{ik}\cos k\,(\lambda - \lambda_{ik})}{J_{ik}}$$

$$\frac{\partial g_E}{\partial J_{ik}} = \frac{-\mu}{r^2}\;\frac{C_{ik}\sin k\,(\lambda - \lambda_{ik})}{J_{ik}}$$

$$\frac{\partial g_N}{\partial J_{ik}} = \frac{-\mu}{r^2}\;\frac{E_{ik}\cos k\,(\lambda - \lambda_{ik})}{J_{ik}}$$

$$\frac{\partial g_U}{\partial \lambda_{ik}} = \frac{-\mu}{r^2}\;kB_{ik}\sin k\,(\lambda - \lambda_{ik})$$

$$\frac{\partial g_E}{\partial \lambda_{ik}} = \frac{+\mu}{r^2} kC_{ik} \cos k (\lambda - \lambda_{ik})$$

$$\frac{\partial g_N}{\partial \lambda_{ik}} = \frac{-\mu}{r^2} kE_{ik} \sin k (\lambda - \lambda_{ik})$$

The component terms are then rotated to the ECI system by the matrix given in Section 3. 6. 1.

T_1, T_2 (Thrust parameters)

$$\frac{\partial F}{\partial T_1} = e^{-T_2(t - t_s)} \frac{\dot{X}}{|\dot{X}|} = \frac{F_4}{T_1}$$

$$\frac{\partial F}{\partial T_2} = -T_1(t - t_s) e^{-T_2(t - t_s)} = -(t - t_s) F_4$$

ω_A (Atmosphere rotation rate)

$$\frac{\partial F}{\partial \omega_A} = -\frac{1}{2} \rho \frac{C_D A}{W} V_A \left[(EX) + \frac{\dot{X}_A^T (EX)}{V_A^2} \dot{X}_A \right]$$

where

$$EX = \begin{pmatrix} y \\ -x \\ 0 \end{pmatrix}$$

3.6.3 Integration Methods

Numerical integration for TRACE-D program purposes, including integration of the differential equations itemized in Sections 3.6.1 and 3.6.2, is accomplished by means of the DE6F subroutine, which is based on the widely known Gauss-Jackson method. This subroutine incorporates variable-step predictor-corrector automatic local truncation-error control as well as double-precision accumulation features, and uses the Runge-Kutta method for obtaining starting values (Ref. 13).

The Gauss-Jackson method, which utilizes 6^{th} differences, has proved to be remarkably effective in the integration of most satellite trajectories, and in some restricted but well-controlled tests involving application of the method to the equations of motion has produced results comparing favorably in both speed and accuracy with more sophisticated special perturbation methods (Ref. 14). A recent simple refinement, wherein rounded values of the dependent variables instead of only their most significant portions are used in calculation of the derivatives, has nearly halved the integration errors previously accumulated in 15 revolutions.

3.6.4 Interpolation

Whenever the position, velocity, and related partial derivatives of a vehicle are required, the applicable quantities X, $\dot{X}$, X_{p_i} and $\dot{X}_{p_i}$ are obtained by Hermite interpolation (Ref. 15) between integration steps. This technique permits an uninterrupted numerical integration, is comparatively rapid, and, as used in this connection, is quite accurate. In particular, function values and their first and second derivatives at two adjacent integration steps are retained to permit 5^{th} and 3^{rd} degree interpolations for position and velocity, respectively.

3.6.5 Trajectory Output

The position and velocity vectors X and $\dot{X}$ of a vehicle constitute the basis of the trajectory output in that evaluations of these quantities as obtained by

interpolation from the results of the numerical integration permit computation of both the spherical coordinates α, δ, β, A, r, v of the vehicle (see Section 3.2.2) and of its geodetic latitude Φ^*, longitude ℓ, and height h. Expressions for the latter three quantities may be written

$$\Phi^* = \tan^{-1}\left[\frac{z}{(x^2 + y^2)^{1/2}(1 - \epsilon)^2}\right]$$

$$\ell = \alpha - \alpha_g - \omega_e(t - t_g)$$

$$h = r - \frac{a_e(1 - \epsilon)}{\left[1 - (2\epsilon - \epsilon^2)\frac{x^2 + y^2}{r^2}\right]^{1/2}}$$

Also, sub-vehicle latitude Φ_v is given by

$$\Phi_v = \delta + \sin^{-1}\left[\frac{\epsilon \sin 2\delta}{r} + \left(\frac{\epsilon^2}{4r}\right)\left(\frac{4}{r} - 1\right)\sin 4\delta\right]$$

Optionally, the elements of the osculating ellipse also are output. Included among these are the elements a, e, i, Ω, ω, and τ (see Section 3.2.4) and also the following: (Ref. 16)

M (Mean anomaly, deg)

$$M = E - e \sin E$$

where

$$E = \cos^{-1}\left(\frac{1 - \frac{r}{a}}{e}\right)$$

f (True anomaly, deg)

$$f = 2 \tan^{-1}\left[\left(\frac{1+e}{1-e}\right)^{1/2} \tan\frac{E}{2}\right]$$

$\dot{\Omega}$ (Regression of the node, deg/day)

$$\dot{\Omega} = \frac{3J_2 a_e^2 \sqrt{\mu}}{2a^{3/2} p^2} \cos i$$

$\dot{\omega}$ (Advance of line of apsides, deg/day)

$$\dot{\omega} = \frac{3J_2 a_e^2 \sqrt{\mu}}{a^{3/2} p^2} \left(1 - \frac{5}{4} \sin^2 i\right)$$

where

$$p = \frac{r^2 v^2 \sin^2\beta}{\mu}$$

r_a (Radius of apogee, n mi)

$$r_a = a(1 + e)$$

r_p (Radius of perigee, n mi)

$$r_p = a(1 - e)$$

P_K (Keplerian period, min)

$$P_K = \frac{2\pi a^{3/2}}{\sqrt{\mu}}$$

P_A (Anomalistic period, min)

$$P_A = P_K \left[1 - \frac{\frac{3}{2} J_2 a_e^2}{a^2} \left(\frac{a}{r}\right)^3 (1 - 3 \sin^2 \delta) \right]$$

where

δ = declination

P_N (Nodal period, min)

$$P_N = P_A - P_K \left[\frac{3J_2 a_e^2 (1 - 5/4 \sin^2 i)}{\sqrt{p}\; a^2 (1 + e \cos \omega)^2} \right]$$

3.6.6 Initial Orbit Condition Derivation (Gaussian Method)

On option, initial conditions of an orbit may be calculated from two sets of range/azimuth/elevation (RAE) observations in accordance with the following adapted procedure (Ref. 17). Letting

X_1 = Cartesian vector associated with first RAE observation point

X_2 = Cartesian vector associated with second RAE observation point

$$W = \frac{X_1 \times X_2}{|X_1 \times X_2|} = \text{unit vector perpendicular to observation plane}$$

$$U_1 = \frac{X_1}{|X_1|} = \text{unit vector parallel to } X_1$$

$$V_1 = W \times U_1 = \text{unit vector perpendicular to } U_1 \text{ and } W$$

values are computed for

$$f = \frac{1}{2}(v_2 - v_1) = \frac{1}{2} \arc\cos \frac{X_1 \cdot X_2}{|X_1|\ |X_2|}$$

$$g_1 = \frac{\sqrt{2\mu}\ (t_2 - t_1)}{[2(|X_1|\ |X_2|)^{1/2} \cos f]^{3/2}}$$

$$g_2 = \frac{|X_1| + |X_2|}{2(|X_1|\ |X_2|)^{1/2} \cos f}$$

Setting $g^{(o)} = f$, g may then be found by means of the iteration

$$g_3 = \sin g^{(i)}$$

$$g_4 = \cos g^{(i)}$$

$$g_5 = \sin^3 g^{(i)}$$

$$g_6 = \sin^4 g^{(i)}$$

$$g_7 = g_2 - g_4$$

$$g_8 = \sqrt{g_7}$$

$$g_9 = (g_7)^2$$

Also,

$$\Delta g = \frac{\frac{1}{g_7}\left(1 - \frac{g_1}{g_8}\right) + \frac{1}{g_5}(g^{(i)} - g_3 g_4)}{\frac{g_3}{g_9}\left(\frac{3g_1}{2g_8} - 1\right) - \frac{1}{g_6}[g_5 + 3(g^{(i)} g_4 - g_3)]}$$

and

$$g^{(i+1)} = g^{(i)} - \Delta g$$

Iteration is continued until $|\Delta g| \leq \epsilon$.

The Keplerian elements are then given by the relationships

$$a = \frac{|X_1| + |X_2| - 2\sqrt{|X_1|\ |X_2|}\cos g \cos f}{2 \sin^2 g}$$

$$e \cos E_1 = 1 - \frac{|X_1|}{a}$$

$$e \cos E_2 = 1 - \frac{|X_2|}{a}$$

$$e \sin E_1 = \frac{\cos 2g}{\sin 2g} \left[e \cos E_1 - (\cos 2g)(e \cos E_2)\right]$$

$$-(\sin 2g)(e \cos E_2)$$

$$E_1 = \tan^{-1} \frac{e \sin E_1}{e \cos E_1} \qquad (0 \leq E_1 < 2\pi)$$

$$e = \left[(e \cos E_1)^2 + (e \sin E_1)^2\right]^{1/2}$$

$$T = t_1 - (E_1 - e \sin E_1) \frac{a^{3/2}}{\sqrt{\mu}} \qquad (a > 0)$$

$$i = \cos^{-1} W_z \qquad (0 \le i < \pi)$$

$$\Omega = \tan^{-1} \frac{W_x}{-W_y} \qquad (0 \le \Omega < 2\pi)$$

$$\cos v_1 = \frac{\cos E_1 - e}{1 - e \cos E_1}$$

$$\sin v_1 = \frac{\sqrt{1 - e^2} \sin E_1}{1 - e \cos E_1}$$

$$P = U_1 \cos v_1 - V_1 \sin v_1$$

$$Q = U_1 \sin v_1 + V_1 \cos v_1$$

$$\omega = \tan^{-1} \frac{P_z}{Q_z} \qquad (0 \le \omega < 2\pi)$$

3.7 DIFFERENTIAL CORRECTION AND ASSOCIATED COMPUTATIONS

The basic problem of differential correction is determination of the change or correction of a given set of parameters that is necessary to permit some specified result to be achieved. For the present case the goal is to minimize the weighted sum of the squares of the differences between the observed radar data and the correspond ing quantities computed from the observational model, which of course includes the trajectory and observation parameters to be corrected.

In this discussion, matrices and vectors in general are denoted by Roman capitals and their components by corresponding lower-case letters with subscripts where appropriate. The following notation and nomenclature als are applicable:

n = number of observed quantities

m = number of parameters

k = number of effective parameters (m minus the number of constraint equations)

o_i = i^{th} observation

σ_{sj} = radar sigma (multiplicative weighting factor) to be applied to data-type j from station s

β_{sj} = radar bias (additive weighting factor) to be applied to data-type j from station s

O_{mc} = vector of weighted residuals (differences between observed and computed radar quantities)

P = vector of parameters

ΔP = correction vector for P

G = diagonal matrix representing bounds on solution ΔP

A = matrix of partial derivatives:

$$a_{ij} = \frac{\partial o_i}{\partial P_j} \ ; \ i = 1, 2, \ldots, n \ ; \ j = 1, 2, \ldots, m$$

A^T = A transpose

W = n × n weighting matrix, diagonal except for 3 x 3 matrices on the diagonal, where the rows correspond to correlated observations. The complete matrix is never formed but is input in inverse form, where observation sigmas are the square roots of the reciprocal diagonal elements and the input observation covariance matrices are the inverse of the 3 × 3 sub-matrices.

B = constraint matrix

3.7.1 Sigmas, Covariances, and Biases

Usually some information is available concerning the characteristics of the observations in terms of random noise, biases, and, in specific cases, correlations. Such information may be included in the least-squares process in the following manner.

If it is known or suspected that a certain set of data exhibits a constant bias in either data or time, the β_{sj} are applied to the appropriate components of O_{mc} before weighting. Biases may be included as parameters to be differentially corrected. If so, the derived value is automatically applied on each subsequent iteration. If a constant bias is to be applied, the parameter may be selected, the desired value entered as the estimate, and the bound set to zero.

The most common form of weighting involves use of a priori standard deviations; in general, one value for each type of data from each station. Under this option the elements of A and O_{mc} are simply divided by the appropriate sigma. In the case of correlated observations (at most three observations of the same set taken at the same time), the weighting is accomplished by means of the inverse of an input 3 × 3 covariance matrix.

3.7.2 The Unconstrained Normal

In its simplest form, differential correction involves the solution of the linearized problem $(A^TWA)\Delta P = A^TWO_{mc}$. A^TWA is the normal matrix. In TRACE-D procedure, if $A^TA = 0$ initially, the normal is formed by accumulating $A^TWA = A^TWA + a^Twa$ and $A^TWO_{mc} = A^TWO_{mc} + a^TwO_{mc}$, where a represents one row of the A matrix and the W is the a priori weighting sigma in the case of an uncorrelated observation. Where covariances among observations are encountered, a is a 3×3 matrix formed with the three appropriate rows of A, and w is the inverse of the appropriate input 3×3 covariance matrix.

The weights and biases as defined in Section 3.7.1 and introduced into the notation of Section 3.7.2 also are implicit in connection with all operations with A and O_{mc} in Sections 3.7.3 through 3.7.7.

3.7.3 The Contrained Normal

It is often desirable to impose linear constraints of the form $\Delta P = B(AP') + C$, where P' is some subset of P and C is a vector of constants, on the solution. For example, if it is assumed that the parameters to be solved for are $p_1 = S_1$ (latitude), $p_2 = S_1$ (longitude), $p_3 = S_2$ (latitude), $p_4 = S_2$ (longitude), and $p_5 = S_2$ (range bias), where S_1, S_2 are two radar stations, the requirement that the positions of the stations relative to each other remain fixed is equivalent to the matrix equation

$$\underbrace{\begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \Delta p_3 \\ \Delta p_4 \\ \Delta p_5 \end{bmatrix}}_{\Delta P} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{B} \times \underbrace{\begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \Delta p_5 \end{bmatrix}}_{\Delta P} + \underbrace{[0]}_{C'}$$

The problem then effectively becomes one of solving the reduced system $(AB)^T(AB)(\Delta P') = (AB)^T O_{mc}$. Thus if a_i is a row of A and if $A^TA \equiv 0$ initially, the constrained normal is formed by accumulating $A^TA = A^TA + (a_iB)^T(a_iB)$.

3.7.4 Bounds

Given a set of bounds g_i, the corrections Δp_i to the components of P are either less in absolute value than g_i if $g_i > 0$, zero if $g_i = 0$, or unrestricted if $g_i < 0$ for i = 1, 2, . . . , m In cases where constraints are to be applied, the bounds are adjusted accordingly. For example, letting

$$h_j = \sum_{i=1}^{m} \frac{(\text{sign } g_i)}{(g_i)^2} b_{ij} \qquad (j = 1, 2, \ldots, k)$$

k new bounds, g_j', are formulated by the program where

$$g_j' = \frac{1}{h_j} \text{ if } h_j > 0$$

$$g_j' = 0 \text{ if } h_j = 0$$

$$g_j' < 0 \text{ if } h_j < 0$$

It should be noted that $g_j' = g_i$ for a variable not appearing in any constraint equation and also that specifying bounds equal in magnitude but opposite in sign for two parameters to be corrected by equal increments will result in a zero correction to both.

3.7.5 Solution of the Normal Equations

With the $m \times m$ matrix A^TA, the vector A^TO_{mc}, and a set of bounds g_i given, and assuming m parameters p_1, p_2, and p_3 to be corrected, the problem

is to minimize $||A\Delta P - O_{mc}||^2$ under the side condition that $\Sigma(\Delta p_i/g_i)^2 \leq 1$, with the sum taken over all i for which $g_i > 0$.

It may be assumed without loss of generality that $g_i \neq 0$, inasmuch as $g_i = 0$ implies that $\Delta p_i = 0$ and also that the i^{th} row and column of the normal matrix may be ignored, which simply reduces the dimension of the problem. Letting G be the diagonal matrix, such that $G_{ii} = 1/g_i$ if $g_i > 0$ and $g_{ii} = 0$ if $g_i \leq 0$, the requirement is to find such a value of z that the solution $\Delta P'(z)$ of the linear system $(A^T A + zG^2)\Delta P = A^T O_{mc}$ satisfies the given side condition. This involves the two processes of choosing the best value for z and accomplishing the actual solution of the system.

It is important to recognize that for the constrained case the only change required in the notation given in the following Sections 3.7.5.1 through 3.7.5.4 is substitution of k (the number of effective parameters) for m.

3.7.5.1 Determination of z

The first step in the determination of z is to obtain $\Delta P'(0)$, or the solution to $(A^T A)\Delta P = A^T O_{mc}$. If then $\Sigma(\Delta p_i/g_i)^2 \leq 1 + \epsilon_1$, the problem is solved. If not, $y(z) = \Sigma(\Delta p_i(z)/g_i)^2 - 1$ must be defined. Noting that $y(0) > \epsilon_1$, computation of y(h), y(10h), and y(100h), where h is some preset constant, is carried out until either a value of z = kh is found such that $-\epsilon_2 \leq y(z) \leq \epsilon_1$, in which case $\Delta P(z)$ is the solution, or two values of z are found such that $y(z_1) > \epsilon_1$ and $y(z_2) < -\epsilon_2$, whereby the required value of z is bracketed. In the latter case a value z_3 is chosen between z_1 and z_2 in accordance with $z = 0.8z_1 + 0.2z_2$, wherein the coefficients 0.8 and 0.2 are fairly arbitrary and z_1 and z_2 may have been interchanged to bring z_3 closest to the value of z giving the smallest y(z).

If $-\epsilon_2 \leq y(z_3) \leq \epsilon_1$, $\Delta P'(z_3)$ then is the solution. Otherwise, inverse quadratic interpolation is used to obtain a new guess, z_4. Similarly, if $-\epsilon_2 \leq y(z_4) \leq \epsilon_1$, $\Delta P'(z_4)$ is the solution, but if not, the two values of z from the set $\{z_1, z_2, z_3, z_4\}$ which bracket the solution most tightly are chosen and the process is

repeated. If more than twenty solutions of the linear system are required, the process is abandoned.

In the foregoing, ϵ_1 and ϵ_2 are suitably small positive constants.

3.7.5.2 Solution of the Linear System - Matrix Operations.

In the expression $(A^T A + zG^2)\Delta P = A^T O_{mc}$, which defines the linear system to be solved, let the term $A^T A + zG^2 = C$.

It is then desired to find a matrix S such that $SCS^T = D$ where S is lower triangular with (-1) on the diagonal and $D = \text{diag}(d_1, \ldots, d_n)$.

$$C' = \begin{pmatrix} C & d \\ d^T & a \end{pmatrix} .$$

Since S' must be lower triangular with (-1) on the diagonal and of the same order as C', it must be of the form

$$S' = \begin{pmatrix} S & 0 \\ W^T & -1 \end{pmatrix}$$

and the requirement $S'C'S'^T = D'$ is equivalent to solving for a vector W and a scalar b such that

$$\begin{pmatrix} S & 0 \\ W^T & -1 \end{pmatrix} \begin{pmatrix} C & d \\ d^T & a \end{pmatrix} \begin{pmatrix} S^T & W \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} D & 0 \\ 0 & b \end{pmatrix} = D'$$

It is easily verified that $W = S^T D^{-1} Sd$ and $b = \alpha - W^T d$ satisfy the foregoing equation.

The computations are carried out in accordance with the above outline, starting with the 2×2 matrix C_{ij}, $i = 1, 2,$; $j = 1, 2$ and continuing until the decomposition

$$\begin{pmatrix} S & 0 \\ W^T & -1 \end{pmatrix} \begin{pmatrix} A^T A + zG^2 & A^T O_{mc} \\ O_{mc}^T A & O_{mc}^T O_{mc} \end{pmatrix} \begin{pmatrix} S^T & W \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} D & 0 \\ 0 & \alpha \end{pmatrix}$$

has been found. Completion of the multiplication indicated on the left side of this equation shows that the m-dimensional vector W is the solution to the linear system.

3.7.5.3 Residuals Prediction

The predicted RMS of the residuals

$$\| O_{mc}^P \| = \| A \Delta P - O_{mc} \|$$

is computed from the augmented normal matrix

$$\| A \Delta P - O_{mc} \|^2 = [(A^T A \Delta P) \cdot \Delta P] - 2[(A^T O_{mc}) \cdot \Delta P] + O_{mc}^T O_{mc}$$

3.7.5.4 The Inverse Normal

$$(A^T A)^{-1} = S^T D^{-1} S$$

wherein S and D are as defined in paragraph 3.7.5.2.

3.7.6 Convergence of the Differential Correction Process

The $\|O_{mc}\|$ is a measure of how well an orbit computed on the basis of a given set of parameters P fits corresponding observed data. $\|O^P_{mc}\|$ computed in accordance with paragraph 3.7.5.3 is an approximation to $\|O_{mc}\|$ which would be obtained by replacing P by P + ΔP. This approximation would be exact if the least squares problem were linear, which is equivalent to P being in a sufficiently small neighborhood of the minimum point.

Convergence therefore is reached when further corrections to P would produce no significant decrease in $\|O_{mc}\|$, or no over-all improvement of the residuals. The criterion is either

$$\frac{\|O_{mc}\| - \|O^P_{mc}\|}{\|O_{mc}\|} \leq \epsilon_I .$$

or

$$\|O_{mc}\| \cdot n^{-1/2} \leq \epsilon_{II}$$

where

n = number of observations

$\epsilon_I, \epsilon_{II}$ = input quantities

If $\|O_{mc}\|$ is decreasing with each iteration, the process is converging and the bounds are expanded at each step by a multiplicative factor β_1 to permit faster convergence. On the other hand, if $\|O_{mc}\|$ is increasing from one iteration to the next, the process is diverging and the last corrections are presumed to have altered P too drastically. In this situation the previous values of P and the corresponding normal matrix are retrieved and resolved with tighter bounds g_i' such that the weighted length $\|G' \cdot \Delta P'\|$ of the solution is reduced to β_2 times its previous value. β_1 and β_2 are input constants.

3.7.7 The Correlation Matrix

It has been shown in Section 2 that if the mathematical model is exact, if the observations are linear functions of the parameters, if the observation errors have mean zero and are independent, and if the input values of σ_{sj} are correct, the inverse of the normal matrix then is the variance-covariance matrix of the parameters that arises because of the random errors in the observations. If the elements of this matrix are given as c_{ij}, the corresponding correlation matrix has elements

$$c'_{ij} = \frac{c_{ij}}{\sqrt{c_{ii}c_{jj}}} \qquad (i, j, = 1, 2, \ldots, m)$$

If all the assumptions noted above are true except that all σ^2_{sj} values are in error by a constant multiplicative factor, then the values in the variance-covariance matrix will also all be in error by the same factor.

3.7.8 The Scatter Coefficient

The scatter coefficient (Ref. 18) may be expressed by

$$S = [\det |c_{ij}|]^{1/2} = \left[\frac{\det |A^T A^{-1}|}{\Pi \sigma_i^2}\right]^{1/2}$$

where

c_{ij} = correlation matrix for the least squares parameters

$[A^T A]^{-1}$ = usual normal matrix inverse or variance-covariance matrix

σ_i^2 = diagonal elements of $[A^T A]^{-1}$

3.8 OTHER TRACKING CALCULATIONS

3.8.1 Proximity Testing

Input quantities are denoted by the set

$$\{\Phi_i^*, \lambda_i, h_i, R_{o_i}^*\}$$

where

Φ^* = geodetic latitude

λ = east longitude

h = altitude (height)

R_o^* = range to be tested against (critical range)

i = 1, 2, 3, = number of site designated by Φ^*, λ, h

3.8.1.1 Input Conversion (INCNI)

Input units must first be converted to working units, wherein degrees and feet/nautical miles are expressed in terms of radians and earth radii, respectively. The input set thus is converted to a set of working parameters denoted by

$$\{X_i, \Psi_i, \lambda_i, R_{o_i}^*\}$$

where X_i, Ψ_i are site (i) coordinates. X_i, Ψ_i are then computed from the relations

$$X_i = \left\{ \frac{a_e}{[(1 - \epsilon^2) \sin^2 \Phi_i^*]^{1/2}} + h \right\} \cos \Phi_i^*$$

and

$$\Psi_i = \left\{ \frac{a_e(1 - \epsilon^2)}{[(1 - \epsilon^2) \sin^2 \Phi_i^*]^{1/2}} + h_i \right\} \sin \Phi_i^*$$

where

a_e = semi-major axis of the earth

ϵ = ellipticiy of reference ellipsoid

3.8.1.2 Proximity Test

After the inertial-coordinate components of a site (i) position vector at time t have been computed, the position vector $\underline{r}_{s_i}$ is determined by

$$\underline{r}_{s_i} = \begin{cases} x_{s_i}(t) = X_i \cos \alpha_{s_i} \\ y_{s_i}(t) = X_i \sin \alpha_{s_i} \\ z_{s_i}(t) = \Psi_i \end{cases}$$

where

$\alpha_{s_i} = \alpha_g + \omega_e(t - t_o) + \lambda_i$

α_{s_i} = right ascension of site (i) at time t

α_{g_i} = right ascension of Greenwich at midnight of epoch

ω_e = rotational rate of the earth

t_o = epoch time in minutes

λ_i = east longitude of site (i).

The proximity indicator is printed if the satellite of interest is within the sphere of influence at a proximity-testing time (integration-step time). It is therefore apparent that a necessary and sufficient condition to print this indicator is

$$|R_i^*| \leq R_{o_i}^*$$

where

R_i^* = magnitude of range vector from site (i) to satellite

$= \underline{r} - \underline{r}_{s_i}$

$= [x(t), y(t), z(t)] - [X_{s_i}(t), Y_{s_i}(t), Z_{s_i}(t)]$

$R_{o_i}^*$ = critical range from site (i)

3.8.1.3 Output Quantities

If proximity with respect to site (i) is detected at time t, the quantities listed in Table 3-1 will be printed.

Table 3-1. Computer Output Quantities for Proximity Condition

Quantity	Computer Symbol	Units
Satellite range from site (i)	R*(i)	nautical miles
Date	XX/YY/ZZ	month/day/year
Time	T	minutes from midnight of day of epoch
Geodetic latitude	LAT(GD)	degrees
East longitude	LONG	degrees
Altitude	ALT	nautical miles
Inertial azimuth of velocity vector	AZ	degrees

3.8.2 Elevation-Angle Refraction Correction

Input elevation-angle observations are corrected for atmospheric refraction effects by

$$E' = E - \eta_{si} \cot E \qquad (E \geq 0.1 \text{ radian})$$

or

$$E' = E - \frac{1}{1000} \cdot \frac{\eta_{si} \times 10^6}{12 + 1000E} + \frac{80}{6 + 1000E} \qquad (E < 0.1 \text{ radian})$$

where

E = input angle

η_{si} = appropriate refractivity index from REFR table (no correction is made if $\eta_{si} = 0$)

3.8.3 Vehicle Height

The height observation and all vehicle-altitude computations are obtained from

$$h = r - \frac{a_e(1 - \epsilon)}{\left[1 - (2\epsilon - \epsilon^2)\frac{x^2 + y^2}{r^2}\right]^{1/2}}$$

3.8.4 Propagation Time Correction

A correction for propagation time may be made for the usual cases of radar observations wherein reported observation times are the times when pulses are sent or received at the station. This adjustment to the input observation time is

$$\Delta t = \frac{R_c}{c}$$

where

R_c = computed range

c = propagation speed with appropriate algebraic sign

In the case of height data,

$$\Delta t = \frac{h}{c}$$

3.9 RESIDUALS ANALYSIS

3.9.1 Orbit-Plane Residuals

The basic purpose of the residuals-analysis link that is presently available in the TRACE-D program is to permit resolution of residual vectors into the orbit-plane coordinate system. At this time, capability exists for resolving three types of residual vectors having the following related data types:

a. $\hat{x}, \hat{y}, \hat{z}$ (rotating earth-centered)

b. α_T, δ_T (topocentric right ascension and declination)

c. R, A, E (range, azimuth, elevation)

The vector and matrix products necessary to obtain the desired orbit-plane resolution for the $\hat{x}, \hat{y}, \hat{z}$ data types are

$$\hat{X}^T = \hat{x}, \hat{y}, \hat{z}$$

and

$$\Delta\hat{X}^T = \Delta\hat{x}, \Delta\hat{y}, \Delta\hat{z}$$

Resolution into the orbit-plane system may then be obtained by

$$\Delta(OP) = M^T A \Delta\hat{X}$$

where

$$A = \begin{bmatrix} \cos\alpha_g & -\sin\alpha_g & 0 \\ \sin\alpha_g & \cos\alpha_g & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (\alpha_g = \text{right ascension of Greenwich})$$

$$M = \begin{bmatrix} r_x & t_x & c_x \\ r_y & t_y & c_y \\ r_z & t_z & c_z \end{bmatrix} \quad (\underline{r} = X/|X|;\ \underline{c} = (X \times \dot{X})/(|X \times \dot{X}|);\ \underline{t} = \underline{c} \times \underline{r})$$

In the case of the α_T, δ_T data types, the calculated slant range R is necessary. Letting

$$S^T = \left[R,\ \alpha_T,\ \delta_T\right]$$

$$\Delta S^T = \left[\Delta R,\ \Delta\alpha_T,\ \Delta\delta_T\right]$$

wherein ΔR usually will be 0, then for resolution,

$$\Delta(OP) = M^T B \Delta S$$

where, dropping the T subscript,

$$B = \begin{bmatrix} \cos\alpha\cos\delta & -R\sin\alpha\cos\delta & -R\cos\alpha\sin\delta \\ \sin\alpha\cos\delta & R\cos\alpha\cos\delta & -R\sin\alpha\sin\delta \\ \sin\delta & 0 & R\cos\delta \end{bmatrix}$$

For the R, A, E data types, letting

$$U^T = \left[R,\ A,\ E\right]$$

$$\Delta U^T = \left[\Delta R,\ \Delta A,\ \Delta E\right]$$

then for resolution,

$$\Delta(OP) = M^T DEF \Delta U^T$$

where

$$F = \begin{bmatrix} \cos A \cos E & -R \sin A \cos E & -R \cos A \sin E \\ \sin A \cos E & R \cos A \cos E & -R \sin A \sin E \\ \sin E & 0 & R \cos E \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & \sin(\Phi^* - \phi) & \cos(\Phi^* - \phi) \\ 1 & 0 & 0 \\ 0 & \cos(\Phi^* - \phi) & -\sin(\Phi^* - \phi) \end{bmatrix}$$

(Φ^* = station geodetic latitude, ϕ = station declination)

$$D = \begin{bmatrix} \cos \alpha \cos \delta & -\sin \alpha & -\cos \alpha \sin \delta \\ \sin \alpha \cos \delta & \cos \alpha & -\sin \alpha \sin \delta \\ \sin \delta & 0 & \cos \delta \end{bmatrix}$$

3.9.2 Time Residuals

The observation-time adjustment which would cause a residual to go to zero may be estimated by

$$\Delta t_i = \frac{\Delta R_i}{\dot{R}_i}$$

where

ΔR_i = representation for any unweighted residual

$\dot{R}_i$ = time derivative of the same observation type computed at the observation time

Δt_i = time residual

The residuals analysis link automatically performs this computation for data types $\hat{x}$, $\hat{y}$, $\hat{z}$, α_T, δ_T, and R, A, E if they are entered and outputs the corresponding time residuals in units of seconds.

3.9.3 Propagation Time Correction

The propagation-time adjustment to observation times in RESIDUE is the same as in the orbit determination links (see Section 3.8.4).

3.9.4 Residual Vector Magnitude

The magnitude of the residual vector for a set of three observations is given by

$$R_T = [\xi^2 + \eta^2 + \zeta^2]^{1/2}$$

where

ξ, η, ζ = radial, in-track, and cross-track vector components

SECTION 4

TRACE-D PROGRAM STRUCTURE

4.1 GENERAL

The TRACE-D program is written in the FORTRAN-II language, which is intended to be used with the IBM 7094 FORTRAN Monitor System. The basic TRACE-D program structure consists of a series of eleven major independent links connected by the CHAIN feature of FORTRAIN-II. Each link in turn incorporates a series of large blocks, or major subroutines, each of which makes use of many smaller subroutines. TRACE-D consequently is an extremely flexible program that not only is easily expanded and modified but also whose flow of computation is easily understood.

The single factor restricting TRACE-D use to the IBM 7094 computer unit is occasional utilization of FAP, which includes the FINP-input, ARDC-1959-atmosphere, numerical-integration, and gravity subroutines.

A general TRACE-D program flow chart is presented in Figure 4-1.

4.2 PROGRAM LINKS

4.2.1 CHAIN

CHAIN is the only link that must be executed regardless of the mode of TRACE-D program usage. This link reads basic data, prints a header, sets several options to their nominal values, and computes the Julian date and the orientation of the earth.

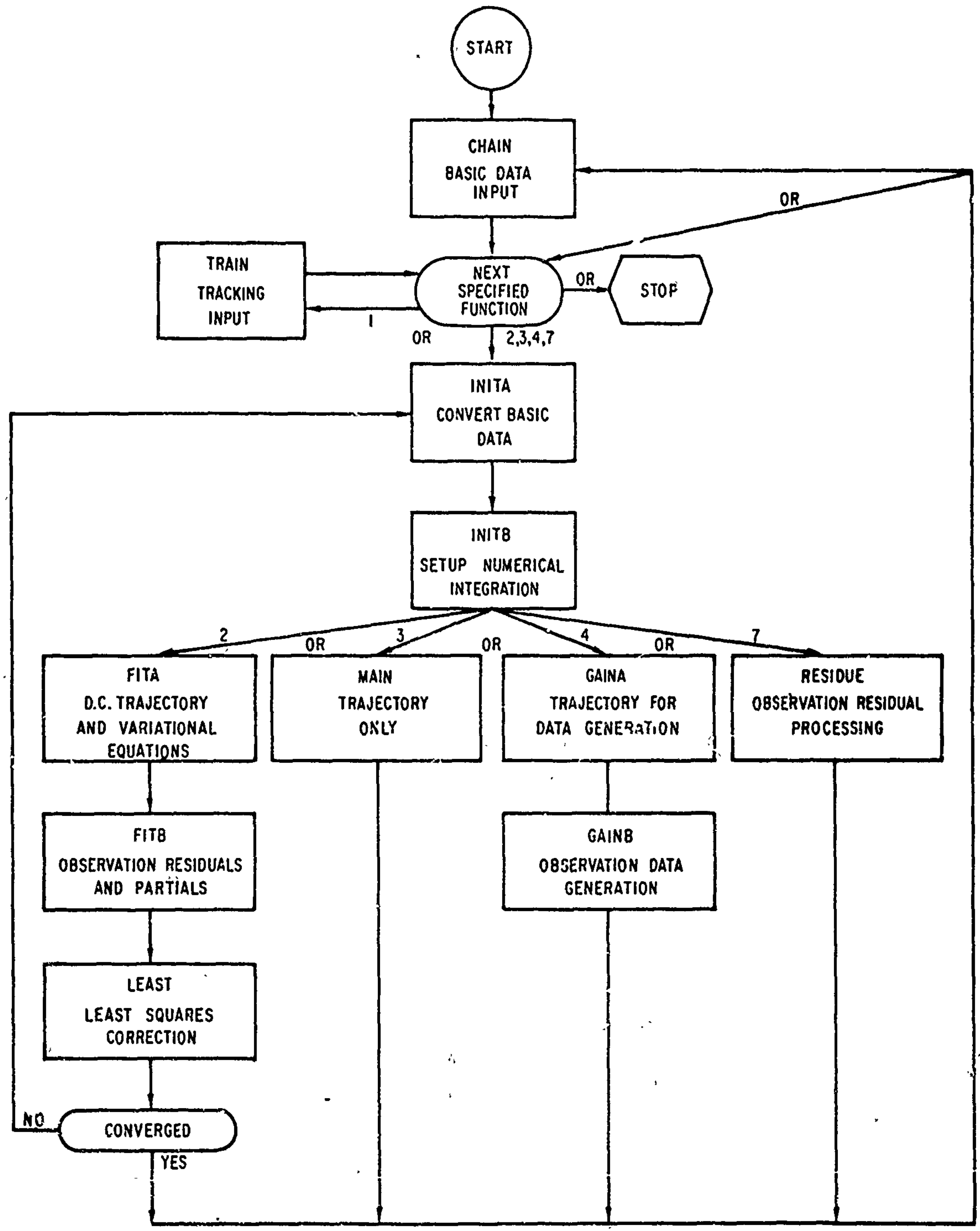


Figure 4-1. General Flow Chart for TRACE D

4.2.2 TRAIN

The TRAIN tracking input link reads radar station location and observation data, which may be on the BCD input tape produced by the IBM 1401 or on a binary tape previously written by TRAIN. The observations not only are sorted chronologically, but a compacted list of the observations is produced which eliminates storage corresponding to blanks in the information reported. In this way approximately two hundred observation times can be handled in core on a 32K machine without resorting to intermediate tapes.

The binary tape produced by TRAIN containing the sorted and compacted observation data may be used either on successive runs or for the next case of the same run. All units of this data will have been converted to an earth-radii/minutes/radians system, with observation times reduced to minutes from midnight of epoch. TRAIN also prints the tracking input and decodes and prints input concerning parameters to be solved for by differential correction.

4.2.3 INITA

The four modes of operation included in INITA are trajectory-only, orbit determination, data generation, and residuals analysis. The trajectory-only mode converts and prints the input initial conditions, sets up the print times specified by the input, and determines the parameters for which the variational equations are to be integrated. In the orbit determination mode, INITA must be entered at the beginning of each iteration. For the first iteration only, the initial conditions are converted and printed and the $n\sigma$ editor is cleared. For all iterations, the tapes are set to their correct positions and the low thrust is reset. In the data generation mode, the initial conditions are converted and printed. In the residuals analysis mode, the tapes to be used are set to the correct position.

4.2.4 INITB

INITB includes the four operational modes noted above for INITA -- trajectory-only, orbit determination, data generation, and residuals analysis.

In the trajectory-only mode, the numerical integration is initialized for the orbit and the variational equations. In the orbit determination mode, the numerical integration is initialized for the orbit and variational equations, certain radar station location quantities are computed, and the nr editor is reset, except on the first iteration. In the data generation mode, station locations and data specifications are read and the numerical integration is initialized. In the residuals analysis mode, certain radar station location quantities are computed.

4.2.5 FITA

The function of FITA is to read the compacted data tape produced by TRAIN, perform the necessary numerical integration to obtain vehicle position and velocity at the observation times, and write a trajectory tape to be used by FITB and RESIDUE.

4.2.6 FITB

The function of FITB is to read the compacted radar data tape and the trajectory tape, compute the radar residuals and partials, and accumulate the $A^T A$ to be used in LEAST.

4.2.7 LEAST

The function of LEAST is to determine and print the corrections to be made in the differential correction procedure.

4.2.8 MAIN

The function of MAIN is to integrate the orbit and variational equations for the trajectory-only option. MAIN interpolates and prints output at the times specified by the input print vector.

4.2.9 GAINA

The function of GAINA is to perform the numerical integration and to write a trajectory tape for the data generation computations that are to be accomplished in GAINB.

4.2.10 GAINB

The function of GAINB is to read the tape produced by GAINA and to compute, sort, and print the radar data observations specified.

4.2.11 RESIDUE

RESIDUE performs a variety of functions associated with error analysis of remaining residuals (i. e., residuals which remain after convergence of the least-squares fitting process). It also performs certain differencing operations which allow for comparison of ephemerides, measured observations, etc.

4.3 STORAGE MAPS

4.3.1 Tab Listings of Key Storage Area Functions

Figures 4-2 through 4-12 illustrate various TRACE-D program tab listings which describe the functions of certain key storage areas. Although these listings are of primary interest to the TRACE-D programmer, they also provide insight into the operating logic of the program and in some cases serve as a summary of input cell usage (see Appendix A for description of INTEG and C storage areas). The listings presented in Figures 4-2 through 4-12 refer to the following storage areas:

a. NUMB
b. IFLAG
c. DRAG
d. CNDT
e. DPRAM
f. KIND
g. XTRA
h. TEMP (as used for integration)
i. TEMP (as used by Subroutine FILL in Link FITB)
j. TUMP
k. PUSH
l. COMMON

It should be noted that storage area blocks within the COMMON storage area (Item l.) that are needed for all links, for fit and data generation links, and for fit links only are listed respectively in Figures 4-13. In these listings the given columns represent the symbolic designation, dimension, and decimal and octal origin of each of the storage area blocks noted.

NUMB	NUMBER OF	SET IN	INPUT NAME
1	RADAR STATIONS	TRAIN	
2	OBSERVATION TIMES OR STATIONS REQUIRING SIGHTING EPHEMERIS DATA OR STATIONS REQUIRING SIMULATION DATA	TRAIN DUM DUM	
3	WORDS IN COMPACTED RADAR OBSERVATION LIST - TOTAL	TRAIN	
4	WORDS OF COMPACTED RADAR OBSERVATIONS IN CORE NOW	TRAIN	
5	DIFFERENTIAL EQUATION PARAMETERS TO BE SOLVED FOR	CHAIN	
6	INITIAL CONDITION PARAMETERS TO BE SOLVED FOR	CHAIN	
7	RADAR STATION PARAMETERS TO BE SOLVED FOR	KING	
8	RADAR OBSERVATION PARAMETERS TO BE SOLVED FOR	KING	
9	PROGRAM TAPE UNIT	REIN -	PTAPE
10	MAXIMUM ITERATIONS ALLOWED	CHAIN-	MAXIT
11	TOTAL PARAMETERS (SUM OF 5,6,7 AND 8)	TRAIN	
12	TRAJECTORY PARAMETERS (SUM OF 5 AND 6)	TRAIN	
13	OBSERVATIONS (TOTAL NUMBER OF MEASUREMENTS)	TRAIN	
14	PRESENT ITERATION	MAIN	
15	BASIC TYPES OF OBSERVATIONS	CHAIN	
16	SIZE OF BUFFERS IN DEAN (Q DATA) - IF = 0, SET TO 5000	DEAN	
17	TOTAL RADAR PARAMETERS (SUM OF 7 AND 8)	TRAIN	
18	TAPE UNIT FOR PLANETARY COORDINATE TAPE	CHAIN-	CTAPE
19	SECOND ORDER DIFFERENTIAL EQUATIONS BEING INTEGRATED (3*(1+NUMB(12)*IFLAG(8)))	INCN	
20	POSSIBLE KINDS OF RADAR PARAMETERS (LAT, LONG, HEIGHT, AND BIASES)	CHAIN	
21	WORDS IN CORE FOR SIGHTING EPHEMERIS BUCKET	CHAIN	
22	POSSIBLE KINDS OF SIGHTING DATA	CHAIN	
23	DATA NOISE CONTROL (ZERO FOR NO NOISE, NON-ZERO STARTS RANDOM NUMBER GENERATOR FOR DATA NOISE)	CHAIN-	NOISE
24	POSITION IN ITIN LIST OF FUNCTION BEING EXECUTED	ITIN	
25	EFFECTIVE PARAMETERS BEING SOLVED FOR (NUMB(11)-(NO.OF CONSTRAINTS))	CHAIN-	KNST
26	TAPE UNIT FOR GENERATING BCD STATION AND OBSERVATION TAPE (IF ZERO, NO TAPE GENERATED)	CHAIN-	ETAPE
27	FLOCKS OF DATA	TRAIN	
28	TAPE UNIT FOR ECLIPSE COORD TAPE	CHAIN	
29	(NOT USED)		
30	ELEMENTS IN ATA	MAIN	
31	TAPE UNIT FOR NOMINAL TRAJ. FOR DIFFERENCING RUN (LOGICAL NO.) MUST BE ON CHANNEL B. IF NOT INPUT, SET = 15 IN SUBROUTINE INPAL WHEN IFLAG(15) NOT = 0.	INPAL-	NTAPE
32	TAPE UNIT FOR DIFFERENCED TRAJ. FOR DIFFERENCING RUN (LOGICAL NO.) MUST BE ON CHANNEL A FOR AUTOMATIC PLOTTING. IF NOT INPUT, SET =14, WHEN IFLAG(15) NON-ZERO.	INPAL-	DTAPE
33	MAXIMUM NUMBER OF 2ND DIFFERENCE EDITOR ITERATIONS		
34	NUMBER OF 4 COLUMN PARAMETERS		
35	NO. OF PERMANENT STATIONS - INPUT POSITIVE IF VARIABLE STATIONS ARE USED, INPUT NEGATIVE IF ALL STATIONS ARE VARIABLE STATIONS		
36	MULTIPLE SATELLITE INDICATOR	TRAIN, INITA,FITA	
37	(USED INTERNALLY)		
38	RESIDUALS ANALYSIS INTERMEDIATE TAPE (LOGICAL NUMBER) - MUST BE ON CHANNEL B.		
39	RESIDUALS ANALYSIS DATA TAPE (LOGICAL NUMBER) - MUST BE ON CHANNEL A.		
40	(USED INTERNALLY)		
41	ZERO, INDICATES DATA TYPE 7 HAS R-DOT RADAR DATA IN FIELD 1. NON-ZERO, INDICATES DATA TYPE 7 HAS DELTA-F DATA IN FIELD 1.		
42	CONVERSION CONSTANT FOR DISTANCES GENERATED IN GAIN. MUST BE INPUT = 3443.9336 NM/ER OR 6975.246 KYD/ER		
43	LOGICAL TAPE NUMBER FOR ANOTHER COMPACTED DATA TAPE TO BE GENERATED IN TRAIN FOR DESIDUALS ANALYSIS.		
44	LOGICAL TAPE NUMBER FOR TRAJECTORY TAPE TO BE USED TO OBTAIN CALCULATED OBSERVATIONS AND REFERENCE ORBIT FOR RESIDUALS - MUST BE ON CHANNEL B.		
45	LOGICAL TAPE NUMBER FOR TRAJECTORY TAPE TO BE USED TO OBTAIN MEASURED OBSERVATIONS FOR RESIDUALS ANALYSIS - MUST BE ON CHANNEL B.		
46	NON-ZERO INDICATES SPECIAL TRAJECTORY TAPE GENERATION =1, FIRST OF MULTIPLE CASES =2, OTHER THAN FIRST OR LAST CASE =3, ONLY CASE =4, LAST OF MULTIPLE CASES		TTAPE
47-50	(NOT USED)		

Figure 4-2. NUMB Storage Area

	IFLAG - OPTION INDICATORS	INPUT NAME
1	CURRENT FUNCTION BEING EXECUTED =1, TRAIN =2, TRACKING =3, TRAJECTORY =4, GAIN =7, RESIDUE	
2	RESTORE (1) LAST GOOD SOLUTION OR CORRECT (0) PRESENT SOLUTION	
3	CURRENT LINK BEING EXECUTED	
4	REASON FOR EXIT FROM MAIN (1-MAXIMUM NUMBER OF ITERATIONS, 2-CONVERGED, 3-TRAJECTORY COMPLETED)	
5	CORRECTIONS ARE HITTING BOUNDS (1) OR NOT (0)	
6	= 0, COMPLETE SIGHTING EPHEMERIS PRINTED = 1, ONLY RISE, SET, MAXIMUM ELEVATION TIMES PRINTED	
7	(USED INTERNALLY)	
8	ANALYTIC TRAJECTORY PARTIALS (0) OR VARIATIONAL EQUATIONS (1)	PARTL
9	BOUNDS PROVIDED FOR LEAST SQUARES SOLUTION (1) OR NO BOUNDS (0)	
10	(NOT USED)	
11	T-MATRIX OPTION IF=0, NO T-MATRIX =1, INPUT DRHODH*H/RHO, NO EARTH FLATTENING =2, INPUT DRHODH*H/RHO, USE EARTH FLATTENING =3, CALC. DRHODH*H/RHO, NO EARTH FLATTENING =4, CALC. DRHODH*H/RHO, USE EARTH FLATTENING	TMATX
12 13	PARAMETER SPECIFYING SEQUENCE OF FUNCTIONS TO BE PERFORMED	ITIN
14	USED IN GAIN. IF = 0, SORT OUTPUT - NON-ZERO, DO NOT SORT	
15	TRAJECTORY COMPARISON OPTION IF IDIFF =0, REGULAR TRAJECTORY =1, WRITE PRESENT TRAJECTORY AS REFERENCE ON NTAPE =2, 1ST COMPARISON,DIFFERENCES WRITTEN ON DTAPE =3, OTHER THAN FIRST OR LAST COMPARISON CASES =4, FOR ONE AND ONLY COMPARISON CASE =5, LAST COMPARISON	IDIFF
16	TAPE UNIT FOR TRAJECTORY TAPE WRITTEN BY FITA(PHYSICAL UNIT ON CHANNEL B) OR GAIN(LOGICAL UNIT)	
17	SET NON-ZERO IN TSET IF TO IS A PARAMETER	
18	NON-ZERO,ACCUMULATE ATA IN DOUBLE PRECISION (FOR ATA LESS THAN 2746)	
19	(USED INTERNALLY)	
20	IF NON-ZERO, OPERATE IN 3 PTS./PASS MODE	
21	OPTION INDICATOR FOR 2ND DIFFERENCE EDITOR, CONTROLS EDITING OF FIRST 2 AND LAST 2 POINTS	
22	=0,A AND E RESIDUALS ARE NOT INCLUDED IN RAE RESIDUALS ANALYSIS =1,INCLUDE RESIDUALS.	
23	(USED INTERNALLY)	
24	(USED INTERNALLY)	
25	SET IN TSET IF ALPHA PARTIALS ARE TO BE COMPUTED.	
26	AFTER COMPLETING FITA, CALL FITB(=0) OR DO NEXT FUNCTION(=1 OR 2)	
27	(USED INTERNALLY)	
28	(USED INTERNALLY)	
29	IF NON ZERO, V,F,U MATRICES ARE PRINTED FROM PERT (TEMPORARY CHECK OUT ONLY)	
30	IF NON ZERO, THE OLD (TRACE OF V MATRIX FORMULATION IS USED (CHECK OUT ONLY)	

Figure 4-3. IFLAG Storage Area

DRAG		INPUT NAME
1	CDA/W	
2	ATMOSPHERE OPTION, IF = 0, ARDC 1959 = 1, LOCKHEED	
3	D1 = 6.83 (LOCKHEED)	
4	D2 = -15.684 (LOCKHEED)	
5	(NOT USED)	
6	INPUT DRHODH*H/RHO FOR H BETWEEN 76 AND 108 N.MI (-8.6)	
7	INPUT DRHODH*H/RHO FOR H BETWEEN 108 AND 376 N MI (-5.55)	
8	CALCULATED DRHODH*H/RHO (SEE IFLAG(11))	
9	USED INTERNALLY TO STORE THE INTERPOLATED VALUE OF CD FROM CD TABLES	
10	IF NON-ZERO, USE DRAG TABLE. =1,DRAG TABLE VS ALT AND MACH NO =2,DRAG TABLE VS TIME	
11	CRITICAL ALTITUDE. ABOVE - USE ALT TABLE BELOW - USE MACH NO TABLE	
12-35	DRAG TABLE - CDA/W VS ALT OR TIME	
36-59	- CDA/W VS MACH NO	
60-99	RESERVED FOR LATER USE	
100	INPUT REVOLUTION NUMBER FOR ITIN = 3	REV
101-120	RESERVED FOR LATER USE	

Figure 4-4. DRAG Storage Area

CNDT - PARAMETER LIST. CODES ARE IN ITRCD		
1-6	INITIAL CONDITIONS	
7	T-ZERO	
8	CDA/W	
9	GM	
10-18	J2-J10	
19-38	J21-J66	ORDERED J21,J31,J41,...J56,J66
39-53	LAMBDA21-LAMBDA66	SAME
59	(USED INTERNALLY)	
60	OMEGASUBA - ATMOSPHERE ROTATION RATE	
61	T1 THRUST (T1*EXPF(-T2*T)	
62	T2	
68-75	ICS, TO, AND DRAG FOR SATELLITE NO. 2	
80-87	ICS, TO, AND DRAG FOR SATELLITE NO. 3	
92-99	ICS, TO, AND DRAG FOR SATELLITE NO. 4	
104-111	ICS, TO, AND DRAG FOR SATELLITE NO. 5	
116-123	ICS, TO, AND DRAG FOR SATELLITE NO. 6	

Figure 4-5. CNDT (Parameter List) Storage Area

DPRAM CODE LIST	1	2	3	4	5	6	7	8	9	10	11	12	POSITION IN CNDT LIST	INPUT NAME
DDPRAM	CDA/W	GM	J2	J3	J4	J5	J6	J7	J8	J9	J10	J21	8-19	
D3	J22	J31	J32	J33	J41	J42	J43	J44	J51	J52	J53	J54	20-31	
D5	J55	J61	J62	J63	J64	J65	J66	L21	L22	L31	L32	L33	32-43	
D7	L41	L42	L43	L44	L51	L52	L53	L54	L55	L61	L62	L63	44-55	
D9	L64	L65	L66		OA	T1	T2						56-67	
D11 *	ALPH	DELT	BETA	A	R	V	TO	CDA/W					68-79	PSAT2
D13 *	ALPH	DELT	BETA	A	R	V	TO	CDA/W					80-91	PSAT3
D15 *	ALPH	DELT	BETA	A	R	V	TO	CDA/W					92-103	PSAT4
D17 *	ALPH	DELT	BETA	A	R	V	TO	CDA/W					104-115	PSAT5
D19 *	ALPH	DELT	BETA	A	R	V	TO	CDA/W					116-127	PSAT6

* REFERS TO SATELLITES 2-6

Figure 4-6. DPRAM (Code List) Storage Area

	KIND - MISCELLANEOUS FIXED POINT INPUT	INPUT NAME
1	UNIT FOR INPUT BCD RADAR DATA TAPE	IBCDI
2	UNIT FOR INPUT BINARY COMPACTED RADAR DATA TAPE	IBINI
3	USED INTERNALLY	
4	EPOCH DATE - YEAR	YEAR
5	- MONTH	MNTH
6	- DAY	DAY
7	PRCDE - OUTPUT CONTROL	PRCDE
8	PARAMETER	
9	FLOCK FLAG	

Figure 4-7. KIND (Miscellaneous Fixed Point Input) Storage Area

	XTRA - MISCELLANEOUS FLOATING POINT INPUT	INPUT NAME
1	ZONE TIME, GMT IS TIME ZONE ZERO	TZNE
2	EPOCH TIME - FROM MIDNIGHT OF EPOCH DATE - HOUR	HR
3	- MINUTES	MIN
4	- SECONDS	SEC
10	PROXIMITY INDICATOR FOR SITE 1- GEOCENTRIC LATITUDE (DEG)	ANOM1
11	- GEOCENTRIC EAST LONGITUDE (DEG)	(2)
12	- HEIGHT (FT)	(3)
13	- PROXIMITY (N MI)	(4)
14-17	PROXIMITY INDICATOR FOR SITE 2-LAT, LONG, HT, PROX	ANOM2
18-21	PROXIMITY INDICATOR FOR SITE 3-LAT, LONG, HT, PROX	ANOM3
22-27	RESERVED FOR LATER USE	
28-50	(NOT USED)	

Figure 4-8. XTRA (Miscellaneous Floating Point Input) Storage Area

USE OF TEMP FOR INTEGRATION (DAUX AND PERT)

(1)	T	
(2)-(4)	X	
(5)-(7)	PSI(P1)	
(3K+2)-(3K+4)	PSI(PK)	
(185)	DELTA-T	
(186)-(188)	X-DOT	
(189)-(191)	PSI-DOT(P1)	
(3K+186)-(3K+188)	PSI-DOT(PK)	
(369)-(371)	X-DOUBLE DOT	
(372)-(374)	PSI-DOUBLE DOT(P1)	
(3K+369)-(3K+371)	PSI-DOUBLE DOT(PK)	
(552)		
(553)	R	
(554)	R**2	
(555)	R**3	
(556)	R**5	
(557)	R	GRAV CALLING SEQUENCE
(558)	SIN(PHI)	GRAV CALLING SEQUENCE
(559)	COS(PHI)	GRAV CALLING SEQUENCE
(560)	SIN(ALPHA(G))	GRAV CALLING SEQUENCE
(561)	COS(ALPHA(G))	GRAV CALLING SEQUENCE
(562)	SIN(ALPHA)	GRAV CALLING SEQUENCE
(563)	COS(ALPHA)	GRAV CALLING SEQUENCE
(564-566)	F(1)	GRAV CALLING SEQUENCE
(567-569)	GLH	GRAV CALLING SEQUENCE
(570-578)	ROT MATRIX	GRAV CALLING SEQUENCE
(579)	U POTENTIAL	GRAV CALLING SEQUENCE
(580-588)	A	GRAV CALLING SEQUENCE
(589-608)	B	GRAV CALLING SEQUENCE
(609-628)	C	GRAV CALLING SEQUENCE
(629-637)	D	GRAV CALLING SEQUENCE
(638-657)	E	GRAV CALLING SEQUENCE
(658-677)	SIN(M(LAMBDA-LAMBDA(MN)))	GRAV CALLING SEQUENCE
(678-697)	COS(M(LAMBDA-LAMBDA(MN)))	GRAV CALLING SEQUENCE
(698-706)	DFDX(1) V-MATRIX	
(707-715)	DFDX(2) T-MATRIX	
(716-724)	DFDXD U-MATRIX	
(725-727)	DF/DX*(PSI(PK))	
(728-730)	DF/DXDOT*(PSI-DOT(PK))	
(731-910)	DF/DB	
(916)	RHO (DENSITY)	
(917)		
(918)	H (ALTITUDE)	
(919)	A (VEL OF SOUND)	
(920)	M (MACH NUMBER)	
(921-923)	X-DOT(A)	
(924)	D/V(A) = RHO/2*V(A)*CDA/W	
(925-927)	F(3)	
(928)	ABSF(X-DOT) = V(A)	
(935-937)	F(4) THRUST FORCE	
(938-943)	ABSF(X(J))**3	
(944-949)	ABSF(X-X(J))	
(950-955)	ABSF(X-X(J))**3	
(956-961)	ABSF(X-X(J))**5	
(962-964)	X-X(1)	
(965-967)	X-X(2)	
(968-970)	X-X(3)	
(971-973)	X-X(4)	
(974-976)	X-X(5)	
(977-979)	X-X(6)	
(980-982)	F(2)	
(983-1000)	INTERPOLATED VELOCITIES OF OTHER BODIES	

Figure 4-9. TEMP Storage Area (As Used for Integration)

USE OF TEMP FOR FILL (FITB)	
(1-552)	POSITION, VELOCITY, ACCELERATION
(553)	W1
(554)	W2
(555)	W3
(556-735)	DW1DP1 THRU DW3DP60
(736)	W1DOT
(737)	W2DOT
(738)	W3DOT
(739-918)	DW1DOTDP1 THRU DW3DOTDP60
(919)	U1
(920)	U2
(921)	U3
(922)	R
(923)	V1
(924)	V2
(925)	V3
(926)	V
(927-936)	JTEMP (SET IN RADR)

Figure 4-10. TEMP Storage Area (As Used by Subroutine FILL in Link FITB)

USES OF TUMP

CELLS	ROUTINES	TYPE	USAGE	EQUIV	INPUT NAME
(1)	CSCP	TEMP			
(1-2)	TSET	TEMP	WHEN COMPUTING ELEMENTS (XY2A)		
(1-5)	CHAIN,INCN1, TRAJ3,TROUT INCN	TEMP	WHEN COMPUTING JULIAN DATE		
(51)	PTRAJ,TRAJ3	PERM	ASCENDING NODE AND SPEC PRINT FLAG	ASCFL	
(52)	MAIN,PTRAJ	PERM	REV NO (FIXED PT)	IREV	
(53)	PTRAJ,TRAJ3	PERM	NODAL CROSSING TIME	ECTIM	
(59-100)		PERM	FOR LATITUDE, LONGITUDE PRINT TABLES		
(59)	INCN,LATPR		NUMBER OF LATITUDES (FIXED PT)	NLAT	LATPR
(60-69)	INCN		LATITUDES IN DEGREES	ELAT	(2-11)
(70-79)	INCN,LATPR		LATITUDES IN RADIANS	FLAT	
(80)	INCN,LATPR		NUMBER OF LONGITUDES (FIXED PT)	NLAM	LONPR
(81-90)	INCN		LONGITUDES IN DEGREES	ELAM	(2-11)
(91-100)	INCN,LATPR		LONGITUDES IN RADIANS	FLAM	
(1-100)	FIT B	PERM	USED THROUGHOUT THE LINK		
(1-100)	RESIDUE	PERM	USED THROUGHOUT THE LINK		
(1-100)	PLAIN	PERM	USED THROUGHOUT THE LINK		

Figure 4-11. TUMP Storage Area

USES OF PUSH

CELLS	ROUTINES	USAGE	INPUT NAME
(1-5)		EXPONENTIAL THRUST	
(1)	INITA,INCN, INCN1,INPUT	T1	THRST
(2)	INITA,INCN, INCN1	T2	
(3-4)	INITA,INCN1, KTIM,TRAJF, TRAJ3,TRAJ4	START AND STOP TIMES, IN SECONDS,FROM MIDNIGHT OF EPOCH	
(5)	INITA,INCN1, PERT,TRAJF, REINT	THRUST INDICATOR, = 0 WHEN NO THRUSTING = RESULTANT OTHERWISE	
(6-19)		(NOT USED)	
(20)	*CMPR,FILL *FILLR,ORES	RANGE REFRACTION CORRECTION	RREFC
(21)	TRAJF,TRAJ3, TSET1,TSET2	FLAG SET IN TRAJF,TRAJ3 AND TESTED IN TSET1,TSET2	
(22-28)		(NOT USED)	
(29)	INCN,INCN1, KTIM,REINT, TRAJF,TRAJ3	INTERNAL COUNT OF ORBIT ADJUSTS FROM XKICK	
(30)	INCN,INCN1, INPUT,KTIM, TRAJ3	INPUT NUMBER OF ORBIT ADJUSTS (KICKS) IN XKICK - MUST BE LESS THAN 51	NXK
(31-130)	INCN,INCN1, INPUT,KTIM, REINT,TRAJF, TRAJ3	BLOCK WHERE ORBIT ADJUSTS ARE INPUT AS FOLLOWS XKICK = TIME OF KICK(1) IN SEC FROM MIDNIGHT OF EPOCH (2) = DELTA VEL(1) IN FT/SEC (3) = TIME OF KICK(2) IN SEC FROM MIDNIGHT OF EPOCH (4) = DELTA VEL(2) IN FT/SEC - - (2*NXK-1) = TIME OF KICK(NXK) (2*NXK) = DELTA VEL(NXK)	XKICK
(131-200)	TRAIN,INPUT	INITIAL CONDITIONS FOR MULTIPLE SATELLITES	
(131-144)		SATELLITE 2	SAT2
(145-158)		SATELLITE 3	SAT3
(159-172)		SATELLITE 4	SAT4
(173-186)		SATELLITE 5	SAT5
(187-200)		SATELLITE 6	SAT6
		SATJ (J=2,3,...,6) INPUT IS AS FOLLOWS ISATJ = YEAR I 2 = MONTH I 3 = DAY 4 = HOUR 5 = MINUTES 6 = SECONDS I 7 = ICTYP 8-13 = INITIAL CONDITIONS 14 = DRAG	

* PUSH(50) REFERENCE MUST BE CHANGED TO RREFC

Figure 4-12. PUSH Storage Area

COMMON LAYOUT

A COMMON A - BLOCKS NEEDED FOR ALL LINKS - 8722 CELLS

ALPH	1	EQU	32561	OCT	77461
AUS	1	-	32560	-	77460
C	100	-	32559	-	77457
CC	3,6	-	32459	-	77313
CO	55	-	32441	-	77271
CI	56	-	32385	-	77201
CNCT	160	-	32329	-	77111
CPRAM	2	-	32169	-	76651
DAVE	100	-	32167	-	76647
DPRAM	40	-	32067	-	76503
DRAG	120	-	32027	-	76433
ELL	20	-	31907	-	76243
FIC	10	-	31887	-	76217
FLEE	100	-	31877	-	76205
HEAD	24	-	31777	-	76041
IDTAPE	17	-	31753	-	76011
IFLAG	30	-	31736	-	75770
ITEMP	45	-	31706	-	75732
ITRCD	62	-	31661	-	75655
KIND	10	-	31599	-	75557
NUMB	50	-	31589	-	75545
PARINT	50	-	31539	-	75463
PKICK	50	-	31489	-	75401
PRTIM	21	-	31439	-	75317
PUSH	200	-	31418	-	75272
REFR	15	-	31218	-	74762
SUS	1	-	31203	-	74743
SUSP	1	-	31202	-	74742
TRAJX	367	-	31201	-	74741
TRJNI	552	-	30834	-	74162
TUMP	100	-	30282	-	73112
XTRA	50	-	30182	-	72746
TEMP	1000	-	30132	-	72664
TREG	5493	-	29132	-	70714

Figure 4-13. COMMON Storage Area Blocks Needed for A All Links, B Fit and Data Generation Links, and C Fit Links Only

(B) COMMON B - BLOCKS NEEDED FOR FIT AND DATA GENERATION LINKS - 4300 CELLS

GCK	1800	DEC	23639	OCT	56127
BIAS	100	-	21839	-	52517
DPAR	100	-	21739	-	52353
ICDLC	100	-	21639	-	52207
IPRCD	500	-	21539	-	52043
ISIG	100	-	21039	-	51057
PBIAS	100	-	20939	-	50713
RPRAM	200	-	20839	-	50547
SIGGY	100	-	20639	-	50237
SIGMY	100	-	20539	-	50073
STAT	11,100	-	20439	-	47727

Figure 4-13. COMMON Storage Area Blocks Needed for (A) All Links, (B) Fit and Data Generation Links, and (C) Fit Links Only (Continued)

C COMMON C - BLOCKS NEEDED FOR FIT LINKS ONLY - 711 CELLS

RLIST	300	DEC	19339	OCT	45613
BNDS	100	-	19039	-	45137
LIL	101	-	18939	-	44773
PAR	100	-	18838	-	44626
PBNDS	100	-	18738	-	44462
RAD	10	-	18638	-	44316

Figure 4-13. COMMON Storage Area Blocks Needed for A All Links, B Fit and Data Generation Links, and C Fit Links Only (Concluded)

4.3.2 Use of TREG For Integration (Subroutine COW)

If n is the total number of initial condition and differential equation parameters, then the quantities X, X_{pi}, $\dot{X}$, $\dot{X}_{pi}$, $\ddot{X}$, and $\ddot{X}_{pi}$ will be located in storage area as specified in Table 4-1.

Table 4-1. Locations of Quantities Used by Subroutine COW

Quantity	Location	
X	TREG(K) to TREG(K-2)	
X_{pi}	TREG(K-3i) to TREG(K-3i-2)	
$\dot{X}$	TREG(K-NEQ) to TREG(K-NEQ-2)	
$\dot{X}_{pi}$	TREG(K-NEQ-3i) to TREG(K-NEQ-3i-2)	
$\ddot{X}$	TREG(K-2NEQ) to TREG(K-2NEQ-2)	
$\ddot{X}_{pi}$	TREG(K-2NEQ-3i) to TREG(K-2NEQ-3i-2)	
NEQ	TREG(K+3)	(scaled at B35, set by COW)
T	TREG(K+2)	(current time)
ΔT	TREG(K+1)	(current integration step size)

Notes:

1. NEQ = 3 (n + 1) (n = total number of initial condition and differential equation parameters)

2. K = dimension of TREG - 3 (The dimension of TREG must be 90 (m + 1) + 3, where m = maximum number of initial condition and differential equation parameters.)

4.3.3 Initial-Condition and Differential-Equation Parameter Code List (ITRCD)

Table 4-2 specifies the coding whereby initial condition and differential equation parameter (ITRCD) storage locations are identified.

Table 4-2. ITRCD Storage Locations

Sequential Locations	Symbol	Description
1	IC	Initial condition parameter type (from CPRAM)
2	n	Total number of initial condition and differential equation parameters
3	P_1	Parameter Code No. 1
4	P_2	Parameter Code No. 2
5	P_3	Parameter Code No. 3
.	.	. . .
.	.	. . .
.	.	. . .
n+2	P_n	Parameter Code No. n

Notes:

1. The P_i correspond to locations in the CNDT list of parameters to be corrected in the PKICK list in the case of an orbit-adjust parameter.
2. The maximum number of initial condition and differential equation parameters is 60.

SECTION 5

USAGE

Section 5 describes the input data, data-deck arrangement, tape requirements, sense-switching controls, and output associated with TRACE-D program applications, and is intended to resolve most questions pertaining to program usage. To facilitate reference to the following information, input-data characteristics have been separately considered with respect to each of the major program functions of trajectory generation, tracking, data generation, and residuals analysis (see Section 1.3 for explanation of these functions).

5.1 INPUT DATA

Input data intended for use in connection with TRACE-D program operation fall into five categories of basic data that are common to the trajectory generation, tracking, data generation, and residuals analysis program functions and into specialized data that are applicable to each of these four functions individually. Section 5.1 presents a detailed description of the features characterizing each of these five principal input data categories.

The FINP, station-location and -identification data, observation data, and data generation specification load sheets are the four types of load sheets used for input by the TRACE-D program. Sample load sheets are included in the following discussions of applicable input data. Use of the FINP load sheets is facilitated by consideration of the following information:

a. Although the load sheet imposes an order on program input, the actual order of the cards is almost immaterial. The only restriction on card order is that cards without symbolic locations must follow the last card carrying the appropriate symbolic location. In the case where the same location symbol appears twice in succession, the last value read constitues the effective input.

b. A prefix appearing in Columns 1, 19, 37, and 55 (Fig. 5-1) determines the mode of input. A blank indicates that the following value is to be read as a floating point number, an I as a fixed point integer, a D as BCD (Hollerith) information, a B as an octal number, and M as a matrix array. The END cards are used because the prefix E terminates the FINP reading function.

c. Any card for which no value appears may be omitted. Blank fields are ignored except for the D prefix (BCD).

d. No plus signs or commas are permitted.

e. A decimal point should appear in each value unless it is an integer, in which case there must be no decimal point.

5.1.1 Basic Data

The term basic data is defined as the data that are common to all the principal TRACE-D program functions. In addition to a list of the functions to be performed, the required basic data also include the specification of the trajectory (date, time, and initial conditions), the force model assumed, and the constants and parameters to be used in the trajectory integration. Although the force model and the trajectory-integration constants and parameters are "required" data, standard values are provided, so that replacement of these quantities thus is "optional" (see Appendix A for a list of the standard values). Certain options common to all functions relating to identification information, specification of the ballistic (drag) coefficient and the atmosphere model, selection of other-body perturbations, low-level thrusting, and orbit adjusts also are contained in the basic input.

Sections 5.1.1.1 and 5.1.1.2 present a line-by-line explanation of the TRACE-D basic data load sheet shown in Figure 5-1.

5.1.1.1 Required Input

Line 1: Functions to be Performed

1	■	D	ITIN	1 2 4	

Line 1 contains the ordered list of all functions to be performed by TRACE-D during a given run. Selection of functions is governed in accordance with the

X-3 7090 INPUT DATA TRACE-D Basic Input AEROSPACE CORPORATION COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ____ KEYPUNCHED ____ VERIFIED ____ DATE ____ PAGE ____ OF ____

1	7	73
H1		
H2		

SYMBOL	M	LOC.	VALUE	EXP.
1	D	ITIN		
2	I	YEAR		
3	I	MNTH		
4	I	DAY		
5		TZNE		
6		HR		
7		MIN		
8		SEC		
9	I	ICTYP		
10		IC		
11		2		
12		3		
13		4		
14		5		
15		6		
16		DRAG		
17	I	2		
18		3		
19		4		
20		6		
21		7		
22	I	TMATX		
23	I	CTAPE		
24	D	PRCDE		
25		THRST		
26		2		
27		3		
28		4		
29		PKICK		
30		2		
31		3		
32		4		
33	I	NXK		
34		XKICK		
35		2		
36		3		
37		4		
38		5		
39		6		
40		DALFG		

AEROSPACE FORM 3717-A

Input Description

H1, H2 Headings which will appear on output
ITIN Itinerary
- 1, 2: Tracking
- 3: Trajectory
- 4: Data Generation
- 7: Residuals Analysis

YEAR/MNTH/DAY/HR/MIN/SEC/TZNE Epoch date and time

ICTYP Type of initial conditions
- 1: $x, y, z, \dot{x}, \dot{y}, \dot{z}$
- 2: $\alpha, \delta, \beta, A, r, V$
- 3: $a, e, i, \Omega, \omega, \tau$
- 4: $\lambda, \delta, \beta, A, r, V$
- 5: Last trajectory point of previous case
- 6: Predicted solution of previous fit case
- 8: Same as CPRAM, but machine units
- 9: Same as 1, but machine units
- 0: Self-initialize

IC Initial-condition input
DRAG C_DA/W (ft^2/lb)
DRAG(2) Atmosphere option*
- 0: ARDC 1959 model
- 1: Lockheed-Jacchia (L-J) model

DRAG(3) d_1 for L-J model*
DRAG(4) d_2 for L-J model*
DRAG(6) Density slope for L-J model, low-alt.*
DRAG(7) Density slope for L-J model, high-alt.*
TMATX T-matrix options*
CTAPE Other body perturbations
PRCDE Print options
- 1: Ephemeris
- 2: Residuals
- 3: Partials
- 4: ATA
- 5: Variational equations
- 6: Elements
- 7: Suppress station print
- 9: Special prints
- 10: Elements at ascending nodes only
- 12: Print at ascending nodes only

(Boxes 8 & 11 not used)

THRST T_1 (amplitude) (ft/sec^2)
THRST(2) T_2 (time constant) (min^{-1})
THRST(3) Start time (sec from midnight of epoch date)
THRST(4) Stop time (sec from midnight of epoch date)
PKICK Time (sec from midnight of epoch date)
PKICK(2) ΔV (ft/sec)
PKICK(3) θ_y (deg)
PKICK(4) θ_p (deg)
NXK Number of extra kicks
XKICK Time, ΔV, time, ΔV . . .
DALFG True equinox correction (deg)

*Input values included in standard constants deck

Figure 5-1. TRACE-D Basic Input Data Load Sheet

Code Numbers 1, 2, 3, 4, or 7, corresponding respectively to tracking data input, tracking computations, trajectory only, data generation, and residuals analysis.

Up to twelve functions may be selected. When this sequence is exhausted, TRACE-D will reset certain standard options, prepare to run another sequence of functions, read basic data, or stop if none is present. The foregoing example would be for a tracking case (Functions 1 and 2) followed by a data generation case (Function 4).

Lines 2 through 8: Epoch

2	I	YEAR	1963	
3	I	MNTH	6	
4	I	DAY	15	
5		TZNE	0	
6		HR	12	
7		MIN	45	
8		SEC	15.5	

In the usual case, wherein the year, month, and day are input with the year positive, the X-axis is directed to the vernal equinox (see Section 3.1.1). Alternatively, if the year is input negative, the X-axis would be directed to the longitude of Greenwich. The hour, minute, and second entries refer to midnight of zone time. GMT is Time Zone 0.

Lines 9 through 15: Initial Conditions

9	I	ICTYP	2	
10		IC	126.1	
11		2	31.23	
12		3	89.	
13		4	14.	
14		5	22600114	
15		6	25117.3	

Line 9 indicates which of the ten IC types (1, 2, ... 9, 0) are entered in Lines 10 through 15. The alternative ICTYP entries are characterized as follows:

a.	IC Type 1	Earth-centered inertial cartesian coordinates (x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ in units of feet and feet per second) (see Section 3. 1. 1. 1).
b.	IC Type 2	Spherical coordinates ($\alpha, \delta, \beta, A, r, v$) in units of degrees, feet, and feet per second (see Section 3. 1. 1. 2). In Line 14, negative r is interpreted as height above the earth's surface in feet. In Line 15, if v is negative, circular velocity is computed and used.
c.	IC Type 3	Orbital elements (a, e, i, Ω, ω, τ) in units of feet, degrees, and minutes (see Section 3. 1. 1. 3).
d.	IC Type 4	Same as Item b. above, with longitude λ replacing right ascension α.
e.	IC Type 5	No IC's input. The last trajectory point of the immediately preceding case is used.
f.	IC Type 6	No IC's input. The corrected initial conditions from the last previous tracking run are used.
g.	IC Type 8	Either Type 1 or 2 above, but in units of earth radii, minutes, and radians. Type number is entered at CPRAM (see Section 5. 1. 2. 2).
h.	IC Type 9	Same as Type 1 above, but in units of earth radii and earth radii perminute.
i.	IC Type 0	No IC's input. For a tracking run, two R, A, E sets are used from the data to calculate a set of initial conditions (see Section 3. 6. 6).

5.1.1.2 Optional Input

Lines 16 through 22: DRAG and T Matrix

16		DRAG	.01	
17	I	2	1	
18		3		
19		4		
20		6	-8.6	
21		7	-5.55	
22	I	TMATX	4	

Line 16 contains the drag parameter $C_D A/W$ in square feet per pound and Line 17 contains the atmosphere model specification (ARDC 59 or Lockheed/ Jacchia). The ARDC model will be used[3] when DRAG (2) is 0 and the Lockheed/Jacchia model when DRAG (2) is 1. If line 17 is a 1, Lines 18 and 19 contain quantities used in the Lockheed/Jacchia model.

Lines 18 and 19 contain d_1 and d_2, respectively. (See Appendix C.) These are values of certain constants in the Lockheed-Jacchia atmospheric density expressions.

An entry in Line 22, TMATX, will cause the T matrix to be used in the variational equations in accordance with the following options:

a.	TMATX = 0	T matrix is not used.
b.	TMATX = 1	Input $\partial\rho/\partial h$ is used with no earth flattening.
c.	TMATX = 2	Input $\partial\rho/\partial h$ is used with earth flattening.
d.	TMATX = 3	$\partial\rho/\partial h$ is calculated with no earth flattening.
e.	TMATX = 4	$\partial\rho/\partial h$ is calculated with earth flattening.

Also, with TMATX non-zero, Lines 20 and 21 should contain input values for $\partial\rho/\partial h$. Lines 20 and 21 contain $\partial\rho/\partial h$ for altitudes between 76 and 108 n mi and between 108 and 376, respectively, (see Appendix C). It should be noted that input values for $\partial\rho/\partial h$ must be used with the ARDC 1959 model.

[3]The notation LLLLL(n) used in this section specifies storage location within an array. The LLLLL portion of the symbol is associated with the first cell in the array and the (n) indicates the n^{th} cell (i.e., Location LLLLL + n - 1).

If a variable C_D term is desired, the drag table option may be utilized. In this case the drag parameter may be considered to consist of the product of two terms, $(C_D A/W) \times C'_D$, wherein $C_D A/W$ is a constant which can be differentially corrected by the use of the variational equation and C'_D may be considered a function of Mach number below a certain altitude and as a function of altitude at points above that altitude. Alternatively, C'_D may be considered a function of time. In either case, use of the drag table is necessary.

When the drag table is not used, input of $C_D A/W$ into the DRAG location does not change usual TRACE operation. In this case C'_D is automatically set equal to 1. If the drag table is used, the following additional inputs are required:

a.	DRAG(10)	If DRAG(10) = 0, tables not used. If DRAG(10) = 1, Mach and altitude tables used. If DRAG(10) = 2, time table only used.
b.	DRAG(11)	Altitude above which altitude table is used and below which Mach table is used (needed if (DRAG(10) = 1).
c.	DRAG(12) = 0	Used by interpolation routine.
	DRAG(13) through (35)	Altitude table, or time table if DRAG(10) = 2 $(h_1, C'_D(h_1), h_2, C'_D(h_2) \; . \; . \; . \; . \; , h_n, C'_D(h_n), 0, 0$ or $t_1, C'_D(t_1), t_2, C'_D(t_2), \; . \; . \; . \; . \; , t_n, C'_D(t_n), 0, 0)$.
d.	DRAG(36) = 0	Used by interpolation routine.
	DRAG(37) through (59)	Mach table (stored as noted in Item c. above).

Line 23: Other-Body Perturbations

23		I	CTAPE	7

If perturbations due to other bodies in the solar system are to be included in the trajectory calculations, a planetary coordinate tape must be mounted and the logical-tape unit number must be entered at CTAPE.

Line 24: Print Code

24	D	PRCDE	1	2	3	4	5	6	7	8	9	10	11	12

The print-code entry consists of two BCD words accommodating six character positions each. Entry of an X at any of these twelve positions will initiate corresponding outputs in accordance with the following:

a. (1) Trajectory (trajectory only option)

b. (2) Residuals (tracking only option)

c. (3) Partials (tracking only option)

d. (4) A^TA after each iteration (tracking only option)

e. (5) Variational equations (trajectory only option)

f. (6) Orbital elements (trajectory only option)

g. (7) Do not print station locations (tracking and data generation)

h. (8) Not used

i. (9) Special trajectory prints[4]

j. (10) Orbital elements at ascending nodes only (trajectory only option)

k. (11) Not used

l. (12) Suppress all trajectory print except ascending nodes (trajectory only option)[5]

Lines 25 through 28: Exponential Thrust

25		THRST		
26		2		
27		3		
28		4		

If an exponential thrust is to be used, the quantities T_1 in units of force/ mass = ft/sec^2, T_2 in units of min^{-1}, and t_s and t_f in seconds from midnight

[4]Prints will occur at times of maximum and minimum altitude above the oblate earth, at times when the flight-path angle equals 90 degrees and at special latitudes and longitudes if values are entered (see Section 5.1.2.2).

[5]If the option to write a binary trajectory tape (B7) has been selected, the writing of that tape is controlled by the PRTIM entries (see Section 5.1.2.1).

of epoch date must be input at THRST, THRST(2), THRST(3), and THRST(4) locations, respectively.

Lines 29 through 32: Instantaneous Orbit Adjusts

29		PKICK	89020.31	
30		2	200.	
31		3	0	
32		4	62.	

Instantaneous orbit adjusts are input at PKICK. Line 29 contains t_1 (time of first orbit adjust, OA_1) in seconds from midnight of epoch, Line 30 contains K (magnitude of velocity change of OA_1) in feet per second, Line 31 contains θ_y (yaw angle for OA_1) in degrees, and Line 32 contains θ_p (pitch angle for OA_1) in degrees.

The TRACE-D program will accommodate up to six orbit adjusts. Additional cards may be added as necessary for OA_2 through OA_6.

Lines 33 through 39: Extra Kicks

33	I	NXK	3	
34		XKICK	63456	
35		2	.2	
36		3	88721	
37		4	10.	
38		5	101018	
39		6	.02	

Up to fifty fixed orbit adjusts (i.e., instantaneous changes of the in-track velocity component) may be input at XKICK. It should be noted that these orbit adjusts are not parameters for differential correction, but are applied in the equations of motion only and are independent of the PKICK inputs. The number of extra kicks (≤ 50) is input at NXK, and the table of times and ΔV values is input beginning at XKICK. The format is time, ΔV, time, ΔV, etc., in units of seconds from midnight of epoch day and feet per second, respectively.

Line 40: Equinox Precession Corrections

40		DALFG	.0002	

The rotational position of the earth with respect to the inertial system is characterized by the right ascension of Greenwich at midnight on the day of epoch (α_g) and is computed with respect to the mean equinox of epoch date. An additional factor for correcting to true equinox of epoch date optionally may be input at DALFG in units of degrees.

5.1.2 Trajectory Only

Since for purposes of this document the terms "trajectory" and "ephemeris" are essentially equivalent, these designations will be used interchangeably, with the selection in a particular context usually historically motivated. A similar situation exists for the terms "tracking" and "orbit determination."

Detailed description of the TRACE-D trajectory input load sheet shown in Figure 5-2 is presented in Sections 5.1.2.1 and 5.1.2.2.

5.1.2.1 Required Input

Lines 1 through 7: Print Time Vector

	1	I	PRTIM		
n	2	I	2		
t_0	3		3		
Δt_1	4		4		
T_1	5		5		
Δt_2	6		6		
T_2	7		7		

The above sequence of print times is for outputs selected by PRCDE entries (Line 24, Fig. 5.1). As many as nine sets of print intervals may exist (Line 2). In the case of the i^{th} set, output is from t_{i-1} to t_i at intervals of Δt_i, with all times in minutes from midnight of epoch date if PRTIM = 1 or from epoch if PRTIM = 0. Additional cards may be inserted if $3 \leq n \leq 9$. It should be noted that a normal print at epoch is automatic.

X-3 7090 INPUT DATA

TRACE-D
Trajectory Input

AEROSPACE CORPORATION
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PROGRAMMER ________ KEYPUNCHED ________ VERIFIED ________ DATE ________ PAGE ____ OF ____

SYMBOL		N_s	LOC.	VALUE	EXP.
	1	I	PRTIM		
n	2	I	2		
t_o	3		3		
Δt_1	4		4		
T_1	5		5		
Δt_2	6		6		
T_2	7		7		
	8	D	CPRAM		
	9	D	DPRAM		
	10	D	3		
	11	D	5		
	12	D	7		
	13	D	9		
	14	D	KPRAM		
	15	D	3		
	16	I	IDIFF		
	17	I	NTAPE		
	18	I	DTAPE		
	19		REV		
	20	I	TTAPE		
n_1	21	I	LATPR		
	22		2		
	23		3		
	24		4		
n_2	25	I	LØNPR		
	26		2		
	27		3		
	28		4		

SPACE FORM 3717-B

Input Description

PRTIM Table of print intervals
- Line 1: If entry = 0, times given are from epoch. If entry = 1, times given are from midnite of epoch.
- Line 2: Number of different Δt intervals ($n \leq 9$)
- Line 3: Printing start time (epoch print is automatic)
- Line 4: Δt_1 (trajectory information printed every Δt_1 minutes until T_1)
- Line 5: T_1 (min)
- Line 6: Δt_2 (trajectory information printed after T_1 every Δt_2 minutes until T_2)
- Line 7: T_2 (min)

CPRAM/ DPRAM/ KPRAM: Integration of variational equations corresponding to boxes with X entries.

IDIFF: Trajectory differencing indicator
- 1: Nominal trajectory for differencing
- 2: First of multiple differencing cases
- 3: 2^{nd} through ($n-1^{st}$) of multiple differencing cases
- 4: First and only differencing case
- 5: Last (n^{th}) of multiple differencing cases

NTAPE: Nominal trajectory tape (logical unit = (15) entered)

DTAPE: Difference tape (logical unit = (14) entered)

REV: Initial revolution number

TTAPE: Standard-format trajectory tape (B7).
- 1: First of multiple cases
- 2: 2^{nd} through ($n-1^{st}$) of multiple cases
- 3: Only case
- 4: Last (n^{th}) of multiple cases

LATPR/ LØNPR: Latitudes and longitudes where special prints should occur ($\leq$10 entries each). Number of special latitudes entered at LATPR, followed by the latitude values. Number of special longitudes entered at LØNPR, followed by the longitude values.

Figure 5-2. TRACE-D Trajectory Input Load Sheet

5.1.2.2 Optional Input

Lines 8 through 15: Variational-Equation Partial Derivatives

8	D	CPRAM
9	D	DPRAM
10	D	3
11	D	5
12	D	7
13	D	9
14	D	KPRAM
15	D	3

An X entered in any CPRAM, DPRAM, or KPRAM character position causes the corresponding variational equation to be solved. Printout of the partial derivatives will occur only if an X is entered at Character Position 5 in the PRCDE print code entry location. The ordering of entries in the CPRAM, DPRAM, and KPRAM character position boxes is as follows:

a. CPRAM (Initial Condition Parameters) (Line 8)

The first position specifies which one of three types of initial conditions is applicable, and succeeding positions indicate the particular parameters that are desired in each case. Ordering of CPRAM parameter entries for initial condition (IC) Types 1, 2, and 3 is shown in Figure 5-3.

	(IC Type)											
(either)	1	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	t_o				
(or)	2	α	δ	β	A	r	v	t_o				
(or)	3	a	e	i	Ω	ω	τ	t_o				

Figure 5-3. Ordering of CPRAM IC Parameter Entries

b. DPRAM and KPRAM (Differential Equation Parameters) (Lines 9 through 15)

The ordering of DPRAM and KPRAM parameter entries is shown in Figure 5-4, wherein T_1 and T_2 are the exponential thrust parameters and a_i, K_i, θ_{y_i}, and θ_{p_i} are the OA number (1, 2, . . . 6), ΔV magnitude, yaw angle, and pitch angle, respectively.

Sixty differential equation parameters is the maximum number which may be selected for any one run.

DPRAM	Drag	μ	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{21}
3	J_{22}	J_{31}	J_{32}	J_{33}	J_{41}	J_{42}	J_{43}	J_{44}	J_{51}	J_{52}	J_{53}	J_{54}
5	J_{55}	J_{61}	J_{62}	J_{63}	J_{64}	J_{65}	J_{66}	λ_{21}	λ_{22}	λ_{31}	λ_{32}	λ_{33}
7	λ_{41}	λ_{42}	λ_{43}	λ_{44}	λ_{51}	λ_{52}	λ_{53}	λ_{54}	λ_{55}	λ_{61}	λ_{62}	λ_{63}
9	λ_{64}	λ_{65}	λ_{66}		ω_a	T_1	T_2					

KPRAM	a_1	K_1	θ_{y_1}	θ_{p_1}	a_2	K_2	θ_{y_2}	θ_{p_2}	a_3	K_3	θ_{y_3}	θ_{p_3}
3	a_4	K_4	θ_{y_4}	θ_{p_4}	a_5	K_5	θ_{y_5}	θ_{p_5}	a_6	K_6	θ_{y_6}	θ_{p_6}

Figure 5-4. Ordering of DPRAM and KPRAM Differential Equation Parameter Entries

Lines 16 through 18: Trajectory Comparison Options

16	I	IDIFF	1	
17	I	NTAPE	15	
18	I	DTAPE	14	

Tape units and case indicators required for the trajectory differencing function are as follows:

a. IDIFF = 0 — A regular trajectory run is indicated.

b. IDIFF = 1 — The reference trajectory will be written on the logical tape specified by NTAPE. If no entry is input at NTAPE, Logical Tape 15 will be used.

c. IDIFF = 2 — The differences between the present and reference cases are computed and written on the logical tape specified by DTAPE. If no entry is input at DTAPE, Logical Tape 14 will be used. The difference tape specified by DTAPE is rewound at the beginning of the case.

d. IDIFF = 3 — Conditions are the same as when IDIFF = 2 except that the tape specified by DTAPE is not rewound.

e. IDIFF = 4 — The tape specified by DTAPE is rewound at the beginning of the case and unloaded upon completion.

f. IDIFF = 5 — The tape specified by DTAPE is unloaded upon completion of the case.

The significance of the foregoing options is that if a single-comparison case is to be processed, IDIFF = 1 is used for the reference case and IDIFF = 4 for the perturbed case. If a series of perturbed cases are to be processed, IDIFF = 1 is used for the reference case, IDIFF = 2 for the first perturbed case, IDIFF = 3 for all intermediate cases, and IDIFF = 5 for the last perturbed case.

Note that the tapes generated by this option cannot be used in the residuals analysis function.

Line 19: Revolution Number

19		REV	7	

If an initial value other than zero is desired for the revolution number, it may be input at the REV location. This value must be reinitialized for each individual case.

Line 20: Trajectory Tape Generation

20	I	TTAPE	3	

If TTAPE is non-zero, a binary trajectory tape will be generated on Logical Tape 15 (physical tape B7) in accordance with the following input options:

a. TTAPE = 0 Tape will not be generated.

b. TTAPE = 1 Tape will be rewound before generating but not unloaded after completion. This entry should be used for the first case when more than one case is involved.

c. TTAPE = 2 Tape will not be rewound before generating and not unloaded after completion. This entry should be used for all intermediate cases.

d. TTAPE = 3 Tape will be rewound before generating and unloaded after completion. This entry should be used when only one case is involved.

e. TTAPE = 4 Tape will not be rewound before generating but will be unloaded after completion. This entry should be used for the last case.

The format of this tape is appropriate for use by the residuals analysis function for differencing trajectories (see Appendix E). However, this tape is not suitable for the trajectory-comparison option of the trajectory-only function.

Lines 21 through 28: Latitude and/or Longitude Prints

n_1	21	I	LATPR	3	
	22		2	10.	
	23		3	15.	
	24		4	20.	
n_2	25	I	LONPR	3	
	26		2	200.	
	27		3	100.	
	28		4	180.	

Line 21 contains n_1 ($n_1 \leq 10$), or the number of special latitudes at which trajectory prints are requested, and Lines 22 through 24 contain the special latitudes. Additional cards may be added if $4 \leq n_1 \leq 10$.

Line 25 contains n_2 ($n_2 \leq 10$), or the number of special longitudes at which trajectory prints are requested, and Lines 26 through 28 contain the special longitudes. Additional cards may be added if $4 \leq n_2 \leq 10$.

Note that an X must be entered in Character Position 9 of the PRCDE entry if either of the foregoing options are selected.

5.1.3 Tracking

Detailed description of the TRACE-D tracking input load sheet shown in Figure 5-5 is presented in Sections 5.1.3.1 and 5.1.3.2.

5.1.3.1 Required Input

A tracking run for the purpose of obtaining residuals only with respect to a known orbit would require only the basic data plus station and observation cards. A more typical tracking run, which would involve differential correction of some number of parameters, would require inputs for parameter specification, bounds, sigmas, maximum number of iterations, station cards, and observations as noted in this Section.

5.1.3.2 Optional Input

Lines 1 through 14: Initial Conditions for Satellite 2

Year 1	I	SAT 2	1964	
Month 2	I	2	2	
Day 3	I	3	10	
Hour 4		4	3.	
Minute 5		5	30.	
Second 6		6	52.	
ICTYP 7	I	7	2	
IC 8		8	352.	
2 9		9	10.	
3 10		10	90.05	
4 11		11	165.	
5 12		12	22580632.	
6 13		13	25205.3	
DRAG 14		14	.015	

If observations for a second satellite are to be input, Lines 1 through 14 are used for entry of epoch, initial conditions, and drag coefficient for Satellite 2. Lines 1 through 3 contain the year, month, and day, and Lines 4 through 6 contain the hour, minute, and second, Greenwich time. Line 7 indicates the type of initial conditions that may be entered in Lines 8 through 13 (Type 1, 2, 3, 4, 8, or 9). Line 14 contains the drag coefficient (C_DA/W).

X-3 7090 INPUT DATA TRACE-D Tracking Input (1)

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PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ______ OF ______

SYMBOL	R	LOC.	VALUE	EXP
Year 1	I	SAT2		
Month 2	I	2		
Day 3	I	3		
Hour 4		4		
Min. 5		5		
Sec 6		6		
ICTYP 7	I	7		
IC 8		8		
2 9		9		
3 10		10		
4 11		11		
5 12		12		
6 13		13		
DRAG 14		14		
15	D	CPRAM		
16	D	DPRAM		
17	D	3		
18	D	5		
19	D	7		
20	D	9		
21	D	PSAT2		
22	D	PSAT3		
23	D	PSAT4		
24	D	PSAT5		
25	D	PSAT6		
26	D	KPRAM		
27	D	3		
28		BNDS		
29		2		
30		3		
31		4		
32		5		
33		6		
34		7		
35		8		
36		9		
37		10		
38		11		
39		12		
40		13		

AEROSPACE FORM 3717-C (1)

Input Description

SAT2 Epoch, initial conditions, and drag for Satellite 2.

CPRAM Initial-condition parameters for Satellite 1.

1	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	t_o				
2	α	δ	β	A	r	v	t_o				
3	a	e	i	Ω	ω	τ	t_o				

DPRAM Differential equation parameters

DPRAM	D	μ	20	30	40	50	60	70	80	90	100	21
3	22	31	32	33	41	42	43	44	51	52	53	54
5	55	61	62	63	64	65	66	21	22	31	32	33
7	41	42	43	44	51	52	53	54	55	61	62	63
9	64	65	66		ω_a	T_1	T_2					

D: Drag for Satellite 1. Numbers are degree and order of zonal coefficients, tesseral coefficients, and tesseral arguments.

ω_a: Atmosphere rotation rate

T_1: Thrust amplitude

T_2: Thrust time constant

PSAT2/
PSAT6

PSATX	α	δ	β	A	r	v	t_o	D				

(Note: Only ADBARV and drag parameters are available for Satellites 2-6)

KPRAM Orbit-adjust parameters

KPRAM	a_1	b_1	c_1	d_1	a_2	b_2	c_2	d_2	a_3	b_3	c_3	d_3
3	a_4	b_4	c_4	d_4	a_5	b_5	c_5	d_5	a_6	b_6	c_6	d_6

a: Orbit-adjust number (≤ 6)

b: ΔV magnitude (K)

c: Yaw angle

d: Pitch angle

BNDS Differential correction bounds (entered in same sequence as parameters above)

Figure 5-5. TRACE-D Tracking Input Load Sheet

PROGRAMMER ________ KEYPUNCHED ________ VERIFIED ________ DATE ________ PAGE ____ OF ____

SYMBOL		LOC.	VALUE	EXP.
41	M	RAPAR	05, 99	
42	D	01, 01		
43		03, 01		
44		04, 01		
45	D	01, 02		
46		03, 02		
47		04, 02		
48	D	01, 03		
49		03, 03		
50		04, 03		
51		SIGMA		
52		2		
53		3		
54		4		
55		5		
56		6		
57	I	ISIG		
58	I	2		
59	I	3		
60	I	4		
61	I	5		
62	I	6		
63	I	MAXIT		
64		CEDIT		
65		2		
66		3		
67		REFR		
68		2		
69		RREFC		
70		SLT		
71	I	IBCDI		
72	I	IBINI		
73		DALFG		
74		2		

AEROSPACE FORM 3717-C (2)

Input Description

RAPAR Radar parameter array

	1	2	3	4	5	6	7	8	9	0		
01, P												
03, P												
04, P												

P	Radar parameter number ($1 \le P \le 99$)
Line 1	Boxes 1-2: Station ID Boxes 3-4: Pass ID Boxes 5-10: Parameter Designation
Line 2	Bound
Line 3	Estimate
SIGMA	Observation sigmas (weighting factors)
ISIG	Table of codes defining use of SIGMA table entries. For each SIGMA entry quantity (100I + K), where I = sigma index (Column 5 of station card) and K = data type, entered in corresponding location of ISIG table.
MAXIT	Maximum number of iterations
CEDIT	Residuals editing (standard entry = 3)
REFR	Table of refraction indices (standard value built in at REFR)
RREFC	Range refraction correction (0 = No, 13 = Yes)
SLT	Velocity for propagation-time correction (E.R./min)
IBCDI	BCD data-tape input (0 = No, 6 = Yes)
IBINI	Compacted data tape input (0 = No, 1 = Yes)
DALFG	Table of additive corrections to initial right ascension of Greenwich (one entry per vehicle) (deg)

Figure 5-5. TRACE-D Tracking Input Load Sheet (Continued)

X-3 7090 INPUT DATA TRACE-D Tracking Input (3) AEROSPACE CORPORATION COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ___ OF ___

SYMBOL		$^{\eta}_{E}$	LOC	VALUE	EXP
ϕ^c	75		ANOM1		
λ	76		2		
h	77		3		
R_o	78		4		
	79		NUMB		
	80	I	35		
	81	I	KNST		
	82	I	BLIST		
	83	I			
	84				
	85	I			
	86	I			
	87				
	88	I			
	89	I			
	90				
	91	I			
	92	I			
	93				
	94	I			
	95	I			
	96				
	97	I			
	98	I			
	99				
	100	I			
	101	I			
	102				

AEROSPACE FORM 37-17-C (3)

Input Description

ANØM 1 Proximity testing
Line 75 Latitude
Line 76 Longitude
Line 77 Altitude
Line 78 Testing distance
NUMB(35) Number of permanent stations (if temporary stations are present)
KNST Number of unconstrained parameters (if constraint matrix is used)
BLIST Constraint matrix (general format is i, j, b_{ij})

Figure 5-5. TRACE-D Tracking Input Load Sheet (Concluded)

The input initial conditions (IC) for Satellites 3 to 6 are the same as those noted above except that the symbol SAT2 is replaced by SAT3, SAT4, SAT5, and SAT6 as appropriate.

Lines 15 through 27: Initial-Condition and Differential Equation Parameter Specification Boxes

15	D	CPRAM	2	x	x	x	x	x	x					
16	D	DPRAM	x											
17	D	3												
18	D	5												
19	D	7												
20	D	9												
21	D	PSAT2												
22	D	PSAT3												
23	D	PSAT4												
24	D	PSAT5												
25	D	PSAT6												
26	D	KPRAM												
27	D	3												

An X entered in any CPRAM, DPRAM, PSAT, or KPRAM character position causes the corresponding parameter to be differentially corrected. The ordering of entries in the CPRAM, DPRAM, PSAT, and KPRAM character position boxes is as follows:

a. CPRAM (Initial Condition Parameters for Satellite 1) (Line 15)

   The first position specifies which one of these types of initial conditions is applicable, and succeeding positions indicate the particular parameters that are desired in each case. Ordering of parameter entries for IC Types 1, 2, and 3 is as previously shown in Figure 5-3.

b. DPRAM (Differential Equation Parameters) (Lines 16 through 20)

   The ordering of DPRAM differential equation parameter entries is as previously shown in Figure 5-4. However, in this application the T_1 and T_2 parameters are associated with Satellite 1 only.

c. PSAT2 through PSAT6 (Initial Condition and Drag Coefficient Parameters for Satellites 2 - 6) (Lines 21 through 25)

The ordering of PSAT IC and drag parameter entries for Satellites 2 through 6 is shown in Figure 5-6.

PSAT2	α_2	δ_2	β_2	A_2	r_2	v_2	t_{o_2}	$Drag_2$
PSAT3	α_3	δ_3	β_3	A_3	r_3	v_3	t_{o_3}	$Drag_3$
PSAT4	α_4	δ_4	β_4	A_4	r_4	v_4	t_{o_4}	$Drag_4$
PSAT5	α_5	δ_5	β_5	A_5	r_5	v_5	t_{o_5}	$Drag_5$
PSAT6	α_6	δ_6	β_6	A_6	r_6	v_6	t_{o_6}	$Drag_6$

Figure 5-6. Ordering of PSAT Initial Condition/ Drag Parameter Entries

It is important to note that only IC Type 2 may be specified for Satellites 2 through 6.

d. KPRAM (Orbit Adjust Parameters for Satellite 1) (Lines 26 and 27)

Ordering of KPRAM orbit adjust parameters is as previously shown in Figure 5-4. It should be noted that the orbit adjusts are for Satellite 1 only.

Sixty trajectory parameters is the maximum number which may be selected for simultaneous solution.

Lines 28 through 40: Bounds

28			BNDS	.5	
29			2	.5	
30			3	.1	
31			4	.5	
32			5	1000.	
33			6	5	
34			7	.05	
35			8		
36			9		
37			10		
38			11		
39			12		
40			13		

A bound must be entered for each parameter selected. These bound entries must be in the same sequence as the parameters. For each iteration of the differential correction process, the change in each parameter is less in absolute value than the corresponding bound if that bound is positive, zero if the corresponding bound is zero, or unrestricted if the corresponding bound is negative.

Lines 41 through 50: Radar Parameter Specification

41		M	RAPAR	05, 99	
42		D	01, 01	BN LAT	
43			03, 01	.01	
44			04, 01		
45		D	01, 02	CK24RBIAS	
46			03, 02	500.	
47			04, 02	750.	
48		D	01, 03	HU TBIAS	
49			03, 03	.001	
50			04, 03	.0035	

Line 41 indicates that the data listed subsequent to RAPAR on the load sheet will be input into a 5 X 99 matrix array which has been preset to zero. The columns of this array correspond to parameters, and the rows correspond to parameter identification (Positions 1 and 2), bounds (Position 3), and bias estimates (Position 4) respectively. Row 5 is currently not used.

Lines 42 through 44 specify the first applicable radar parameter. Line 42 contains the station name (Positions 1 and 2), pass identification (Positions 3 and 4), and parameter name (Positions 5 through 10). Lines 43 and 44 contain the bound and initial value, respectively, in feet, degrees, and minutes except for $\dot{R}$, $\dot{P}$, and $\dot{Q}$, which are in feet per second. If the parameter is station latitude, longitude, or altitude, the initial value is taken from the station location card.

If the pass identification character position is left blank, all data with the indicated station name will be used to correct the parameter. If the pass identification is not omitted, only data that are identified by both the indicated station name and indicated pass identification will be used to correct the parameter. If the radar parameter specified is station latitude, longitude, or altitude, the pass identification is ignored and all data with the station name are used for the parameter correction.

Lines 45 through 47 and Lines 48 through 50 specify the second and third radar parameters respectively. Station names, pass identifications, parameter names, bounds, and initial values are treated in the same manner as the inputs in Lines 42 through 44 described above. Additional cards may be added in cases where more than three radar parameters are involved. Available radar parameters are listed in Table 5-1.

Note that the total number of parameters which may be selected for simultaneous solution must be less than one hundred.

Table 5-1. Available Radar Parameters

Parameter	Symbol
Station latitude	LAT
Station longitude	LONG
Station altitude	ALT
Time bias	TBIAS
Range bias	RBIAS
Azimuth bias	ABIAS
Elevation bias	EBIAS
Topocentric right ascension bias	RTBIAS
Topocentric declination bias	DTBIAS
Topocentric hour angle bias	HABIAS
Geocentric right ascension bias	RGBIAS
Geocentric declination bias	DGBIAS
Argument of latitude (u) bias	UBIAS
Cross plane (v) bias	VBIAS
Height bias	HBIAS
$\hat{x}$ bias	XBIAS
$\hat{y}$ bias	YBIAS
$\hat{z}$ bias	ZBIAS
P bias	PBIAS
Q bias	QBIAS
Range-rate bias	RDBIAS
$\dot{P}$ bias	PDBIAS
$\dot{Q}$ bias	QDBIAS
Range scale factor (K_R)	KR
Range-rate scale factor (K_D)	KD

Lines 51 through 62: Observation Sigmas

51			SIGMA	100.	
52			2	.5	
53			3	.5	
54			4	1000.	
55			5	200.	
56			6	300	
57		I	ISIG	1	
58		I	2	2	
59		I	3	3	
60		I	4	113	
61		I	5	114	
62		I	6	115	

Lines 51 through 56 contain the observation data weighting factors. For each SIGMA entry, a corresponding entry defining the sigma set and data type appears in ISIG Lines 57 through 62. The ISIG entries are of the form 100I + K, where I is the observation set number and K is the data type. Ten sets, corresponding to I = 0, 1, 2, . . . , 9, may be entered. This selected value of I is the same as the entry in Column 5 of the station location card. The data type, K, must be one of those listed in Table 5-2.

Table 5-2. Data Types for ISIG Entries

Data Type (K)	Data Description	Symbol
1	Range	R
2	Azimuth	A
3	Elevation	E
4	Topocentric right ascension	α_T
5	Topocentric declination	δ_T
6	Topocentric hour angle	HA
7	Geocentric right ascension	α_g
8	Geocentric declination	δ_g
10	Argument of latitude	u
11	Cross plane	v
12	Height	h
13	$\hat{x}$	$\hat{x}$
14	$\hat{y}$	$\hat{y}$
15	$\hat{z}$	$\hat{z}$
17	Range difference	P
18	Range difference	Q
19	Range rate	$\dot{R}$
20	$\dot{R}$ difference	$\dot{P}$
21	$\dot{R}$ difference	$\dot{Q}$

Line 63: Maximum Number of Iterations

63		I	MAXIT	4	

If the differential correction process has not converged at the end of MAXIT iterations, the process will be terminated.

Line 64 through 66: Residual Editing

64			CEDIT	3	
65			2	.9	
66			3	1	

If CEDIT is zero, no editing is done. If CEDIT is non-zero, residuals will be edited in accordance with the following:

a. CEDIT < 0 Residuals greater than (input sigma × |CEDIT|) will be discarded.

b. CEDIT > 0 Residuals greater than (statistical sigma from the previous iteration × CEDIT) will be discarded. No editing is done on Iteration 1. Sigmas are computed for the first five data types encountered for each station.

Line 65 represents a scale factor such that if CEDIT (2) is non-zero, CEDIT is replaced by (CEDIT × CEDIT (2)) at the end of each iteration. If CEDIT (2) is zero, CEDIT is not modified.

Line 66 is a special option wherein if CEDIT (3) is non-zero, Iteration 1 will be repeated with editing performed with the sigmas computed during the first pass through Iteration 1. This will allow editing to be done on all iterations with computed sigmas.

Sigmas (rms) for the first five data types for each station are computed and printed at the end of each iteration regardless of the residuals-editing option selected.

Lines 67 and 68: Elevation-Angle Refraction Index Table

67			REFR		
68			2	4.0	-6

A table of refraction indices η_i, which may contain up to ten values, may be input starting at REFR. The entry used to compute refraction corrections for radar elevation observations is determined by the type number contained in Column 6 of the corresponding station location card. A zero in Column 6 causes the entry at REFR to be used, a 1 in Column 6 causes the entry at REFR + 1 to be used, etc.

If the table contains no entries, the value 312.0×10^{-6}, which is built in at location REFR, will be used to compute refraction corrections for all data whose station location cards contain zero in Column 6. All other positions of the table are assembled as zeros.

Line 69: Range Refraction Correction

69			RREFC	1	

Refraction corrections to all range observations will be computed and applied if RREFC is non-zero.

Line 70: Propagation Time Correction

70			SLT	-2820.1763	

The velocity to be used in calculating the observation time correction due to propagation time is entered at SLT in units of earth radii per minute. In the absence of an entry, no correction will be applied. If an entry is present the correction will be applied to times associated with R, A, E, $\dot{R}$, h, P, Q, $\dot{P}$ and $\dot{Q}$ data only.

Lines 71 and 72: Data-Tape Options

71		I	IBCDI	6	
72		I	IBINI	1	

If the radar observation and station location information is to be input via a BCD tape other than the A3 normal FORTRAN system input tape, the tape number must be specified at IBCDI. If a binary tape containing compacted radar data produced by a previous run is to be input, IBINI must be non-zero and the tape must be mounted on Logical Unit B5.

Lines 73 and 74: True Equinox Correction

73			DAFLG	-.004	
74			2		

An additive factor may be applied to the computed right ascension of Greenwich at midnight of epoch day by entering the appropriate value in units of degrees at DALFG for Vehicle 1, at DAFLG(2) for Vehicle 2, etc. This entry usually is used to correct from mean to true equinox reference coordinates.

Lines 75 through 78: Proximity Indicator

Φ^* 75			ANOM1		
λ 76			2		
h 77			3		
R_o 78			4		

During an orbit determination run, an indicator may be obtained whenever the trajectory passes within a given distance (range) of a point on the surface of the earth by input of geodetic latitude (deg), east longitude (deg), and altitude (n mi) of the point at ANOM1 and the succeeding two positions and of the testing distance (the range from the point to the vehicle) at ANOM1(4). Testing and printing is done in the FITA link. Up to three such sets may be input at ANOM1, ANOM2, and ANOM3.

Lines 79 and 80: Number of Permanent Stations

79			NUMB		
80		I	35	30	

NUMB(35) must be used whenever the temporary station option is exercised (see Section 5.1.3.3.2). If all stations are to be handled as temporary, a negative entry at NUMB(35) is required. If some stations are to be permanent (locations held in core while all data are processed), the number of permanent stations must be entered at NUMB (35).

Line 81 and Subsequent: Constraint Matrix

	81	I	KNST	4	
	82	I	BLIST	1	
	83	I		1	
b_{11}	84			1	
	85	I		2	
	86	I		2	
b_{22}	87			1	
	88	I		3	
	89	I		3	
b_{33}	90			1	
	91	I		4	
	92	I		4	
b_{44}	93			1	
	94	I		5	
	95	I		1	
b_{51}	96			-1	
	97	I		6	
	98	I		2	
b_{62}	99			.5	
	100	I		5	
	101	I		5	
c_{51}	102			6	
	103	I		7	
	104	I		5	
	105			1	

If it is assumed that n parameters are to be solved for $(p_1, p_2, \ldots, p_n) = p$, the ordering of the p_i corresponds to the order of the X's for the CPRAM, DPRAM, and KPRAM and the RAPAR arrays. Further assuming that these parameters are to be subjected to m linear constraints, which, for example for $n = 6$ and $m = 2$ might be $p_1 + p_5 = 6$, $p_2 - 2p_6 = 0$, KNST would be equal to $(n - m) = 4$, or the number of effective (unconstrained) parameters.

NUMB(35) must be used whenever the temporary station option is exercised (see Section 5.1.3.3.2). If all stations are to be handled as temporary, a negative entry at NUMB(35) is required. If some stations are to be permanent (locations held in core while all data are processed), the number of permanent stations must be entered at NUMB (35).

Line 81 and Subsequent: Constraint Matrix

	81	I	KNST	4	
	82	I	BLIST	1	
	83	I		1	
b_{11}	84			1	
	85	I		2	
	86	I		2	
b_{22}	87			1	
	88	I		3	
	89	I		3	
b_{33}	90			1	
	91	I		4	
	92	I		4	
b_{44}	93			1	
	94	I		5	
	95	I		1	
b_{51}	96			-1	
	97	I		6	
	98	I		2	
b_{62}	99			.5	
	100	I		5	
	101	I		5	
c_{51}	102			6	
	103	I		7	
	104	I		5	
	105			1	

If it is assumed that n parameters are to be solved for $(p_1, p_2, \ldots, p_n) = p$, the ordering of the p_i corresponds to the order of the X's for the CPRAM, DPRAM, and KPRAM and the RAPAR arrays. Further assuming that these parameters are to be subjected to m linear constraints, which, for example for $n = 6$ and $m = 2$ might be $p_1 + p_5 = 6$, $p_2 - 2p_6 = 0$, KNST would be equal to $(n - m) = 4$, or the number of effective (unconstrained) parameters.

observations are to be processed. This information will previously have been accumulated on the station-card load sheet illustrated in Figure 5-7.

5.1.3.3.1 Station Card Format

For stations associated exclusively with geocentric or with vehicle-centered observations, only the information in Columns 1 through 6 of Figure 5-7 must be entered. Up to 100 stations may be entered at any one time (see Section 5.1.3.3.2). The last station card must be followed by a card carrying the designation TS in Columns 1 and 2. Specific information categories contained in the station-card load sheet are:

a. Columns 1 and 2(ST): Station identification symbol. No two stations may be identified by the same symbol or any one station by the symbol TS.

b. Column 5: Sigma index identifying observation-sigma set to be applied to data from corresponding station. The sets of sigmas are input with the FINP data (see Lines 51 through 62, SIGMA/ISIG).

c. Column 6: Type of refractivity correction to be used for elevation readings from this station. Refractivities are numbered in their input order within the FINP Data (see Line 67, REFR).

d. Columns 9 through 17: North latitude of station in degrees

e. Columns 19 through 27: East longitude of station in degrees

f. Columns 29 through 36: Altitude of station in feet

g. Columns 38/39 and 41/42: If a station reports P, Q or $\dot{P}$, $\dot{Q}$ data, Columns 38/39 and 41/42 contain the two letter symbols for the associated station(s) of the tracking configuration. Each such associated station must be represented by a separate station card, but it is not necessary for Columns 38/39 and 41/42 to be filled out on the latter.

5.1.3.3.2 Temporary Station Option

The 100 allowable station locations may be classified either as permanent or as temporary stations. Permanent station locations are input in the manner outlined in Section 5.1.3.3.1 and are held in core at all times during a fit. Temporary station locations are input with the data to which they

TRACE-D
Station Cards

AEROSPACE CORPORATION
COMPUTATION AND DATA PROCESSING CENTER

PROGRAMMER ________ KEYPUNCHED ________ VERIFIED ________ DATE ________ PAGE ____ OF ____

1 2		5 6	9–17	19–27	29–36	38 39	41 42		73–80
S T		* *	Latitude (deg)*	Longitude (deg)*	Altitude (ft)*	P Ṗ (ST)	Q Q̇ (ST)	NOTES:	Identification

NOTES:

1. ST is the station-identification code
2. Station codes must be entered in Columns 38-39 and 41-42 if P, Q or Ṗ, Q̇ data are input.
3. An asterisk (*) following a unit value indicates that a decimal point is necessary for that quantity.

AEROSPACE FORM 3709

Figure 5-7. Station-Card Specification Load Sheet

correspond, subject to the flocking restrictions subsequently described. The number of permanent stations, which remains constant throughout the run, is input at NUMB(35) in the FINP data. The number of temporary stations may vary from flock to flock but the total number of permanent plus temporary stations must not exceed 100 for any flock. Radar parameters may be determined for permanent stations only (see Section 5.2.2.6 for the particular deck setup required).

5.1.3.4 Observation Data Cards

Observations are input by means of observation cards punched with information previously accumulated on the observation-data specification load sheet illustrated in Figure 5-8. Symbols identifying information categories noted on this load sheet form are:

a. Columns 1 and 2 (ST): Station identification symbol, which must correspond to a station-location card.

b. Columns 3 and 4 (PS): Pass identification (optional)

c. Columns 6 through 23: MO, DAY, HR, MIN, and SEC entries indicate the date and time (GMT) of corresponding observations

d. Column 24: Observation-set number

e. Column 72: Card number indicating observations, variances, or covariances

Contents of the various observation-load-sheet information fields are itemized in Table 5-3, wherein input units are feet, degrees, and seconds.

The last observation card must be followed by a card carrying the symbol TR in Columns 1 and 2. The TR card must be followed by an END card.

5.1.3.5 Flocking Option

For numbers of observations greater than 200, the data must be divided into "flocks." Flocks may be of arbitrary size, but also must not include more than 200 observations each. A control card with the letters TF in Columns 1 and 2 is used to signal the end of a flock, and any number of these may be placed among the observation cards. However, the last flock must be terminated by a TR card. The only restriction imposed is that the obser-

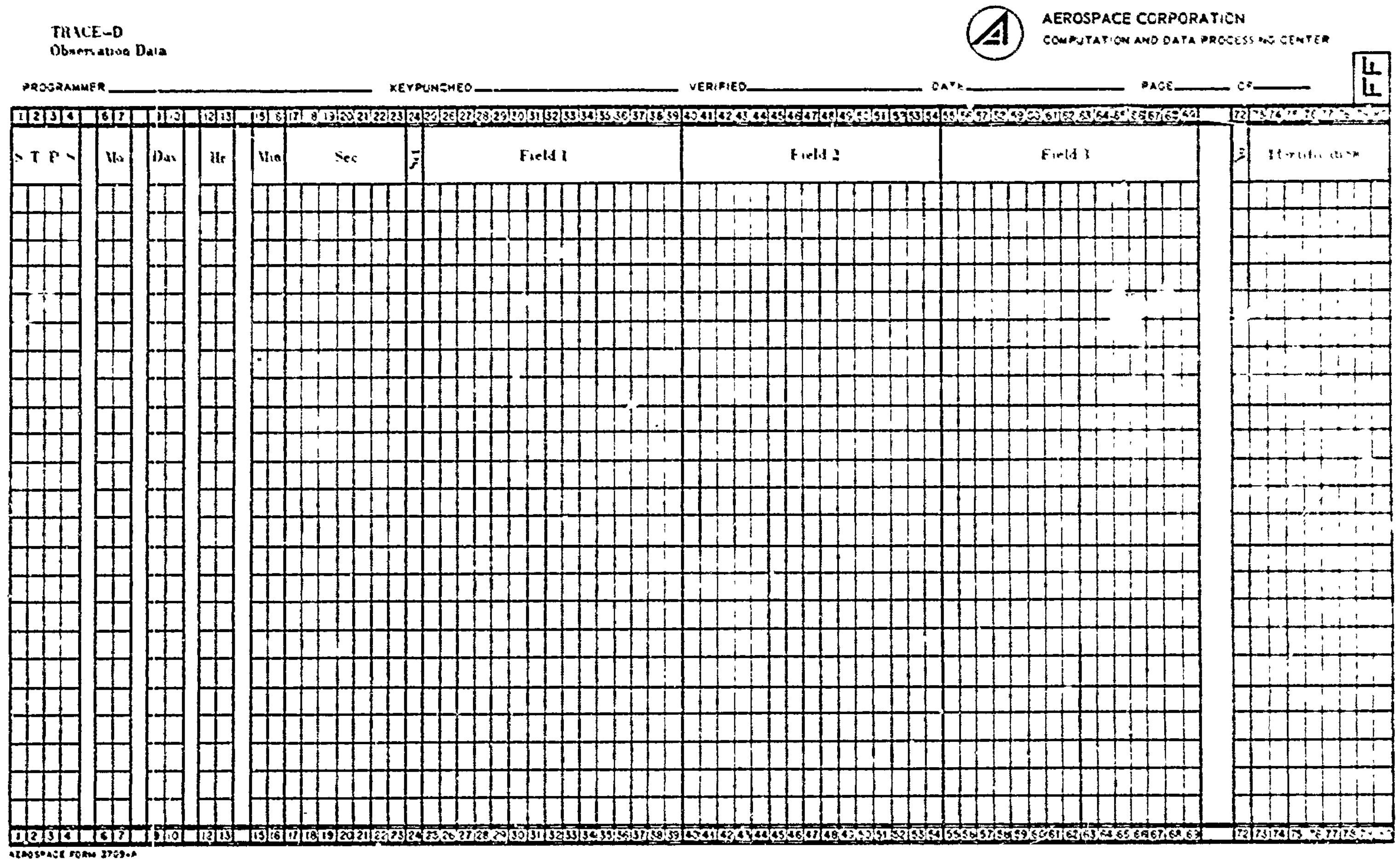

Figure 5-8. Observation Data Specification Load Sheet

Table 5-3. Information Accumulated on Observation Load Sheet

Observation Set Number (Column 24)	Observation Card Type (Column 72)	Field 1	Field 2	Field 3
1	0	Slant range(R)	Azimuth(A)	Elevation(E)
1	1	Variance(R)	Variance(A)	Variance(E)
1	2	Covariance(R, A)	Covariance(R, E)	Covariance(A, E)
2	0	Topocentric right ascension(α_T)	Topocentric declination(δ_T)	Topocentric hour angle(HA)
2	1	Variance(α_T)	Variance(δ_T)	Variance(HA)
2	2	Covariance(α_T, δ_T)		Covariance(δ_T, HA)
3	0	Geocentric right ascension(α_g)	Geocentric declination(δ_g)	
3	1	Variance(α_g)	Variance(δ_g)	
3	2	Covariance(α_g, δ_g)		
4	0	Argument of latitude(u)	Cross plane(v)	Altitude(h)
4	1	Variance(u)	Variance(v)	Variance(h)
4	2	Covariance(u, v)	Covariance(u, r)	Covariance(v, h)
5	0	$\hat{x}$	$\hat{y}$	$\hat{z}$
5	1	Variance($\hat{x}$)	Variance($\hat{y}$)	Variance($\hat{z}$)
5	2	Covariance($\hat{x}$, $\hat{y}$)	Covariance($\hat{x}$, $\hat{z}$)	Covariance($\hat{y}$, $\hat{z}$)
6	0	Slant range(R)	Range difference(P)	Range difference(Q)
6	1	Variance(R)	Variance(P)	Variance(Q)
6	2	Covariance(R, P)	Covariance(R, Q)	Covariance(P, Q)
7	0	Range rate($\dot{R}$)	Range rate difference($\dot{P}$)	Range rate difference($\dot{Q}$)
7	1	Variance($\dot{R}$)	Variance($\dot{P}$)	Variance($\dot{Q}$)
7	2	Covariance($\dot{R}$, $\dot{P}$)	Covariance($\dot{R}$, $\dot{Q}$)	Covariance($\dot{P}$, $\dot{Q}$)

vations must be in partial chronological order such that every data time of a given flock is later than all times in all previous flocks.

The mechanics of this option involve use of TRAIN, which causes the observations to be read, sorted, processed, and written on tape one flock at a time. If more than one flock is present, the differential correction process first reads the tape and then computes residuals and the normal matrix for one flock at a time.

5.1.3.6 Multiple Vehicle Option

If data from more than one vehicle are to be used, the Vehicle-1 data are set up in the manner outlined in Sections 5.1.3.3 and 5.1.3.5, except that the TR card must be replaced by a TT card. The data for Vehicle 2 are then similarly arranged. If Vehicle 2 is the last vehicle, corresponding data are followed by a TR card: if it is not the last vehicle, data are followed by a TT card. Data for Vehicles 3 through 6, as applicable, are added in the same manner. A TR card rather than a TT card must follow the data corresponding to the last vehicle, and an END card must follow the TR card.

5.1.3.7 Correlated Observations

Observations may be weighted by means of the covariance matrix as well as by the usual normalization based on the a priori standard deviation. The allowable covariance inputs are shown in Table 5-3.

The type of information content carried by each observation data card is identified by the 0, 1, or 2 symbol contained in Column 72 of the observation, variance, and covariance cards, respectively. Columns 1 through 24 of all three cards must be identical, and all cards associated with the same observation time must be input in the same flock.

The only covariances accepted are between observations of the same type number which occur at the same time. For example, in the case of an $\hat{x}$ (Type 5) observation at time t, the only quantities with which it could be correlated would be $\hat{y}$ at time t, $\hat{z}$ at time t, or both.

5.1.4 Data Generation

Detailed description of the TRACE-D data generation input load sheet shown in Figure 5-9 is presented in Sections 5.1.4.1 and 5.1.4.2.

5.1.4.1 Required Input

The only required input in addition to the basic data are the station location data and the data tabulated on Data-Generation Specification Load Sheets I and II as outlined in Sections 5.1.4.3 through 5.1.3.5.

5.1.4.2 Optional Input

5.1.4.2.1 Group 1

Line 2: Output Options

1			IFLAG		
2		I	6	1	

If IFLAG(6) = 0, all generated data are printed. If IFLAG(6) = 1, rise, maximum elevation, and set times only are printed and the Data-Generation Specification Load Sheet II is not necessary except for listing of a card carrying TR in Columns 1 and 2.

Line 3: Order of Output

3		I	14		

If IFLAG(14) = 0, data are generated in time sequence until the available core space (bucket) is full. This output is then separated and printed by station in the same sequence as that of the input station cards. Further data are then generated until the bucket again is full, and the sort/print cycle is repeated. If IFLAG(14) = 1, data are printed as they are generated (i.e., in time sequence).

X-3 7090 INPUT DATA TRACE-D Data-Generation Input (1) AEROSPACE CORPORATION COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ___ OF ___

SYMBOL		I R E	LOC.	VALUE	FXP
	1		IFLAG		
	2	I	6		
	3	I	14		
	4	I	ETAPE		
	5		REFR		
θ_y	6		YAW		
θ_p	7		PITCH		
θ_r	8		RØLL		
	9		YAW		
	10		811		
	11		812		
	12		813		
	13		814		
	14		815		
	15		816		
	16		817		
	17		818		
	18		819		
	19		820		
	20		821		
	21		822		
	22		323		
	23		824		
	24		825		
	25		IFLAG		
	26	I	16		

AEROSPACE FORM 3717-D (1)

Input Description

IFLAG(6) Output option
- 0: Print at every Δt and at rise/set and maximum elevation times
- 1: Print at rise/set and maximum elevation times only

IFLAG(14) Sequence option
- 0: Bucket used, data sorted and output by station
- 1: Data output in straight time sequence

ETAPE BCD data-tape generation (0 = No, 6 = Yes)

REFR Refraction index table (standard value built in at REFR)

YAW/ PITCH/ RØLL Attitude specification (θ_y, θ_p, θ_r entered in degrees)

YAW(811) Maneuver table (start time, stop time, yaw rate, pitch rate, roll rate entered in minutes from epoch and degrees. Up to three data sets may be input)

IFLAG(16) Logical unit for trajectory (GAINA) tape (logical unit = (10) entered)

Figure 5-9. TRACE-D Data-Generation Input Load Sheet

X-3 **7090 INPUT DATA** TRACE-D Data-Generation Input (2) **AEROSPACE CORPORATION** COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ____ OF ____

SYMBOL		LOC	VALUE	EXP
41	B	NØISE		
42	M	RAPAR	05, 99	
43	D	01, 01		
44		04, 01		
45	D	01, 02		
46		04, 02		
47		SIGMA		
48		2		
49		3		
50		4		
51	I	ISIG		
52	I	2		
53	I	3		
54	I	4		

AEROSPACE FORM 3717-D (2)

Input Description

NOISE — Random-noise generator option (positive octal number entered to start random-number generator)

RAPAR — Radar parameter array (bias specification)

01, P	1	2	3	4	5	6	7	8	9	10	
04, P											

P — Radar parameter number ($1 \le P \le 99$)

Line 1 — Boxes 1-2: Station identification
Boxes 3-4: Not used
Boxes 5-10: Parameter designation

Line 2 — Value of bias to be applied to generated data

SIGMA — Table of standard deviations (These deviations multiplied by the random numbers which have standard deviation = 1 and zero mean. Resulting noise added to the generated data.)

ISIG — Table of codes defining use of SIGMA table entries. For each SIGMA entry quantity (100I + K), where I = sigma index (Column 5 of station card) and K = data type, entered in corresponding location of ISIG table.

Figure 5-9. TRACE-D Data-Generation Input Load Sheet (Continued)

X-3 7090 INPUT DATA TRACE-D Data-Generation Input (3) AEROSPACE CORPORATION COMPUTATION & DATA PROCESSING CENTER

PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ______ OF ______

SYMBOL	R	LOC	VALUE	EXP
81		ATAS		
82		2		
83		3		
84		4		
85		5		
86		6		
87		7		
88		8		
89		9		
90		10		
91		11		
92		12		
93		13		
94		14		
95		15		
96		16		
97		17		
98		18		
99		19		
100		20		
101		21		
102	D	CPRAM		

AEROSPACE FORM 3117-D (3)

Input Description

ATAS $[A^TA]^{-1}$ for epoch ADBARV vector. Lower triangle of $[A^TA]^{-1}$ entered in row sequence, i.e.,

1					
2	3				
4	5	6			
7	8	9	10		
11	12	13	14	15	
16	17	18	19	20	21

Figure 5-9. TRACE-D Data-Generation Input Load Sheet (Concluded)

Line 4: Observation Tape Generation

4		I	ETAPE	6		

If ETAPE is non-zero, a BCD radar observation tape will be generated on the logical tape unit entered at ETAPE. The tape format will be the same as that of the tracking input data. including station locations and TF and TR cards.

Line 5: Refractivity

5			REFR		

The computed elevation, E, is altered to account for refraction, using either

$$E' = E + \eta_{si} \cot E$$

if $E \geq 0.1$ radian, or

$$E' = E + \frac{1}{1000} \frac{\eta_{si} \times 10^6}{12+1000E} - \frac{30}{6+1000E}$$

if $E < 0.1$ radian and $\eta_{si} \neq 0$.

REFR contains the η_{si} term, wherein i = 0, 1, 2, ... 9. The appropriate value of i is entered on the station card of Column 6. Nominally $\eta_{so} = 3.12 \times 10^{-6}$. Rise, set, and maximum elevation values are determined from the geometric E which represents the elevation before refraction correction is applied. Additional cards may be inserted at this location if necessary.

Lines 6 through 8: Vehicle Attitude Specification

θ_y	6		YAW	1.	
θ_p	7		PITCH	90.	
θ_r	8		RØLL	180.	

Vehicle attitude may be specified by inputting yaw, pitch, and roll angles in degrees in the manner shown above. These entries normally are introduced in conjunction with aspect angle computations.

Lines 9 through 24: Vehicle Attitude Manuevers

9		YAW		
10		811	1.	
11		812	120.	
12		813	.01	
13		814	.1	
14		815	0	
15		816	120.	
16		817	121.	
17		818	-.9	
18		819	-9.0	
19		820	0	
20		821	1420.	
21		822	1420.5	
22		823	360.	
23		824	0	
24		825	0	

The time history of vehicle attitude maneuvers may be specified by means of a table entered at YAW(811). The format of this table, which is used in connection with generation of radar aspect angles and may consist of up to three sets of five entries each, is itemized in Table 5-4.

Table 5-4. YAW(811) Table Format

Entry	Description
YAW(811)	Start time in minutes from epoch for first set of angular rates
YAW(812)	Stop time in minutes from epoch for first set of angular rates
YAW(813)	Yaw rate in degrees per minute
YAW(814)	Pitch rate in degrees per minute
YAW(815)	Roll rate in degrees per minute
YAW(816)	Start time in minutes from epoch for second set of angular rates
YAW(817)	Stop time in minutes from epoch for second set of angular rates
YAW(818)	Yaw rate in degrees per minute
etc.	etc.

The yaw-, pitch-, and roll-angle values that make up these input sets of angular rates are assumed to change at the rate given over the time interval defined by the start and stop times. Nominal orientation is zero yaw, pitch, and roll, which corresponds to the condition where the vehicle body axis is normal to the geocentric radius vector, the nose of the vehicle is in the in-track direction, and the top of the vehicie is in the direction of the extended radius vector.

Vehicle attitude at time of epoch and for the case where the entries in Table 5-4 are all zero (i. e., when nothing is input) is assumed to be the attitude specified at YAW, PITCH, and ROLL. If gaps in the time entries of the table are present, the angles are held constant at the last computed values.

Lines 25 and 26: Trajectory Tape Unit

25			IFLAG		
26		I	16	10	

The logical unit of a scratch tape to be used for the GAINA-generated trajectory should be input at IFLAG(16). An END card should follow the foregoing input.

5.1.4.2.2 Group 2

The data generation input deck contains two END cards. In the event the following Group 2 input is used, this deck must be placed between these two END cards (see Section 5.2.3).

Line 41: Noise Generator

41		B	NØISE	3 7 7 7 7 7 7 7 7 7	7 7

If NOISE is non-zero (a positive octal number), normally distributed random noise with standard deviation and mean value specified by input at RAPAR is added to the generated data (see Section 5.1.4.2.2, Lines 42 through 46, RAPAR). The entry at NOISE is used to start the random-number generator.

Lines 42 through 46: Bias Specifications

42		M	RAPAR	05,99	
43		D	01,01	HU ABIAS	
44			04,01	.057	
45		D	01,02	AL HBIAS	
46			04,02	200.	

The RAPAR array (see Section 5.1.3.2) is used on data generation runs to indicate the stations for which biased data are to be generated and also as a means of entering the value of the bias.

Lines 47 through 54: Standard Deviations

47			SIGMA	200	
48			2	.1	
49			3	.1	
50			4	50.	
51		I	ISIG	1	
52		I	2	2	
53		I	3	3	
54		I	4	112	

Standard deviations for the noise added to generated observations are entered via the SIGMA table. The usage of the SIGMA, ISIG, and station-card index numbers is the same as that previously described in connection with tracking input instructions (see Sections 5.1.3.2 and 5.1.3.3.1) In the example noted above, output R, A, and E data would contain noise with sigmas of 200 feet, 0.1 degree, and 0.1 degree, and with means of zero feet, 0.057 degree, and zero degrees respectively. Also, the input HU station card would carry a zero as the sigma index in Column 5. The AL station card would contain a 1 in Column 5, and the generated output data would contain noise with a 50-foot sigma and a 200-foot mean.

Lines 81 through 101: Orbit Covariance Matrix

81			ATAS	1.	-4
83			3	.5	-7
86			6	2.	-2
90			10	4.	-2
95			15	2000.	
101			21	1.	

If observation uncertainties are to be calculated, a covariance matrix for the ADBARV elements at epoch must be input in lower triangular form at ATAS. The order of the elements is Row 1/Column 1 at ATAS, Row 2/ Column 1 at ATAS(2), Row 2/Column 2 at ATAS(3), etc.

<u>Line 102: Parameter Specification</u>

102	■	D	CPRAM	2	x	x	x	x	x	x					

If observation uncertainties are to be computed, the ADBARV parameters must be selected in accordance with the usage described in Section 5.1.3.2. However, only the ADBARV parameters may be indicated since their associated covariance matrix $(A^T A)^{-1}$ is the only one which may be used in computing the uncertainties.

5.1.4.3 Station Cards

Data-generation station card format and usage is the same as that previously outlined in connection with the tracking function (see Section 5.1.3.3.1) except that, in the case of a data generation run, the station cards follow the second END card and are in turn followed by a TS card.

5.1.4.4 Data-generation Specification Load Sheet I Input

Specific information categories contained in Data-Generation Specification Load Sheet I as shown in Figure 5-10 are:

a. <u>Columns 1-2 (ST)</u>: Station identification symbol. Must correspond to symbol letters appearing on station cards for that station.

b. <u>Columns 9 through 16</u>: Time interval in seconds at which data for a given station are to be generated and testing interval for rise/set only option.

c. <u>Columns 18 through 23</u>: Minimum elevation at which vehicle is visible.

d. <u>Columns 25 through 30</u>: Maximum elevation at which vehicle is visible. Zero value set to 90 degrees.

TRACE-D
Data Generation I

AEROSPACE CORPORATION
COMPUTATION AND DATA PROCESSING CENTER

PROGRAMMER ________ KEYPUNCHED ________ VERIFIED ________ DATE ________ PAGE ______ OF ______

FF

1–2	9–16	18–23	25–30	32–40	51–58			60–67			
	Interval (sec)*	Minimum Elevation (deg)*	Maximum Elevation (deg)*	Maximum Range (n mi)*	Start Time			Stop Time			NOTES:
					day	hr	min	day	hr	min	
ST											

NOTES:

1. ST is the station-identification code
2. Start and Stop times are from midnight of epoch
3. If Start time is zero epoch is defined as Start time
4. An asterisk (*) following a unit value indicates that a decimal point is necessary for that quantity

AEROSPACE FORM 3709-B

Figure 5-10. Data-Generation Specification Load Sheet I

e. Columns 32 through 40: Maximum range in nautical miles to which vehicle is visible. Zero value causes this test to be ignored.

f. Columns 51 through 58: Start time from midnight of start date. Zero value implies epoch is start time (Columns 51-52, 54-55, and 57-58, days, hours, and minutes, respectively.)

g. Columns 60 through 67: Stop time from midnight of start date (Columns 60-61, 63-64, and 66-67, days, hours, and minutes respectively).

The last card carrying Data-Generation Load Sheet I data must be followed by a card identified by the letters TR in Columns 1 and 2.

5.1.4.5 Data-Generation Specification Load Sheet II Input

Except for a TR card, Data-Generation Load Sheet II as shown in Figure 5-11 is not used for the rise/set only option. Specific information categories contained in Load Sheet II are as follows:

a. Columns 1-2 (ST): Station identification symbol. Must correspond to symbol letters appearing on station cards for that station.

b. Columns 7 through 33: An X entry in the appropriate column will initiate output of quantities itemized in Table 5-5.

TRACE-D
Data Generation II

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PROGRAMMER ______ KEYPUNCHED ______ VERIFIED ______ DATE ______ PAGE ____ OF ____

Column	Entry
1	S
2	T
7	Range
8	Azimuth
9	Elevation
10	Range Rate
11	P Dot
12	Q Dot
13	P
14	Q
15	Azimuth Rate
16	Elevation Rate
17	R Double Dot
18	Mutual Visibil.
19	Latitude
20	Longitude
21	Surface Range
22	Height
23	Doppler Rate
24	Look Angle
25	Obs. Variances
26	Kappa
27	Aspect Angles
28	Attenuation
29	ξ, η, ζ
30	Topocentric RA,D
31	Geocentric RA,D
32	Hour Angle
33	u, v
73–80	Identification

NOTES:

1. ST is the station-identification code
2. An X in a column causes the corresponding quantity to be output.
3. If P, $\dot{P}$, Q, or $\dot{Q}$ are selected, the specification of an additional station-identification code is necessary (Ref. Figure 5-7).

AEROSPACE FORM 3709-C

Figure 5-11. Data-Generation Specification Load Sheet II

Table 5-5. Output Quantities Corresponding to Columns 7 Through 33 of Data-Generation Specification Load Sheet II

Column	Output Quantity	Unit
7*	Range	n mi (ft on ETAPE)
8*	Azimuth	deg
9*	Elevation	deg
10*	Range rate	ft/sec
11-14*	$\dot{P}$, $\dot{Q}$, P, Q	ft/sec, ft
15	Azimuth rate	deg/min
16	Elevation rate	deg/min
17	Range acceleration	ft/sec^2
18	Mutual visibility (Output will be a list of numbers of stations visible at output time. Stations numbered in order of input on station cards. Number of stations, 8 maximum.)	
19	Geodetic latitude of vehicle	deg
20	Longitude of vehicle	deg
21	Surface range from station to subvehicle point	n mi
22*	Altitude of vehicle	n mi (ft on ETAPE)
23	Doppler rate	
24	Look angle (Angle between a vehicle axis and the station/vehicle line of sight. The direction cosines of the vehicle axis in the basic inertial system must be entered in C(37), C(38), and C(39). These	deg

*These quantities are output on ETAPE.

Table 5-5. Output Quantities Corresponding to Columns 7 Through 33 of Data-Generation Specification Load Sheet II (Continued)

Column	Output Quantity	Unit
	quantities may be input as constant or the user may provide a subroutine (FANG) to compute the direction cosines at each output point.)	
25	Observation uncertainties (If inverse A^TA matrix for initial conditions is input at ATAS, the ADBARV elements are selected as parameters, and if an X is entered in Column 25, the $[A^TA]^{-1}$ is updated to observation times and the standard deviations in the quantities R, A, E, $\dot{R}$, $\dot{A}$, $\dot{E}$ are derived and printed. The uncertainties are only those due to the uncertainty in the ephemeris which is implied by the given $[A^TA]^{-1}$ for the epoch conditions.)	Same units as observations
26	Angle kappa (K) (Angle between station line-of-sight and geocentric radius vectors.)	deg
27	Aspect angles (Angle 1 (Φ) is defined as the angle between the vehicle yaw axis and projection of the station line-of-sight vector in the roll plane. Angle 2 (θ) is defined as the angle between the vehicle roll axis and the line-of-sight vector to the station (See Section 5.1.4.2.1 for description of vehicle attitude-control options.)	deg
28	Signal attenuation = $-40 \log_{10} R$, where R is slant range in feet	db
29*	$\hat{x}$, $\hat{y}$, $\hat{z}$ (Same rectangular earth-fixed (X through Greenwich) geocentric quantities accepted as Type 5 observations for orbit determination.)	n mi (ft on ETAPE)

*These quantities are output on ETAPE

Table 5-5. Output Quantities Corresponding to Columns 7 Through 33 of Data-Generation Specification Load Sheet II (Concluded)

Column	Output Quantity	Unit
30*	Topocentric right ascension and declination	deg
31*	Geocentric right ascension and declinasion	deg
32*	Topocentric hour angle	deg
33*	Vehicle-centered argument of latitude and cross-plan angle	deg

*These quantities are output on ETAPE

5.1.5 Residuals Analysis

Detailed description of the TRACE-D residuals analysis input load sheet shown in Figure 5-12 is presented in Sections 5.1.5.1 and 5.1.5.2.

5.1.5.1 Required Input

Line 1: Itinerary

1		D	ITIN	127	

Residuals analysis runs normally require an Itinerary-127 sequence, which causes the TRACE-D program to carry out a 1-iteration fit sequence and then to call the RESIDUE link. Since no differential correction is carried out, the only required FINP entries are the basic input and IFLAG(26). The optional tracking-run input items which apply to residuals analysis runs are listed in Section 5.1.5.2. For most runs, station cards and observation cards are required, with formats identical to those previously described in connection with tracking input instructions (see Sections 5.1.3.3.1 and 5.1.3.4).

Lines 2 and 3: Residuals Analysis Options

2			IFLAG		
3		I	26		

IFLAG(26) selects the particular function to be performed by the RESIDUE link. Options are as follows:

a. IFLAG(26) = 1 TRACE-D will calculate observations corresponding to the given ephemeris (FITA output tape); compute the difference with the measured observations (B5 tape); resolve the residuals into radial, in-track, and cross-track components; compute time residuals; and accumulate a statistical summary by station and data type.

b. IFLAG(26) = 2 TRACE-D will accomplish IFLAG(26) functions noted in Item a. above and also will compute the difference from the mean for each of the resolved residual components. This option requires an entry to be input at NUMB(38) (see Section 5.1.5.2).

X-3 7090 INPUT DATA TRACE-D Residuals-Analysis Input

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PROGRAMMER ________ KEYPUNCHED ________ VERIFIED ________ DATE ________ PAGE ____ OF ____

SYMBOL		LOC.	VALUE	EXP.
1	D	ITIN		
2		IFLAG		
3	I	26		
4		C		
5		29		
6		NUMB		
7	I	38		
8	I	43		
9	I	44		
10	I	45		
11		IFLAG		
12	I	22		

AEROSPACE FORM 3717-E

Input Description

ITIN Itinerary (Itinerary 127 or 7 input)

IFLAG(26) Option indicator
- 1: Standard functions
- 2: Standard functions plus computation of differences from the mean

C(29) Edit and punch option ($\hat{x}, \hat{y}, \hat{z}$ data) (threshold magnitude of residual vector entered in feet)

NUMB(38) Scratch-tape unit designation (If IFLAG(26) = 2, logical unit = (9) entered.)

NUMB(43) Data-tape designation (If computed observations are supplied by a second compacted data tape, logical unit = (16) entered)

NUMB(44) Ephemeris input tape designation (If ephemeris in standard even-minute format is input, logical unit = (15) entered.)

NUMB(45) Ephemeris input-tape designation (If a second ephemeris is to be input for differencing, logical unit = (14) entered.)

IFLAG(22)
- 0: A and E residuals used for vector resolution
- 1: A and E residuals not used for vector resolution

Figure 5-12. TRACE-D Residuals Analysis Input Load Sheet

It should be noted that residuals analysis processing is presently restricted to Data Set 1 (R, A, E), Set 2 (α_T δ_T), and Set 5 ($\hat{x}$, $\hat{y}$, $\hat{z}$).

5.1.5.2 Optional Input

Lines 4 and 5: Edit and Punch Option

4		C		
5		29	1	

Type-5 observations ($\hat{x}$, $\hat{y}$, $\hat{z}$) may be edited with respect to a specified ephemeris by entering a value at C(29). All sets of ($\hat{x}$, $\hat{y}$, $\hat{z}$) which produce a value of R_T (the magnitude of the residual vector) less than the number entered at C(29) will be output on the punch tape in observation-card format.

Lines 6 and 7: Intermediate Tape Unit Specification

6		NUMB		
7	I	38	9	

If residuals analysis Option 2 is selected, a logical tape unit for intermediate input/output must be specified at NUMB(38).

Line 8: Observation Differences

8	I	43	16	

If the logical number of a tape unit is entered at NUMB(43), RESIDUE will not expect a FITA ephemeris tape and will not compute observations, but will expect a second compacted data tape on the unit specified and will proceed as outlined for IFLAG(26) options (see Section 5.1.5.1).

Line 9: Ephemeris Input

9	I	44	15	

The FITA output tape, which usually supplies the ephemeris input to RESIDUE, may be replaced by a direct tape input from the logical unit specified at NUMB(44). The format must be the even-minute option of the binary trajectory tape format (see Appendix D).

Line 10: Ephemeris Differences

10		I	45	14	

Two ephemeris tapes in the even-minute format (see Appendix D) may be differenced and the residuals may be processed in the same manner as $\hat{x}$, $\hat{y}$, $\hat{z}$ data residuals. The logical designations of the units holding the two tapes are entered at NUMB (44) and NUMB(45). Ephemeris points are handled as computed and measured observations, respectively.

Since it is not necessary to utilize the observation processing or fit links for this particular option, a single Itinerary Number 7 may be used.

Lines 11 and 12: Range-Only Option

11			IFLAG		
12		I	22	1	

If it is desired to use only range residuals for resolution into the orbit plane (i.e., A and E residuals are assumed to be zero), a non-zero entry should be made at IFLAG(22).

Additional optional input items which may be applicable to residuals analysis runs are the following:

a. Initial conditions for multiple satellites
b. Refraction corrections with respect to elevation and range
c. Propagation-time correction
d. Data-tape input options
e. Proximity testing option
f. Number of permanent stations

Instructions for use of optional input items are outlined in Section 5.1.3.2.

5.2 DATA-DECK ARRANGEMENT

This section describes the sequence of input data and control cards for the most frequent types of TRACE-D program runs. Notations relating to specific data-deck setups are given as appropriate for cases where known program restrictions may be troublesome and/or where tape input options are available. Each data control card is punched with its applicable letter symbol only (*DATA, END, TF, TS, TT, TR), starting in Column 1.

Cards preceding the *DATA card will be omitted in all but the first data-deck setups described in this section, wherein it also will be assumed that the constants deck is included in the FINP data.

5.2.1 Trajectory Only

5.2.1.1 Single Case

Except for control card changes (tape setup, priority, etc.), the portion through the constants of the trajectory card deck shown in Figure 5-13 normally remains unchanged from run to run. This remains true regardless of the itinerary sequence selected and is the basis for the term "basic running deck."

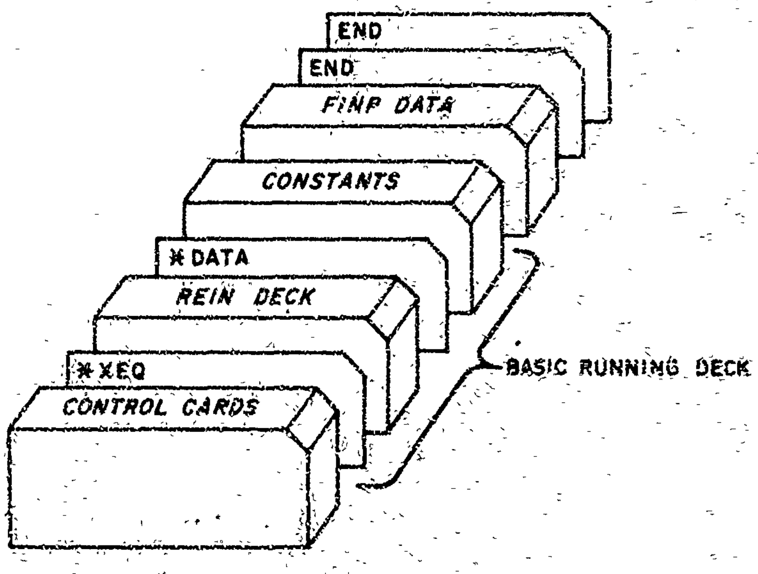

Figure 5-13. Deck Arrangement for Single Trajectory Case

Note: The constants deck includes the standard INTEG and C (constants) entries as well as some inputs relating to other locations where standard values have been selected (see Appendix A for listing of standard constants deck currently in use).

5.2.1.2 Stacked Cases (Itinerary 333)

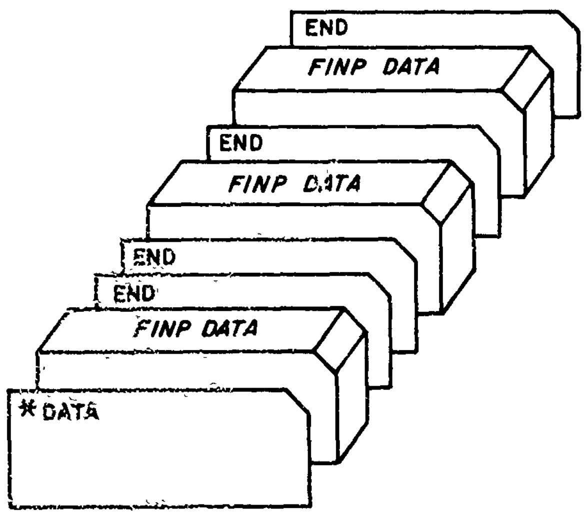

Figure 5-14. Deck Arrangement for Stacked Trajectory Cases

Note: In general, FINP input remains unaltered, and any quantity which is not overwritten will be used on the subsequent case. Exceptions are ICTYP, CTAPE, and the PKICK, XKICK, THRST, and PRTIM tables. Entries at these locations must be loaded for each case.

5.2.2 Tracking

5.2.2.1 Basic Arrangement

Figure 5-15 shows the tracking input deck arrangement in terms of its major components. The detailed deck contents for various special tracking applications are described in Sections 5.2.2.2 through 5.2.2.7.

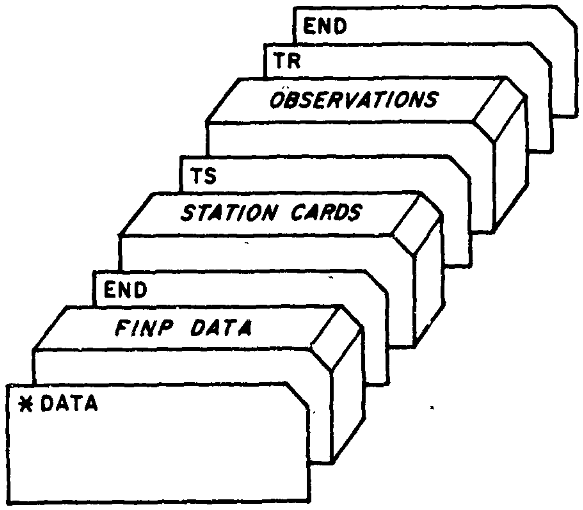

Figure 5-15. Deck Arrangement for Tracking Input

Note: The observation deck may include variance and covariance cards and/or temporary-station cards.

5.2.2.2 Flocked Observations

In the event that more than two hundred observations cards are present, the cards must be separated into flocks, as indicated in Figure 5-16. In Figures 5-17 through 5-21 it may be assumed that observation decks should be similarly flocked unless otherwise specified.

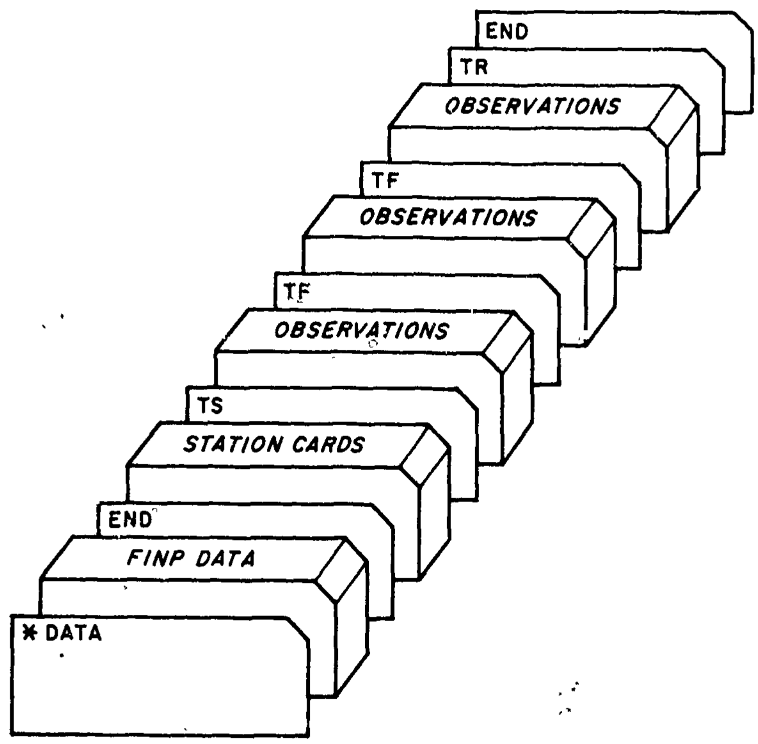

Figure 5-16. Deck Arrangement for Flocked Observations

Notes:
1. All observation times in any flock must be earlier than all observation times in all succeeding flocks.
2. The last flock is followed by TR and END cards only (no TF card)

5.2.2.3 BCD Input Observation Tape

If a BCD input observation tape is used, the station and observation card decks are absent, as shown in Figure 5-17.

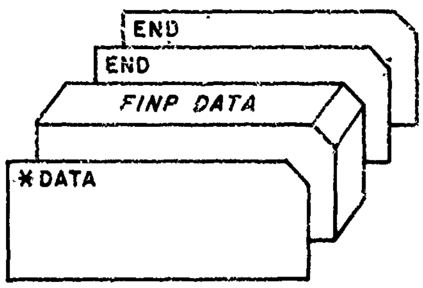

Figure 5-17. Deck Arrangement for Tracking Input with BCD Observation Tape

Notes:
1. The FINP data includes an entry at IBCDI.
2. The ETAPE prepared by GAIN is suitable for input as a BCD observation tape.
3. The BCD input tape contains the card images of the station- and observation-card deck that would be used if those cards were input directly (see Section 5.2.2.2). The first image on the BCD tape is of the first station card and the last image is of a TR card.

5.2.2.4 Input Compacted Data Tape

If the tracking observations are input by means of a previously prepared compacted data tape, the tracking input deck will be structured as shown in Figure 5-18.

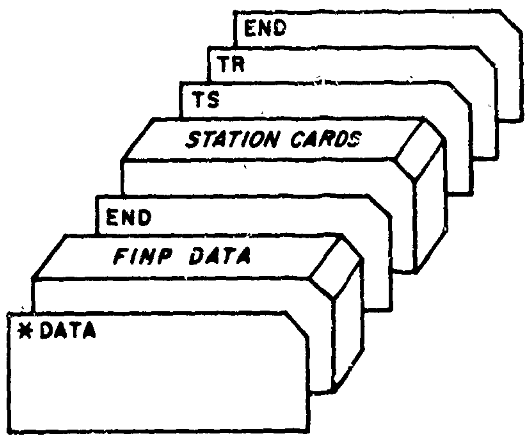

Figure 5-18. Deck Arrangement for Tracking Input with Compacted Data Tape

Notes:
1. The FINP data includes a non-zero entry at IBINI.
2. The input RAPAR matrix must be identical to the one entered on the run which generated the binary data tape.
3. The station-card deck must include all of the station cards in the same sequence as those appearing on the run which generated the binary data tape. The 2- and 4-column identifications also must be the same.

5.2.2.5 Multiple Vehicles

Observations for different vehicles are separated by TT cards, as shown in Figure 5-19.

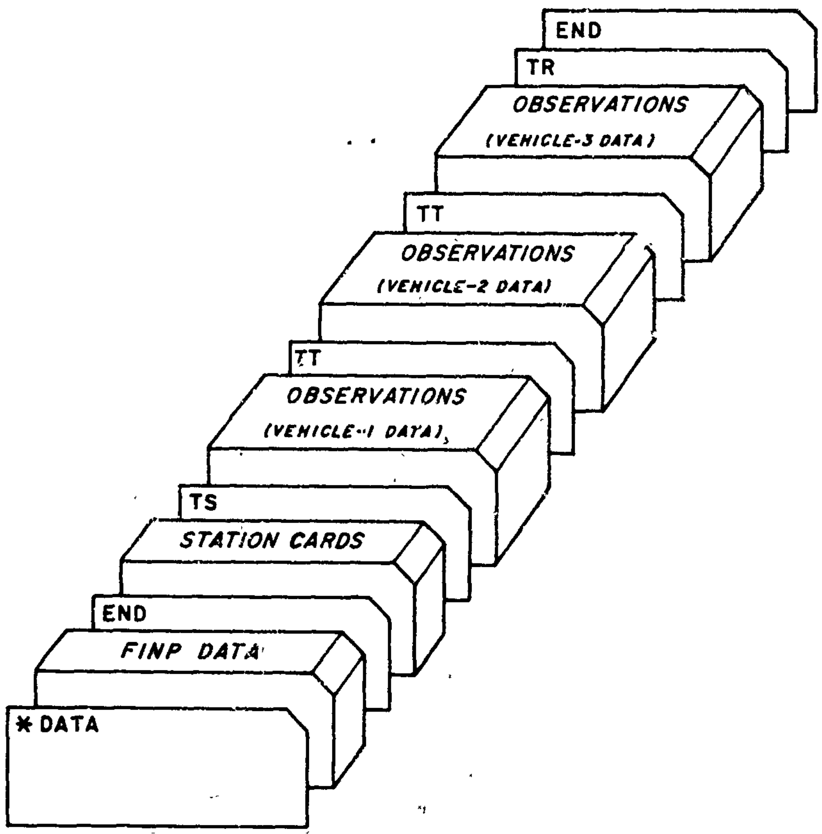

Figure 5-19. Deck Arrangement for Multiple-Vehicle Observations

Notes:
1. The observation time sequence restriction for flocking applies only within the data for each vehicle.
2. No TT card is required after the data for the last vehicle.
3. If either of the tape input options is used, all observations for all vehicles must be on the tape.
4. If flocking is used, the TF card should be omitted after the last flock for a vehicle (i.e., immediately before the TT card).
5. RMS of residuals for temporary-station data may be obtained by setting NUMB(35) = 0 before the final END card.

5.2.2.6 Temporary Stations

Figures 5-13 through 5-19 are based on the assumption that all stations are of the permanent type. In the event that the temporary-station option is used, the station cards for the temporary stations are included in the flock containing the observations to which they correspond.

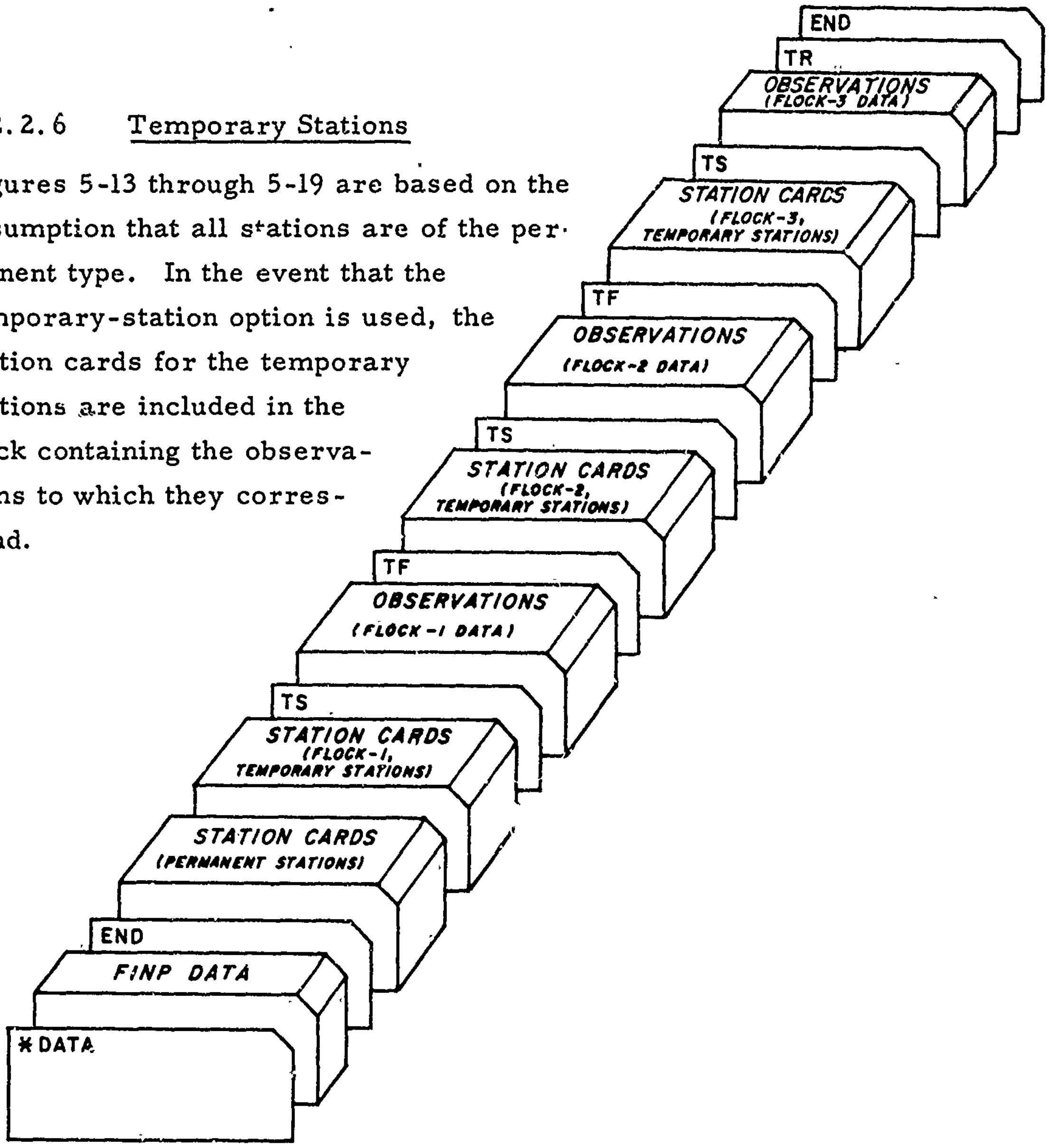

Figure 5-20. Deck Arrangement for Temporary-Station Option

Notes:
1. The number of flocks is not restricted.
2. The number of temporary stations in any flock plus the number of permanent stations may not exceed 100.
3. The FINP input includes the number of permanent stations entered at NUMB(35).
4. A TS card must be present in every flock if the temporary-station option is selected.

5.2.2.7 Temporary Stations with Input Compacted Data Tape

If a previously prepared binary compacted data tape is used, the tracking input deck will be structured as shown in Figure 5-21.

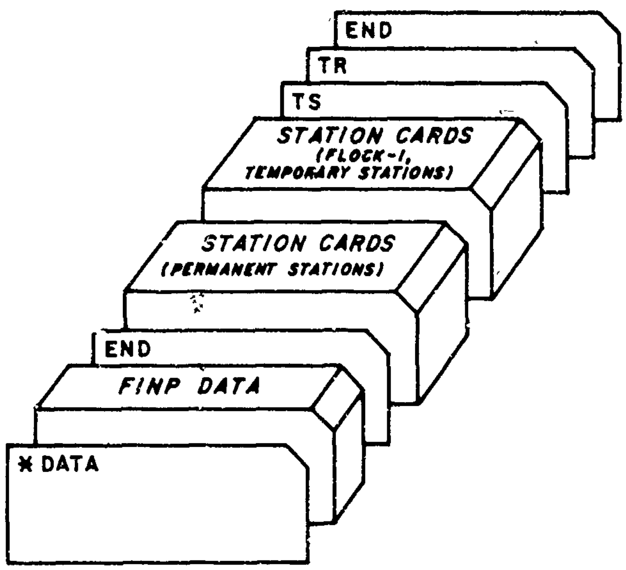

Figure 5-21. Deck Arrangement for Temporary-Station Option and Binary Compacted Input Tape

Note: Temporary-station cards for all flocks other than the first are contained on the binary compacted data tape.

5.2.3 Data Generation

5.2.3.1 Single Case

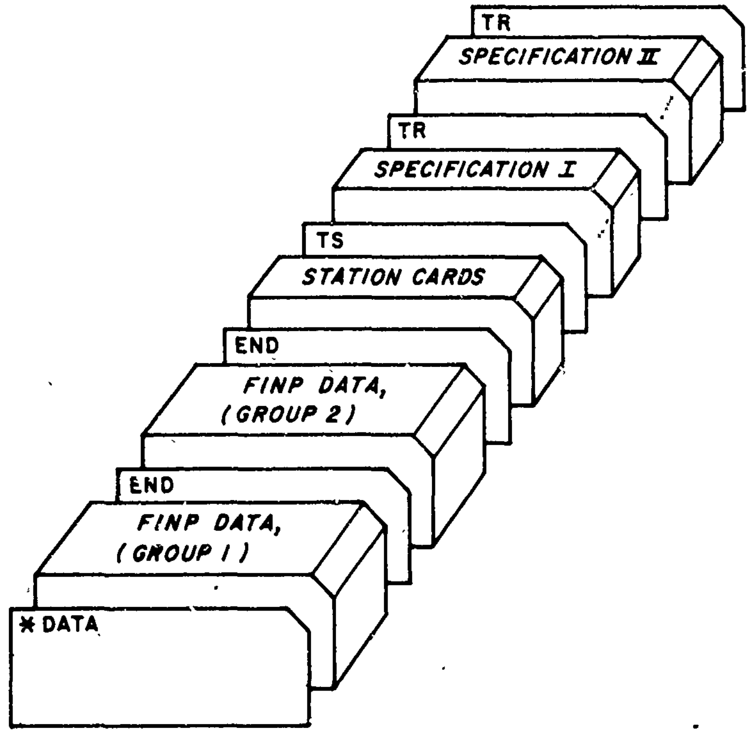

Figure 5-22. Deck Arrangement for Data Generation, Single Case

Note: For a rise/set only run, Specification II cards require only the station identification symbol.

5.2.3.2 Stacked Cases (Itinerary 44)

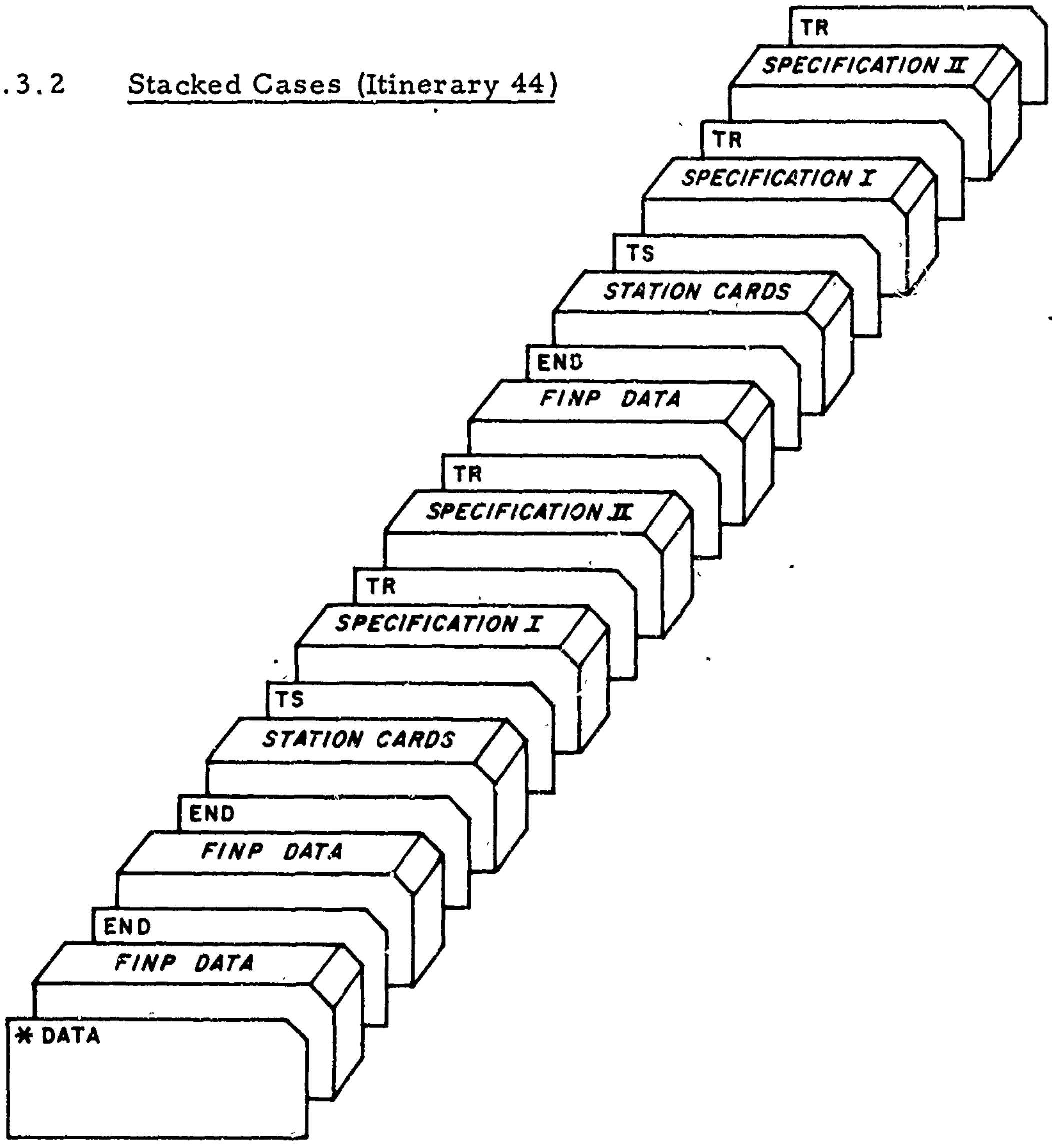

Figure 5-23. Deck Arrangement for Data Generation, Stacked Cases

Notes: 1. The only input carried over from case to case is part of the FINP data (see Section 5.2.1.2). Complete station and data generation specification decks must be loaded for each case. The RAPAR array must be re-input if used.

2. Since the TRACE-D program rewinds the tape specified by ETAPE at the start of each case, use of this option with Itinerary 44 is not recommended.

5.2.4 Residuals Analysis

Deck configurations for residuals analysis runs in general are identical to corresponding tracking-run configurations due to the fact that the TRACE-D program processes the Itinerary 127 sequence as if it were a 1-iteration tracking run followed by a transfer to the RESIDUE link. The single exception to this rule is the setup for the Itinerary 7 ephemeris differencing run, shown in Figure 5-24.

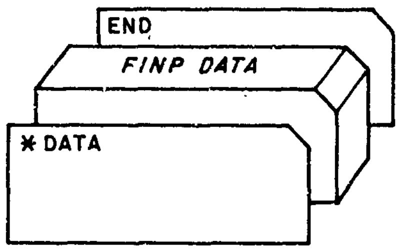

Figure 5-24. Deck Arrangement for Residuals-Analysis Ephemeris Tape Differencing

Note: On all residuals analysis runs, accumulated statistics (RMS, etc) are obtained for permanent stations only.

5.2.5 Mixed Itinerary Runs

5.2.5.1 Itinerary 123

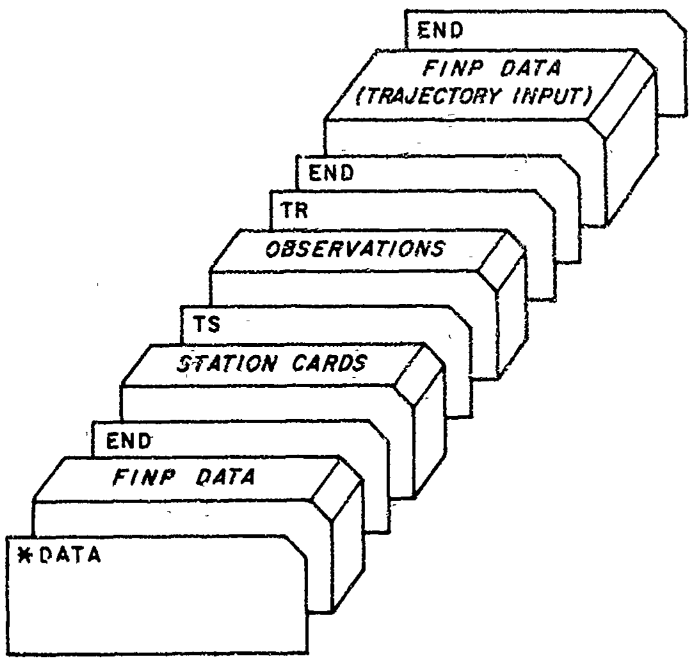

Figure 5-25. Deck Arrangement for Itinerary 123 Sequence

Notes:
1. The maximum amount of FINP data should be input for the trajectory portion of the Itinerary 123 run. Except for constants and the required basic input, it should be assumed that none of the FINP input to the tracking run is carried over.
2. The ICTYP6 option may be used to obtain the ephemeris corresponding to the fit solution by loading the ICTYP6 entry with the trajectory input.
3. If PKICK parameters were determined in the fit, the solution values for the ΔV will be used for the trajectory portion of the run, but the times in the PKICK table must be reloaded with the trajectory input.

5.2.5.2 Itinerary 1243

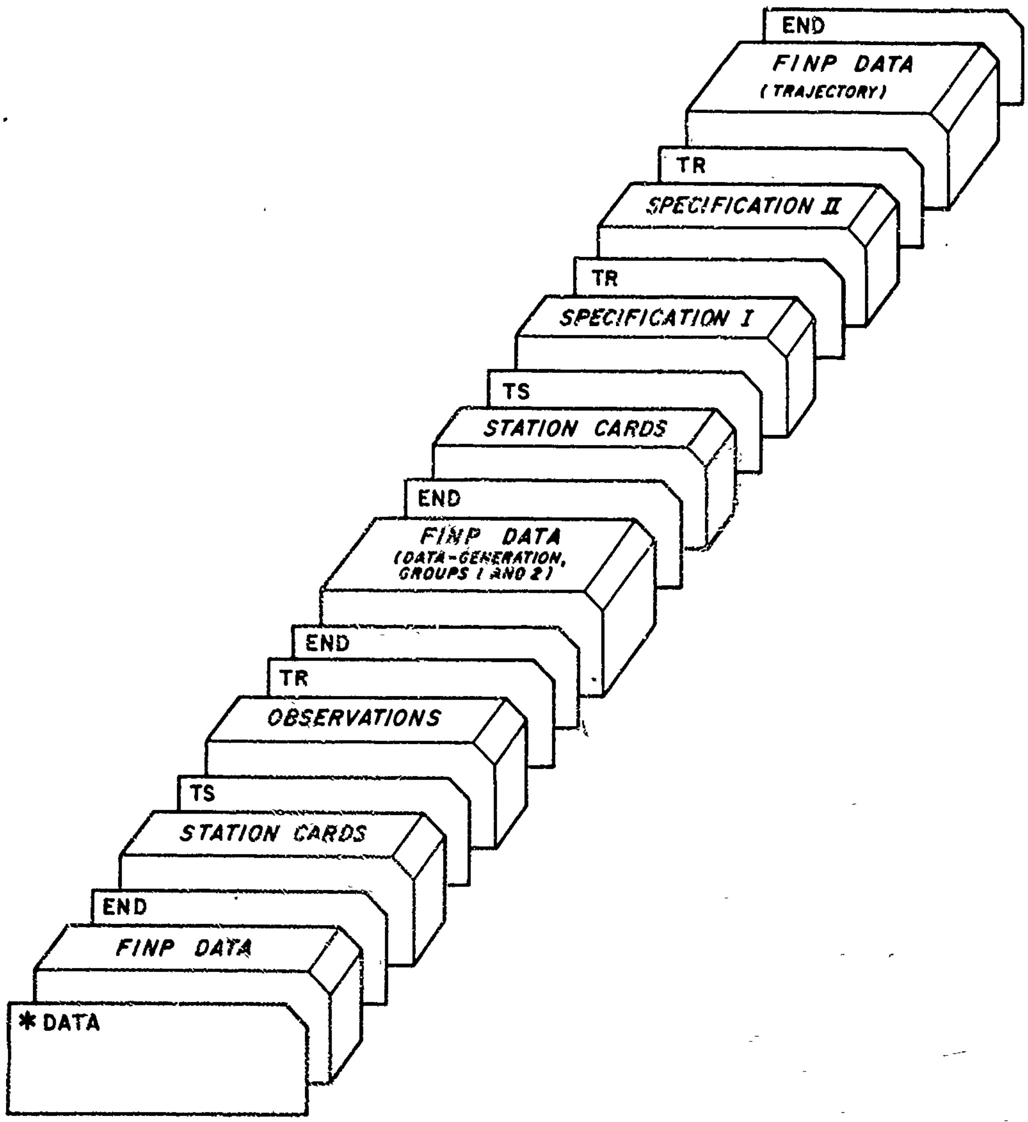

Figure 5-26. Deck Arrangement for Itinerary 1243 Sequence

Note: A data generation run (Itinerary 4) should not follow a trajectory run (Itinerary 3).

5.2.5.3 Itinerary 412

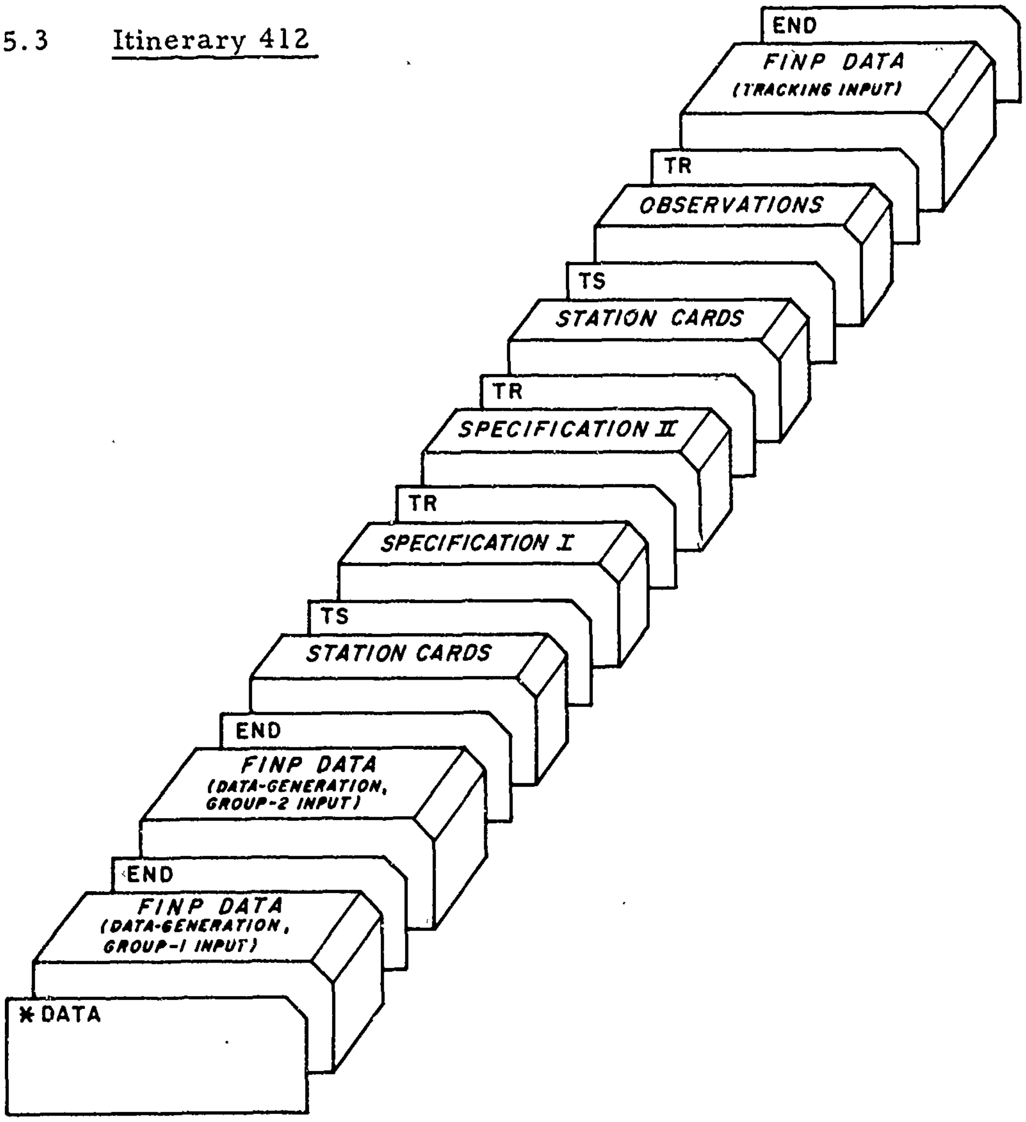

Figure 5-27. Deck Arrangement for Itinerary 412 Sequence

Notes: 1. The Itinerary-412 sequence normally is used with the ETAPE option in order to generate and fit observation data.

2. It should be noted that the FINP data for the tracking run is loaded after the observations. This is necessary because the CHAIN link is not called after the Itinerary 4 function. The TRAIN link, which reads station cards and observations, is called first, followed by the INITA link, which calls the FINP routine.

5.2.5.4 Itinerary 312

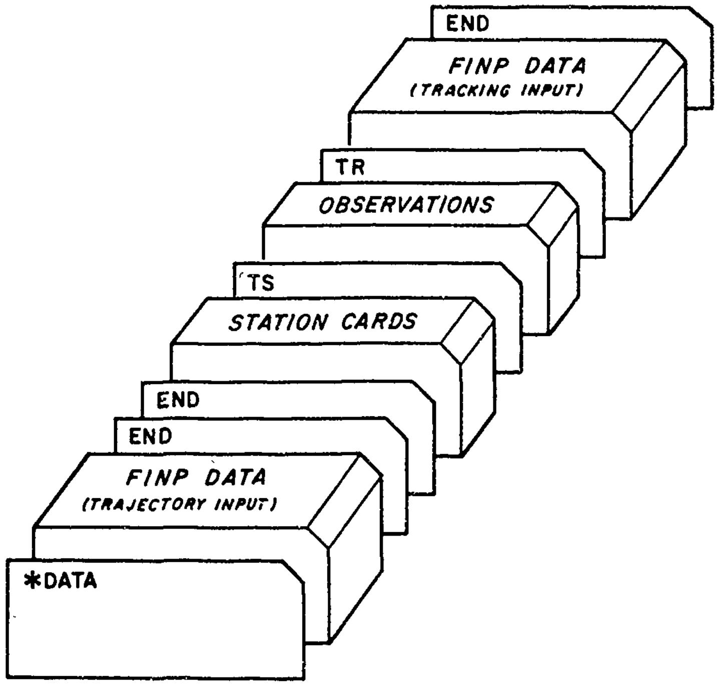

Figure 5-28. Deck Arrangement for Itinerary 312 Sequence

Note: 1. The Itinerary-312 sequence using the ICTYP5 feature may be used to shift epoch for the fit.

5.3 TAPE UNIT REQUIREMENTS

The information tabulated in Tables 5-6 through 5-9 delineates the tape unit requirements that are applicable to the four principal types of TRACE-D program runs. A summary of tape unit requirements by logical unit is given in Table 5-10.

These requirements are based on the assumption that a program tape mounted on Tape Unit 8 is used. However, if the program is loaded by cards, Tape Unit 8 is replaced by Tape Unit 11. The appropriate tape unit number may be assigned by input at points where symbolic locations are given.

In Tables 5-6 through 5-10 the logical unit given is standard for TRACE-D program operation at Aerospace Corporation. Also, it should be noted that standard system tapes are omitted in Tables 5-6 through 5-9 but are included in Table 5-10 for completeness.

Table 5-6. Tape Unit Requirements for Trajectory Runs

Logical Unit	Symbol	Function
7	CTAPE	Planetary-coordinate input
8*		Program input
14	DTAPE	Binary difference tape for trajectory differencing
15	NTAPE	Binary trajectory tape (nominal) for trajectory differencing
15		Binary trajectory tape in standard format

*Use of corresponding tape is mandatory.

Table 5-7. Tape-Unit Requirements for Tracking Runs

Logical Unit	Symbol	Function
6	ETAPE	BCD data output
7	CTAPE	Planetary coordinate input
8*		Program input
9*		Scratch tape (A^TA)
10*		Scratch tape (FITA trajectory)
13*		Binary compacted data (I/O)

Table 5-8. Tape-Unit Requirements for Data Generation Runs

Logical Unit	Symbol	Function
6	ETAPE	BCD data output
7	CTAPE	Planetary coordinate input
8*		Program input
10	IFLAG(16)	Scratch tape (GAINA trajectory)

Table 5-9. Tape-Unit Requirements for Residuals-Analysis Runs

Logical Unit	Symbol	Function
8*		Program input
10*		Scratch tape (FITA trajectory)
13	NUMB(43)	Compacted data input (differencing)
14	NUMB(45)	Trajectory input (differencing)
15	NUMB(44)	Trajectory input

*Use of corresponding tape is mandatory.

Table 5-10. Summary of Tape-Unit Requirements by Logical Unit

Logical Unit	Symbol	Function
1		FORTRAN monitor system*
2		System input*
3		System print output*
4		Not used
5		Not used
6	IBCDI	BCD observation input
6	ETAPE	Data generation output
7	CTAPE	Planetary coordinate input
8		Program input*
9		Scratch tape (tracking*)
9	NUMB(38)	Scratch tape (residuals analysis)
10		Scratch tape (tracking*, residuals analysis, data generation)
11		System chain tape
12		System punch output*
13		Compacted data input/output (tracking*)
14	DTAPE	Differences (used with TTAPE)
14	NUMB(45)	Trajectory input (residuals analysis differencing)
15	NTAPE	Trajectory (nominal) for trajectory link differencing
15		Trajectory in standard format, output from trajectory
15	NUMB(44)	Trajectory in standard format, input to residuals analysis differencing
16	NUMB(43)	Compacted data input, residuals analysis differencing

*Use of correponding tape is mandatory

5.4 IBM 7094 SENSE SWITCH CONTROLS

The sense switch controls on the IBM 7094 computer console may be used to control TRACE-D program operation. Table 5-11 itemizes resulting action for Sense Switches 1 through 6 in the On position. The itinerary types with which the use of each sense switch is permissible are noted in all cases.

Table 5-11. Action Resulting from IBM 7094 Sense Switch Use

Sense Switch Number	Function
1	Termination Options (DCS System only) When subroutine EXIT is called, the message "SET 7094 CONSOLE KEYS, PRESS START TO CONTINUE" is printed on-line and the computer halts. When START is pressed, the program tests the console key positions and accomplishes one of the following functions: a. Terminates normally if the Q key is down. b. Empties the output buffers, rewinds and unloads the A2 list output tape, and calls Link 1 if Key No. 1 is down. c. Rewinds and unloads the A3 input tape and calls Link 1 if Key No. 2 is down. d. Rewinds and unloads punch tape and calls Link 1 if Key No. 3 is down. Used for all itinerary functions.
2	Iteration Print Results of each iteration, corrections, an RMS summary, etc., are printed on-line and the convergence test is bypassed. Used for orbit determination only.
3	Forced Termination The observation time that is encountered during the FITA integration process after Sense Switch 3 is turned on is taken as the last one for the current iteration and the least-squares process is initiated. If Sense Switch No. 2 also is down (On) the corrections are applied and integration for the next iteration is initiated in the normal manner if

Table 5-11. Action Resulting from 7094 Sense Switch Use (Concluded)

Sense Switch Number	Function
3 (continued)	MAXIT has not been exceeded. If Sense Switch No. 2 is not down, the run is terminated after the least-squares process is completed. In the case of a trajectory, or ephemeris generation run, integration is terminated as soon as Sense Switch No. 3 is turned on. If a data generation is in process, the GAINA integration is terminated and the GAINB link is called to compute and output the generated data. Used for orbit determination, ephemeris generation, and data generation.
4	<u>Impact Indication</u> A message is printed on-line whenever the input altitude at C(35) is reached either from above or below. Used for orbit determination and ephemeris generation.
5	<u>Residuals Print</u> All residuals are printed on-line. Used for orbit determination only.
6	<u>Apsis Print</u> Position information is printed on-line and off-line whenever the flight-path angle (β) passes through 90 degrees. Used for orbit determination and ephemeris generation.

5.5 OUTPUT

This section describes the printed output produced by the TRACE-D program for typical trajectory, orbit determination, data generation, and residuals analysis runs. The samples of actual output listings which are included are annotated to detail specific portions of the output data, and also are cross-referenced against applicable equations and definitions given elsewhere in this document. In the case of items occurring within more than one sample listing, citations are given for their first appearance only.

5.5.1 Output Common to All TRACE-D Runs

The initial pages of output generated from any TRACE-D run will present material similar to that tabulated in the sample output listing shown in Figure 5-29. Supplementary descriptive information relating to indicated areas of this listing is annotated in Table 5-12.

```
 •    XEQ
ENTRY POINTS TO SUBROUTINES REQUESTED FROM LIBRARY, 1
    (FPT)        SETH          LINK
      EXECUTION 19.585
CHAIN.   LINK NO.  1
  GINTEG      TRACE-C STANDARD INTEGRATION CONSTANTS                                        FINP INPUT CARD   1
  11    1             12   2             13   0                   STDINTEG 01            FINP INPUT CARD   2
   4    1              5   1             11   1.00002516          STDINTEG 02            FINP INPUT CARD   3
  23    .1          -9 26   1             30   1                   STDINTEG 03            FINP INPUT CARD   4
  31    .015625       32   64            33   1           -7      STDINTEG 04    2       FINP INPUT CARD   5
 134    4            135   1             37   .001                STDINTEG 05            FINP INPUT CARD   6
  38    6.            40   2820.1763    141   1                   STDINTEG 06            FINP INPUT CARD   7
  GC          TRACE-D STANDARD PHYSICAL CONSTANTS    10/ 1/65                              FINP INPUT CARD   8
   1    .43752691 -2  2    .55303935 -2   6   .5                  STDCONST 01            FINP INPUT CARD   9
   8    .43752691 -2  9    1.            10   10                  STDCONST 02            FINP INPUT CARD  10
  13    3260.8399     14   57.2957795    15   20925738            STDCONST 03            FINP INPUT CARD  11
  16    332951.3      17   .0122999      18   .814979             STDCONST 04            FINP INPUT CARD  12
  19    .107821       20   317.887       21   95.129              STDCONST 05    3       FINP INPUT CARD  13
  22    23454.865     23   3443.9336     24   20925738            STDCONST 06            FINP INPUT CARD  14
  25    348762.3      26   32.174        30   348762.3            STDCONST 07            FINP INPUT CARD  15
  31    1.5           32   1.0471976     33   3.14159265          STDCONST 08            FINP INPUT CARD  16
  34    298.3         35   300000.       36   82505.922           STDCONST 09            FINP INPUT CARD  17
```

Figure 5-29. Sample TRACE-D Program Common Output Listing

```
IIOB 4          I2    0          I3    4                  STDCONST 10        FINP INPUT CARD  18
I4   0          IOBJZ            2     .10823     -2      STDCONST 11        FINP INPUT CARD  19
3    -.23      -5 4    -.18      -5ITMAIX4                STDCONST 12        FINP INPUT CARD  20
NUMB            42    3443.9336   DRAG                    STDCONST 13        FINP INPUT CARD  21
I2   1          3     6.83       4     -15.684            STDCONST 14        FINP INPUT CARD  22
RREFC1.                                                   STDCONST 15        FINP INPUT CARD  23
GC   THE PREVIOUS CARDS WERE READ FROM FILE SERVICE                          FINP INPUT CARD  24
GC   STANDARD DECK            6,TH ORDER EARTH MODEL      10/15/65           FINP INPUT CARD  25
1    .043752695-1 2    .55304505 -2 15    20925741.              66 01       FINP INPUT CARD  26
24   20925741.    OBJT             2      1.723      -6          66 02       FINP INPUT CARD  27
3    1.856      -6 4    .4482     -6 5    .1643      -6          66 03       FINP INPUT CARD  28
6    .7278      -6 7    .1536     -6 8    .05719     -6          66 04       FINP INPUT CARD  29
9    .004781    -6 10   .1509     -6 11   .05329     -6          66 05       FINP INPUT CARD  30
12   .005842    -6 13   .004093   -6 14   .000846    -6          66 06       FINP INPUT CARD  31
15   .06611     -6 16   .03746    -6 17   .01166     -6          66 07       FINP INPUT CARD  32
18   .002256    -6 19   .0003328  -6 20   .00005153  -6          66 08       FINP INPUT CARD  33
OBLT               2    -13.18       3    5.31                   66 09       FINP INPUT CARD  34
4    -13.33        5    16.00        6    -136.68                66 10       FINP INPUT CARD  35
7    27.47         8    -1.10        9    36.01                  66 11       FINP INPUT CARD  36
10   -81.86        11   -2.60        12   -3.45                  66 12       FINP INPUT CARD  37
13   58.25         14   -15.32       15   157.21                 66 13       FINP INPUT CARD  38
```

Figure 5-29. Sample TRACE-D Program Common Output Listing (Continued)

```
16   113.15      17   -1.75        18    60.74                  66 14     FINP INPUT CARD  39
19   -17.15      20   -14.34       OBJZ                         66 15     FINP INPUT CARD  40
2    1082.76  -6 3    -2.693    -6 4     -1.56    -6            66 16     FINP INPUT CARD  41
5    -.006    -6 6    .39       -6 7     -.633    -6            66 17     FINP INPUT CARD  42
8    0.          9    .210      -61108  9                       66 18     FINP INPUT CARD  43
12   6           13   9            14    6                      66 19     FINP INPUT CARD  44
H1   ITINERARY 3 EXAMPLE.                                                 FINP INPUT CARD  45
DITIN 3          IYEAR 1965      IMNTH 4          IDAY  26                FINP INPUT CARD  46
 TZNE 0            HR  8           MIN  30         SEC  0                 FINP INPUT CARD  47
IICTYP4            IC  -135        2    32         3    90                FINP INPUT CARD  48
4    8             5   -668372.71  6    -1         DRAG .01               FINP INPUT CARD  49
DPRCDEX      X   IPRTIM1           I2   1          3    530               FINP INPUT CARD  50
4    20            5   1310                                               FINP INPUT CARD  51
END                                                                       FINP INPUT CARD  52
```

Figure 5-29. Sample TRACE-D Program Common Output Listing (Continued)

```
                                          +++++
                                      ++++     ++++
                                   +++             +++
   TTTTTTTTTTTT   RRRRRRRRRR     +                AA   +     CCCCCCCCCCC   EEEEEEEEEEE
   TTTTTTTTTTTT   RRRRRRRRRRR   +               AAAA    +    CCCCCCCCCCC   EEEEEEEEEEE
        TT        RR        RR  +             AA  AA    +    CC            EE
        TT        RRRRRRRRRRR  +            AA    AA     +   CC            EEEEEE
        TT        RRRRRRRRRR   +         AAAAAAAAAAAA    +   CC            EEEEEE
        TT        RR      RR   +       AAAAAAAAAAAAAA    +   CC            EE
        TT        RR        RR  +    AA             AA  +    CCCCCCCCCCC   EEEEEEEEEEE
        TT        RR         RR  + AA               AA  +    CCCCCCCCCCC   EEEEEEEEEEE
                                   +                  +
                                    +++             +++
                                       ++++     ++++          AD0140
                                           +++++              REFERENCE  6

ITINERARY 3 EXAMPLE.  7                                                       8

EPOCH         YEAR     MONTH     DAY     TZONE     HOUR     MIN       SEC  9
              1965.    4.        26.     0.        8.       30.000    0.

ENTERING LINK NO.  3 INITA  10
```

Figure 5-29. Sample TRACE-D Program Common Output Listing (Continued)

```
 END                                                                                              FINP INPUT CARD    1
 ITINERARY 3 EXAMPLE. 11

                                              INITIAL CONDITIONS
 X    =      -0.16354387E 08          ALPHA=        0.20663586E 03          A=        0.21574342E 08
 Y    =      -0.32024749E 07          DELTA=        0.31999999E 02          E=        0.20808183E-07
 Z    =       0.11432660E 08          BETA =        0.89999999E 02          I=        0.83221839E 02   12
 XDOT=        0.13575507E 05          AZ   =        0.79999999E 01          O=        0.20237660E 03
 YDOT=        0.28316946E 04          R    =        0.21574342E 08          U=        0.29668523E 03
 ZDOT=        0.21451353E 05          V    =        0.25543559E 05          T=        0.48652040E 03

   ATMOSPHERE - LOCKHEED 13      CDA/W = 0.09999999E-01 14  W/CDA = 0.99999999E 02 15
                                 D1    = 0.68299999E 01     D2    =-0.15684000E 02 16
                17
                                              EARTH MODEL
 GM=  0.55304505E-02  J2=  0.10827599E-02  J3= -0.26930000E-05  J4= -0.15600000E-05  J5= -0.59999999E-08 18
 J6=  0.38999999E-06  J7= -0.63299999E-06  J8=  0.                     J9=  0.20999999E-06 J10=  0.
      J21=  0.                    L21=  0.                    J52=  0.53290000E-07      L52= -0.26000000E 01
      J22=  0.17230000E-05        L22= -0.13180000E 02        J53=  0.58419999E-08      L53= -0.34499999E 01
      J31=  0.18560000E-05        L31=  0.53100000E 01        J54=  0.40929999E-08      L54=  0.58250000E 02
      J32=  0.44819999E-06        L32= -0.13330000E 02        J55=  0.84620000E-09      L55= -0.15320000E 02
      J33=  0.16429999E-06        L33=  0.16000000E 02        J61=  0.66110000E-07      L61=  0.15720999E 03   19
      J41=  0.72780000E-06        L41= -0.13668000E 03        J62=  0.37459999E-07      L62=  0.11314999E 03
      J42=  0.15360000E-06        L42=  0.27469999E 02        J63=  0.11660000E-07      L63= -0.17500000E 01
      J43=  0.57190000E-07        L43= -0.10999999E 01        J64=  0.22560000E-08      L64=  0.60739999E 02
      J44=  0.47809999E-08        L44=  0.36010000E 02        J65=  0.33279999E-09      L65= -0.17749999E 02
      J51=  0.15090000E-06        L51= -0.81859999E 02        J66=  0.51530000E-10      L66= -0.14340000E 02

 THE FOLLOWING BODIES ARE USED FOR PLANETARY PERTURBATIONS
 NONE 20
 FORMULATION
          COWELL (EQS.OF MOTION) 21
 DIFFERENTIAL EQUATION SUBROUTINE
```

Figure 5-29. Sample TRACE-D Program Common Output Listing (Continued)

```
GAUSS-JACKSCN          E BAR=  0.1000E-C9     A= 1.000    RATIO OF COWELL STEP TO RUNGE KUTTA  4
STEP SIZE -     INITIAL=     0.100CE 01     MINIMUM=     0.1563E-01     MAXIMUM=     0.64C0E 02
DO NOT RECOMPUTE PERTURBATIONS FCR CORRECTOR
```
(22)

EQUATIONS OF MOTION   USE N1 = 9 N2= 6 (23)

CONSTANTS

	DECIMAL	OCTAL		DECIMAL	OCTAL
OMEGA E	0.43752695E-02	171436571536	GM	0.55304505E-02	171552343100
ALPHA G	0.37312833E 01	202735465305			
FT/KM	0.32808398E 04	214632065602	EARTH RADIUS - FT	0.20925741E 08	231477232264
E.R./A.U.	0.23454864E 05	217556366727	N.M./E.R.	0.34439336E 04	214656373600
I-O, DISTANCE	0.20925741E 08	231477232264	I-O, VELOCITY	0.34876230E 06	223524455115
G	0.32173999E 02	206401310550	FT/SEC//E.R./N.M.	0.34876230E 06	223524455115
DEG/SEC//RAD/MIN	0.10471976E 01	201414052221	1/EPS.	0.29830000E 03	211452231463
RELATIVE MASS(SUN)	0.33295130E 06	223505113515	(MOON)	0.12299900E-01	172623026050
(VENUS)	0.81497900E 00	200641211666	(MARS)	0.10782100E-00	175671505015
(JUPITER)	0.31788700E 03	211475706112	(SATURN)	0.95129000E 02	207574410142

(24)

Figure 5-29. Sample TRACE-D Program Common Output Listing (Concluded)

Table 5-12. Common Output Listing Description

Item	Description	Page Reference
1	FORTRAN II monitor message giving names of subroutines called from library by REIN link.	4-1
2	Card images of the first seven FINP input cards. These input data contain the standard entries for the integration constants array.	3-63 Appendix A Appendix F
3	Card images for FINP Input Cards 8 through 22. Standard values of physical constants and option indicators are shown.	4-7 Appendix A Appendix F
4	Card images for FINP input cards numbers 23 through 42. Numbers shown are entries for a gravity field model including non-zero co-efficients through Degree 9 for the zonal and through Degree and Order 6 for the tesseral and sectoral terms.	3-46/3-49 Appendix A Appendix F
5	Card images for FINP Input Cards 43 through 50. These input data specify a single-case trajectory run.	5-1/5-17 Appendix F
6.	Program identification. AD014D is the Aerospace Corporation Computation and Data Processing Center program number for accounting purposes. The word REFERENCE means that the program which produced this output is associated with the basic or reference version of TRACE-D, or the version wherein no modifications are included. Output produced by a modified program version would reflect the appropriate modification numbers in lieu of the word REFERENCE. It should be noted that all material contained in the present document is directed toward description of the reference program version.	
7	H1 header card entry.	Figure 5-1
8	If any header information had been input on an H2 card, the entry would have been printed at this location.	Figure 5-1

Table 5-12. Common Output Listing Description (Continued)

Item	Description	Page Reference
9	Epoch time	5-4
10	Program link identification. Each time a different link of the program is called a remark of this nature is printed.	4-1/4-5
11	Card image of the final (second) END card in the input deck. This card is required because the FINP routine is called in the INITA link as well as in the CHAIN link. Under certain conditions additional FINP input data may be read at this point.	4-1/4-5 5-60 Appendix F
12	Trajectory initial conditions in three coordinate systems. Initial-condition values as shown are the result of transformations which have been applied to the input values. The transformation for the input coordinate set (α, δ, β, A, r, v in this case) consists of conversion from decimal to octal numbers, conversion of units from feet, degrees, and seconds to earth radii, radians, and minutes, and performance of corresponding inverse conversions for output. The two other types of elements sets also require accomplishment of coordinate-system transformations in addition to the number- and units-systems conversions noted above. Accuracy of the values as printed therefore is subject to numerical and roundoff errors. Quantities in the left-hand column are position and velocity components in the basic vernal equinox coordinate system, with units of feet and of feet per second. The center column gives the usual ADBARV spherical system coordinates (i.e., Type-2 initial conditions in units of feet, feet per second, and degrees). The right-hand column from top to bottom contains orbit semi-major axis, eccentricity, inclination, right ascension of ascending node, argument of perigee, and time of last perigee passage in minutes from midnight of epoch day. Other units are feet and degrees.	3-1/3-5 3-9/3-12 5-4, 5-5

Table 5-12. Common Output Listing Description (Continued)

Item	Description	Page Reference
13	Identification of atmosphere model to be used in computing drag force. LOCKHEED and ARDC 1959 refer to the Lockheed-Jacchia and the ARDC 1959 model atmospheres, respectively.	3-50, 3-51 5-6
14	Reciprocal of ballistic coefficient, $C_D A/W$	3-50 5-6
15	Ballistic coefficient, $W/C_D A$	3-50 5-6
16	d_1 and d_2 are values of certain constants in the Lockheed-Jacchia atmosphere density expressions.	5-6 Appendix C
17	The product GM, or the universal gravitational constant times the mass of the earth (frequently designated by μ), expressed in units of earth radii cubed per minute squared.	3-46, 3-47 Appendix A
18	Unitless coefficients of the zonal harmonic terms in the earth potential field expansion (i.e., J_2 through J_{10}).	3-47 Appendix A Appendix F
19	Coefficients and longitudinal arguments of tesseral and sectoral terms in earth potential field expansion, with J indicating a coefficient and L an argument in degrees. The digits following J or L are the associated Legendre polynominal degree and order, respectively.	3-47 Appendix A Appendix F
20	Names of solar-system bodies included in computation of perturbative accelerations.	3-49 5-7 Appendix A
21	Trajectory method indicator. The Cowell formulation of the equations of motion is the only trajectory method available in the present TRACE-D program.	2-2 3-46
22	Numerical integration parameters. The Gauss-Jackson method (subroutine COW) is the only integration method available in the present TRACE-D program.	3-63 Appendix A Appendix F

```
                                    +++++
                                 ++++     ++
                               +++           +++
  TTTTTTTTTTTT   RRRRRRRRR    +          AA     +    CCCCCCCCCCC   EEEEEEEEEEE
  TTTTTTTTTTTT   RRRRRRRRRR  +         AAAA      +   CCCCCCCCCCC   EEEEEEEEEEE
       TT        RR      RR  +       AA  AA      +   CC            EE
       TT        RRRRRRRRRR +      AA    AA       +  CC            EEEEEE
       TT        RRRRRRRRR  +    AAAAAAAAAA       +  CC            EEEEEE
       TT        RR     RR  +  AAAAAAAAAAAA       +  CC            EE
       TT        RR      RR  + AA          AA    +   CCCCCCCCCCC   EEEEEEEEEEE
       TT        RR       RR + AA          AA    +   CCCCCCCCCCC   EEEEEEEEEEE
                               +               +
                                +++         +++
                                  ++++   ++++      AD014D
                                     +++++         REFERENCE


EPOCH         YEAR     MONTH    DAY    TZONE    HOUR    MIN      SEC
              1964.     2.      30.     0.       0.      0.       0.


 ENTERING LINK NO.  2 TRAIN

PERMANENT STATIONS
      SIG    REF    LATITUDE          LONGITUDE         HEIGHT

AA     0.     0.    0.75000000E 02    0.30000000E 03    0.09999999E 04
BB     0.     0.    0.45000000E 02    0.23999999E 03    0.09999999E 03
CC     0.     0.    0.                0.31500000E 03    0.50000000E 03
DD     0.     0.    0.                0.15000000E 02    0.20000000E 03
EE     0.     0.   -0.15000000E 02    0.13500000E 03    0.09999999E 03
FF     0.     0.   -0.65000000E 02    0.75000000E 02    0.50000000E 04

OBSERVATIONS
 MO DY HR MN  SEC        STA  TYP
  3  1  1 41  0.          CC   10   0.98321319E 07   0.11314791E 03   0.60135999E 01
  3  1  1 41  0.          CC   70  -0.13903117E 05  -0.                -0.
  3  1  1 41  0.          CC   50   0.21862530E 08  -0.91494808E 07  -0.38542949E 07
          2                3    4                          5
                                                                                (1)
```

Figure 5-31. Sample TRACE-D Program Tracking Output Listing

Table 5-12. Common Output Listing Description (Concluded)

Item	Description		Page Reference
24 (cont)	m. N.M./E.R.	Number of nautical miles per earth radius	
	n. I-O, VELOCITY	Conversion factor for input/ output in feet per second/ earth radii per minute	
	o. FT/SEC// E.R./MIN.	Units conversion factor in feet per second/earth radii per minute	
	p. 1/EPS (1/ε)	Earth flattening (reciprocal of earth ellipticity)	
	q. (MOON)	Mass of moon relative to mass of earth	
	r. (MARS)	Mass of Mars relative to mass of earth	
	s. (SATURN)	Mass of Saturn relative to mass of earth	

5.5.2 Trajectory Output

The listed output created by a typical trajectory run is shown in Figure 5-30. Supplementary descriptive information relating to the indicated areas of this listing is annotated in Table 5-13.

```
ENTERING LINK NO.  4 INITB

ENTERING LINK NO.  8

  4/26/65    8 HRS.  30 MIN.      -0.        SEC. 1
ME,MH,ST,DT          X,R            XDOT,V          LAT,LONG,H,SBV   ALPHA,DELTA,BETA,A    REV,NPD,NPDK,NREG
    0.          -0.16354387E 08    0.13575507E 05        32.17318      206.63586044          0.
  510.00000     -0.82024749E 07    C.28316946E 04       225.0000C       31.99999928          0.
30599.99976  2   0.11437660E 08  3 C.21451353E 05  4    169.99993  5    89.99999809  6       0.
    0.25000      0.21574342E 08    0.25543559E 05        32.16780        7.99999994          0.

  4/26/65    8 HRS.  50 MIN.      -0.        SEC.
ME,MH,ST,DT          X,R            XDOT,V          LAT,LONG,H,SBV   ALPHA,DELTA,BETA,A    REV,NPD,NPCK,NREG
   20.00000      0.88748144E 07    0.21173423E 05        65.65669        7.21385860          0.
  530.00000      0.11233307E 07    0.10008825E 05        20.55430       65.51185417          0.
31799.99951      0.19640148E 08   -C.10122698E 05       117.48816       89.96440029          0.
    1.00000      0.21581465E 08    0.25513906E 05        65.65191      163.44273567          0.

  4/26/65    9 HRS.   6 MIN.      20.31693 SEC.
ME,MH,ST,DT          X,R            XDOT,V          LAT,LONG,H SBV   ALPHA,DELTA,SETA,A    REV,NPD,NPDK,NREG
```

Figure 5-30. Sample TRACE-D Program Trajectory Listing

```
   36.33862    0.19969025E 08   -0.11518399E 04          0.00008     22.34449363          0.50000
  546.33862    0.82080155E 07    0.27833810E 04         31.59920      0.00007777          0.
32780.31641    0.29306004E 02   -0.25357457E 05        109.34348     90.01619530          0.
    1.00000    0.21590124E 08    0.25535751E 05          0.00008    173.22540474          0.

    4/26/65   9 HRS.  10 MIN.     -0.      SEC.
ME,MM,ST,DT          X,R              XDOT,V      LAT,LONG,H,SBV  ALPHA,DELTA,BETA,A  REV,NPD,NPDK,NREG
   40.00000    0.190475[illegible] 08   -0.71894251E 04        -14.87704     24.14084196          0.54136
  550.00000    0.853670[illegible]E 07    0.19215893E 03         32.47760    -14.78183973          0.
32999.99951   -0.55078332E 07   -0.24502593E 05        109.67533     90.03582287          0.
    1.00000    0.21587552E 08    0.25536284E 05        -14.87409    172.99185562          0.

    4/26/65   9 HRS.  30 MIN.     -0.      SEC.
ME,MN,ST,DT          X,R              XDOT,V      LAT,LONG,H,SBV  ALPHA,DELTA,BETA,A  REV,NPD,NPDK,NREG
   60.00000   -0.31340595E 07   -0.23349[illegible]E 05        -80.84501    155.16534996          0.76736
  570.00000    0.14504394E 07   -0.99444223E 04        158.48841    -80.78437138          0.
34199.99951   -0.21285260E 08    0.27671982E 04        136.22682     90.01528549          0.
    1.00000    0.21563589E 08    0.25528986E 05        -80.04303     47.53156471          0.

    4/26/65   9 HRS.  50 MIN.     -0.      SEC.
ME,MM,ST,DT          X,R              XDOT,V      LAT,LONG,H,SBV  ALPHA,DELTA,BETA,A  REV,NPD,NPDK,NREG
   80.00000   -0.19980005E 08    0.18433792E 03         -2.30664    202.03945732          0.99358
  590.00000   -0.80884813E 07   -0.31801306E 04        200.34884     -2.29121947          0.
35399.99951   -0.86243754E 06    0.25357323E 05        106.45209     89.98230743          0.
    1.00000    0.21572443E 08    0.25556624E 05         -2.30618      6.77947885          0.

    4/26/65   9 HRS.  50 MIN.    33.99216 SEC.
ME,MM,ST,DT          X,R              XDOT,V      LAT,LONG,H,SBV  ALPHA,DELTA,BETA,A  REV,NPD,NPDK,NREG
   80.56654   -0.19957592E 08    0.11376827E 04         -0.00005    202.31175423          1.00000
  590.56654   -0.81899897E 07   -0.27915415E 04        200.47911     -0.00005180          0.
35433.99170   -0.15506885E 02    0.25377961E 05        106.47580     89.98370457          0.
    1.00000    0.21572700E 08    0.25556367E 05         -0.00005      6.77400285          0.

 A,E,I,O,U,T          MEAN ANM=   17.04108       APOGEE= 3557.14874
0.21592714E 08        TRUE ANM=   17.07367          HT  =  114.20410
0.96155182E-03            ODOT=   -1.05319  (8)  PERIGEE= 3550.25778  (9)
0.83225997E 02  (7)       UDOT=   -4.15987          HT  =  107.31314
0.20231175E 03                                 PERIOD(K)=   88.56033
0.34292626E 03                                 PERIOD(A)=   88.42487
0.58437441E 03                                 PERIOD(N)=   88.48662

    4/26/65  10 HRS.  10 MIN.     -0.      SEC.
```

Figure 5-30. Sample TRACE-D Program Trajectory Listing (Continued)

ME,MM,ST,DT	X,R	XDOT,V	LAT,LONG,H,SBV	ALPHA,DELTA,BETA,A	REV,NPD,NPDK,NREG
100.00000	-0.28308333E 07	0.23396437E 05	77.25204	233.78476715	1.21967
610.00000	-0.38656866E 07	0.89774919E 04	227.08046	77.16896820	0.
36599.99951	0.21036513E 08	0.48048424E 04	117.87643	89.98526669	0.
1.00000	0.21575265E 08	0.25516174E 05	77.24929	32.13441420	0.

4/26/65 10 HRS. 30 MIN. -0. SEC.

ME,MM,ST,DT	X,R	XDOT,V	LAT,LONG,H,SBV	ALPHA,DELTA,BETA,A	REV,NPD,NPCK,NREG
120.00000	0.19142365E 08	0.68615822E 04	19.59969	19.87085485	1.44549
630.00000	0.69184285E 07	0.58768047E 04	8.15285	19.47840810	0.
37800.00000	0.71991809E 07	-0.22879769E 05	110.59309	89.99060440	0.
1.00000	0.21589882E 08	0.25531579E 05	19.59590	172.81200790	0.

4/26/65 10 HRS. 34 MIN. 49.54860 SEC.

ME,MM,ST,DT	X,R	XDOT,V	LAT,LONG,H,SBV	ALPHA,DELTA,BETA,A	REV,NPD,NPCK,NREG
120.82581	0.19977789E 08	-0.11488482E 04	0.00007	22.27862501	1.50000
634.82581	0.81847834E 07	0.27840728E 04	9.35086	0.00007069	0.
38089.54834	0.26637282E 02	-0.25358191E 05	109.22637	90.01715755	0.
1.00000	0.21589413E 08	0.25536420E 05	0.00007	173.22672272	0.

4/26/65 10 HRS. 50 MIN. -0. SEC.

ME,MM,ST,DT	X,R	XDOT,V	LAT,LONG,H,SBV	ALPHA,DELTA,BETA,A	REV,NPD,NPDK,NREG
140.00000	0.36072009E 07	-0.21354848E 05	-61.14668	34.65077686	1.67144
650.00000	0.59490443E 07	-0.72052163E 04	17.91908	-60.98375130	0.
38999.99951	-0.18863338E 08	-0.11990058E 05	115.01337	90.05194378	0.
1.00000	0.21570867E 08	0.25528536E 05	-61.14142	165.91061783	0.

4/26/65 11 HRS. 10 MIN. -0. SEC.

ME,MM,ST,DT	X,R	XDOT,V	LAT,LONG,H,SBV	ALPHA,DELTA,BETA,A	REV,NPD,NPDK,NREG
160.00000	-0.16547703E 08	-0.13283781E 05	-36.75079	197.19019318	1.89758
670.00000	-0.51192676E 07	-0.80389226E 04	175.44482	-36.56649208	0.
40199.99951	-0.12848350E 08	0.20290399E 05	109.56472	89.97178555	0.
1.00000	0.21566489E 08	0.25549626E 05	-36.74509	8.44873822	0.

4/26/65 11 HRS. 19 MIN. 3.09402 SEC.

ME,MM,ST,DT	X,R	XDOT,V	LAT,LONG,H,SBV	ALPHA,DELTA,BETA,A	REV,NPD,NPDK,NREG	
169.05157	-0.19966959E 08	0.11340065E 04	-0.00009	202.24542809	2.00000	
679.05157	-0.81668408E 07	-0.27925831E 04	171.23096	-0.00009321	88.48503	10
40743.09375	-0.35094570E 02	0.25377961E 05	106.45809	89.98305702	0.	
1.00000	0.21572593E 08	0.25556317E 05	-0.00009	6.77306515	-0.06633	
A,E,I,O,U,T	MEAN ANM= 17.83829		APOGEE= 3557.08099			

Figure 5-30. Sample TRACE-D Program Trajectory Listing (Concluded)

Table 5-13. Trajectory Output Listing Description

Item	Description	Page Reference
1	Date and time of day (Greenwich Mean Time) with which the quantities in the print block following are to be associated.	5-11
2	Minutes from epoch, minutes from midnight of current day, system time (i.e., seconds from midnight of current day), and current integration step size in minutes.	5-11
3	x, y, z, r: Components and magnitude of the radius vector from geocenter to satellite in the basic coordinate system in units of feet.	3-2
4	$\dot{x}$, $\dot{y}$, $\dot{z}$, v: Components and magnitude of the inertial velocity vector with respect to the basic coordinate system in units of feet per second.	3-2, 3-3
5	Geodetic latitude, in degrees, of the point where the radius vector intersects the ellipsoidal surface of the earth, geographic longitude measured east from Greenwich in degrees, altitude of the satellite above the oblate earth in nautical miles, and geodetic latitude of the subvehicle point in degrees. All latitude quantities are considered positive north of the equator and negative south of the equator.	3-64
6	α, δ, β, A: Right ascension of satellite position, declination of satellite position, flight path angle, and inertial azimuth of velocity vector in units of degrees.	3-3 3-9
7	a, e, i, Ω, ω, τ: The classical elements as computed from X, $\dot{X}$ at the ascending node in units of feet, degrees, and minutes from midnight of epoch.	3-4 3-11 5-8
8	M, ν, $\dot{\Omega}$, $\dot{\omega}$: Mean anomaly and true anomaly in degrees, and nodal regression rate and rate of advance of the line of apsides in degrees per day as computed from X, $\dot{X}$ at the ascending node using closed-form expressions.	3-4, 3-5 3-64, 3-65 5-8

Table 5-13. Trajectory Output Listing Description (Concluded)

Item	Description	Page Reference
9	Radial distance at apogee, height of apogee above the oblate earth, and radial distance and altitude of perigee in nautical miles; Keplerian period, anomalistic period, and nodal period in minutes as computed from X, $\dot{X}$ at the ascending node using closed-form expressions.	3-65, 3-66 5-8
10	Revolution number, nodal period in minutes, nodal period decay rate in minutes per revolution, and nodal regression rate in degrees per revolution. Due to the fact that the nodal period is calculated by simple subtraction of ascending-node crossing times, this quantity cannot be determined until two ascending node crossings have been detected. Also, since the nodal-period decay rate is computed by differencing the nodal-period values at successive ascending nodes, this rate cannot be calculated until three ascending nodes have been crossed. The nodal regression rate is computed by differencing values of right ascension at successive ascending nodes.	3-66 5-15

5.5.3 Tracking Output

Listed output produced by a typical tracking (orbit determination) run is shown in Figure 5-31. Supplementary descriptive information relating to the indicated areas of this listing is annotated in Table 5-14.

```
TTTTTTTTTTTT     RRRRRRRRR                 +++++
TTTTTTTTTTTT     RRRRRRRRRR            ++++     ++
     TT          RR       RR        +++            +++
     TT          RRRRRRRRRR        +          AA      +     CCCCCCCCCCC   EEEEEEEEEEE
     TT          RRRRRRRRR        +          AAAA      +    CCCCCCCCCCC   EEEEEEEEEEE
     TT          RR      RR       +        AA  AA      +    CC            EE
     TT          RR       RR      +       AA   AA       +   CC            EEEEEE
     TT          RR        RR     +     AAAAAAAAAA      +   CC            EEEEEE
                                  +    AAAAAAAAAAAA     +   CC            EE
                                  +   AA          AA    +   CCCCCCCCCCC   EEEEEEEEEEE
                                  + AA            AA    +   CCCCCCCCCCC   EEEEEEEEEEE
                                   +                   +
                                    +++             +++
                                       ++++     ++++       AD014D
                                           +++++           REFERENCE

EPOCH         YEAR     MONTH     DAY     TZONE     HOUR     MIN     SEC
              1964.    2.        30.     0.        0.       0.      0.

ENTERING LINK NO.  2 TRAIN

PERMANENT STATIONS
      SIG   REF    LATITUDE           LONGITUDE           HEIGHT

AA    0.    0.     0.75000000E 02     0.30000000E 03      0.09999999E 04
BB    0.    0.     0.45000000E 02     0.23999999E 03      0.09999999E 03
CC    0.    0.     0.                 0.31500000E 03      0.50000000E 03
DD    0.    0.     0.                 0.15000000E 02      0.20000000E 03   (1)
EE    0.    0.    -0.15000000E 02     0.13500000E 03      0.09999999E 03
FF    0.    0.    -0.65000000E 02     0.75000000E 02      0.50000000E 04

OBSERVATIONS
MO DY HR MN  SEC       STA  TYP
 3  1  1 41  0.         CC   10   0.98321319E 07   0.11314791E 03   0.60135999E 01
 3  1  1 41  0.         CC   70  -0.13903117E 05  -0.              -0.
 3  1  1 41  0.         CC   50   0.21862530E 08  -0.91494808E 07  -0.38542949E 07
      (2)              (3)  (4)                      (5)
```

Figure 5-31. Sample TRACE-D Program Tracking Output Listing

```
       (2)                     (3)(4)                 (5)
 3  1  1 44 30.0000         CC  20  0.21609362E 03  0.76283599E 01  0.28381418E 03
 3  1  1 44 30.0000         CC  30  0.15823904E 03  0.24654999E 01 -0.
 3  1  1 44 30.0000         CC  40  0.25692700E 01  0.17050250E 03 -0.
 3  1  1 44 45.0000         CC  10  0.77532610E 07  0.79464659E 02  0.14204849E 02
 3  1  1 44 45.0000         CC  70 -0.32062510E 04 -0.              -0.
 3  1  1 44 45.0000         CC  50  0.21350726E 08 -0.10895403E 08  0.13896960E 07
 3  1  1 44 45.0000         CC  20  0.21596242E 03  0.10287820E 02  0.28414629E 03
 3  1  1 44 45.0000         CC  30  0.15810767E 03  0.32327599E 01 -0.
 3  1  1 44 45.0000         CC  40  0.34359200E 01  0.16739210E 03 -0.
 3  1  1 45  0.             CC  10  0.77125780E 07  0.76455370E 02  0.14104610E 02
 3  1  1 45  0.             CC  70 -0.22362710E 04 -0.              -0.
 3  1  1 45  0.             CC  50  0.21274888E 08 -0.10990801E 08  0.17395010E 07
 3  1  1 45  0.             CC  20  0.21563589E 03  0.13108400E 02  0.28451524E 03
 3  1  1 45  0.             CC  30  0.15779090E 03  0.41204900E 01 -0.
 3  1  1 45  0.             CC  40  0.43025799E 01  0.16435810E 03 -0.
 3  1  1 45 15.0000         CC  10  0.76864460E 07  0.73666049E 02  0.14379250E 02
 3  1  1 45 15.0000         CC  70 -0.12524389E 04 -0.              -0.
 3  1  1 45 15.0000         CC  50  0.21197424E 08 -0.11086029E 08  0.20902880E 07
 3  1  1 45 15.0000         CC  20  0.21535423E 03  0.15749940E 02  0.28490523E 03
 3  1  1 45 15.0000         CC  30  0.15758481E 03  0.49793799E 01 -0.
 3  1  1 45 15.0000         CC  40  0.51692399E 01  0.16141474E 03 -0.
 3  1  1 45 30.0000         CC  10  0.76749189E 07  0.70750369E 02  0.14425959E 02
 3  1  1 45 30.0000         CC  70 -0.26039699E 03 -0.              -0.
 3  1  1 45 30.0000         CC  50  0.21109664E 08 -0.11177154E 08  0.24395909E 07
 3  1  1 45 30.0000         CC  20  0.21517614E 03  0.18466400E 02  0.28512391E 03
 3  1  1 45 30.0000         CC  30  0.15739927E 03  0.58595099E 01 -0.
 3  1  1 45 30.0000         CC  40  0.60359100E 01  0.15857341E 03 -0.
 3  1  1 45 45.0000         CC  10  0.76784709E 07  0.67895159E 02  0.14538459E 02
 3  1  1 45 45.0000         CC  70  0.73383900E 03 -0.              -0.
 3  1  1 45 45.0000         CC  50  0.21019173E 08 -0.11265577E 08  0.27869489E 07
 3  1  1 45 45.0000         CC  20  0.21488728E 03  0.21205939E 02  0.28544910E 03
 3  1  1 45 45.0000         CC  30  0.15715138E 03  0.66778099E 01 -0.
 3  1  1 45 45.0000         CC  40  0.69025800E 01  0.15584276E 03 -0.

OBSERVATIONS                                                            (6)
NO DY HR MN   SEC       STA  TYP
 3  1  1 46  0.             CC  10  0.76969180E 07  0.65183309E 02  0.14347190E 02
 3  1  1 46  0.             CC  70  0.17242870E 04 -0.              -0.
 3  1  1 46  0.             CC  50  0.20924477E 08 -0.11350807E 08  0.31355510E 07
 3  1  1 46  0.             CC  20  0.21470694E 03  0.24153209E 02  0.28564927E 03
 3  1  1 46  0.             CC  30  0.15693141E 03  0.74402098E 01 -0.
 3  1  1 46  0.             CC  40  0.77692299E 01  0.15322871E 03 -0.
```

Figure 5-31. Sample TRACE-D Program Tracking Listing (Continued)

3	1	17	10	0.	EE	30	0.16732769E 03	-0.25370070E 02	-0.
3	1	17	10	0.	EE	40	0.24331030E 02	0.12113716E 03	-0.
3	1	17	10	15.0000	EE	10	0.10346130E 08	0.24395946E 03	0.44196099E 01
3	1	17	10	15.0000	EE	70	0.95745400E 03	-0.	-0.
3	1	17	10	15.0000	EE	50	-0.74369539E 07	0.20534982E 08	-0.99715868E 07
3	1	17	10	15.0000	EE	20	0.99387829E 02	-0.26233800E 02	0.92726439E 02
3	1	17	10	15.0000	EE	30	0.16711248E 03	-0.24479650E 02	-0.
3	1	17	10	15.0000	EE	40	0.25464349E 02	0.12193254E 03	-0.
3	1	17	10	30.0000	EE	10	0.10365885E 08	0.24585570E 03	0.44875199E 01
3	1	17	10	30.0000	EE	70	0.16694479E 04	-0.	-0.
3	1	17	10	30.0000	EE	50	-0.73651569E 07	0.20712515E 08	-0.96525369E 07
3	1	17	10	30.0000	EE	20	0.99948139E 02	-0.24321640E 02	0.92214099E 02
3	1	17	10	30.0000	EE	30	0.16672221E 03	-0.23675629E 02	-0.
3	1	17	10	30.0000	EE	40	0.24597680E 02	0.12277120E 03	-0.
3	1	17	10	45.0000	EE	10	0.10396121E 08	0.24805012E 03	0.43028200E 01
3	1	17	10	45.0000	EE	70	0.23755059E 04	-0.	-0.
3	1	17	10	45.0000	EE	50	-0.72929390E 07	0.20885453E 08	-0.93314240E 07
3	1	17	10	45.0000	EE	20	0.10053124E 03	-0.22263350E 02	0.91747560E 02
3	1	17	10	45.0000	EE	30	0.16646255E 03	-0.22851489E 02	-0.
3	1	17	10	45.0000	EE	40	0.23730999E 02	0.12365611E 03	-0.
3	1	17	11	0.	EE	10	0.10437164E 08	0.24996119E 03	0.40536299E 01
3	1	17	11	0.	EE	70	0.30729339E 04	-0.	-0.
3	1	17	11	0.	EE	50	-0.72165290E 07	0.21051491E 08	-0.90104569E 07
3	1	17	11	0.	EE	20	0.10097981E 03	-0.20339239E 02	0.91376829E 02
3	1	17	11	0.	EE	30	0.16622707E 03	-0.22047610E 02	-0.
3	1	17	11	0.	EE	40	0.22864330E 02	0.12459043E 03	-0.
3	1	17	11	15.0000	EE	10	0.10488293E 08	0.25189796E 03	0.38566599E 01
3	1	17	11	15.0000	EE	70	0.37600999E 04	-0.	-0.
3	1	17	11	15.0000	EE	50	-0.71399729E 07	0.21214408E 08	-0.86866589E 07

THE FOLLOWING MEASUREMENTS ARE FROM SATELLITE NO. 2

7

OBSERVATIONS

MO	DY	HR	MN	SEC	STA	TYP			
2	23	12	22	30.0000	AA	10	0.58759200E 07	0.34020685E 03	0.94766000E 00
2	23	12	22	45.0000	AA	10	0.55810819E 07	0.33785684E 03	0.15088599E 01
2	23	12	23	0.	AA	10	0.52955820E 07	0.33542579E 03	0.21839800E 01
2	23	12	23	15.0000	AA	10	0.50210720E 07	0.33276861E 03	0.30408499E 01
2	23	12	23	30.0000	AA	10	0.47596360E 07	0.32954984E 03	0.41588699E 01
2	23	12	23	45.0000	AA	10	0.45135390E 07	0.32616091E 03	0.49468699E 01
2	23	12	24	0.	AA	10	0.42860350E 07	0.32235774E 03	0.58443999E 01

Figure 5-31. Sample TRACE-D Program Tracking Listing (Continued)

```
 3  1  1 48 15.0000      CC  10  0.84911240E 07  0.41768279E 02  0.10975160E 02
 3  1  1 48 30.0000      CC  10  0.86410510E 07  0.39422979E 02  0.10359350E 02
 3  1  1 48 45.0000      CC  10  0.88012770E 07  0.37307169E 02  0.97085898E 01
 3  1  1 49  0.          CC  10  0.89714640E 07  0.35315679E 02  0. 1296698E 01
 3  1  1 49 15.0000      CC  10  0.91505419E 07  0.33332289E 02  0.83591498E 01
 3  1  1 49 30.0000      CC  10  0.93383089E 07  0.31427220E 02  0.77406099E 01
 3  1  1 49 45.0000      CC  10  0.95343018E 07  0.29774369E 02  0.70112099E 01
 3  1  1 50  0.          CC  10  0.97377059E 07  0.28083420E 02  0.62559299E 01
 3  1  1 50 15.0000      CC  10  0.99481539E 07  0.26310389E 02  0.57063799E 01
 3  1  1 50 30.0000      CC  10  0.10165156E 08  0.24615350E 02  0.49160099E 01
 3  1  1 50 45.0000      CC  10  0.10388323E 08  0.23178619E 02  0.41711599E 01
 3  1  1 51  0.          CC  10  0.10617165E 08  0.21794420E 02  0.34308999E 01
 3  1  1 51 15.0000      CC  10  0.10851245E 08  0.20539980E 02  0.282467[illegible]E 01
 3  1  1 51 30.0000      CC  10  0.11089943E 08  0.19250680E 02  0.21415799E 01
 3  1  1 51 45.0000      CC  10  0.11333571E 08  0.17896549E 02  0.16304100E 01
 3  1  1 52  0.          CC  10  0.11581053E 08  0.16795029E 02  0.97368000E 00

PARAMETERS TO BE CORRECTEC

 ALPHA CELTA BETA  A      R      V    THRST1THRST22 ALPH3 ALPH  8

    3 ITERATICNS (MAXIMUM),    10 PARAMETERS

  858 OBSERVATION TIMES, 3487 OBSERVATIONS,    6 STATIONS, 6061 CELLS IN COMPACTED DATA LIST
    11 FLOCKS (IF BCD INPUT)  9

MEASUREMENTS FROM    3  SATELLITES ARE BEING USED IN THIS FIT

 ENTERING LINK NO.  3 INITA
```

Figure 5-31. Sample TRACE-D Program Tracking Listing (Continued)

```
END                                                          FINP INPUT CARD   1

BOUNDS
0.100000E 010.100000E 010.100000E 010.100000E 010.100000E 030.100000E 020.500000E-040.500000E-030.100000E 01
0.100000E 01

                         SIGMA TABLE
  1 .1000000E 03   2 .1000000E-00   3 .1000000E-00 19 .1000000E 01

                               INITIAL CONDITIONS

X    =      -0.22390211E 08        ALPHA=      0.15882923E 03        A=     0.24010736E 08
Y    =       0.86714486E 07        DELTA=      0.                    E=     0.14526680E-06
Z    =       0.                    BETA =      0.89999999E 02        I=     0.10500000E 03
XDOT=        0.22632358E 04        AZ   =      0.34499999E 03        O=     0.15882923E 03
YDOT=        0.58438147E 04        R    =      0.24010739E 08        U=     0.18615689E 03
ZDOT=        0.23387897E 05        V    =      0.24212933E 05        T=    -0.50146602E 02

  ATMOSPHERE - LOCKHEED        CDA/W = 0.20000000E-01     W/CDA = 0.49999999E 02
                               D1    = 0.68299999E 01     D2    =-0.15684000E 02

                                   EARTH MODEL
GM=  0.55304505E-02  J2=  0.10827599E-02  J3= -0.26930000E-05  J4= -0.15600000E-05  J5= -0.59999999E-08
J6=  0.38999999E-06  J7= -0.63299999E-06  J8=  0.                J9=  0.20999999E-06  J10=  0.
   J21=  0.                  L21=  0.                  J52=  0.53290000E-07    L52= -0.26000000E 01
   J22=  0.17230000E-05      L22= -0.13180000E 02      J53=  0.58419999E-08    L53= -0.34499999E 01
   J31=  0.18560000E-05      L31=  0.53100000E 01      J54=  0.40929999E-08    L54=  0.58250000E 02
   J32=  0.44819999E-06      L32= -0.13330000E 02      J55=  0.84620000E-09    L55= -0.15320000E 02
   J33=  0.16429999E-06      L33=  0.16000000E 02      J61=  0.66110000E-07    L61=  0.157'0999E 03
   J41=  0.72780000E-06      L41= -0.13668000E 03      J62=  0.37459999E-07    L62=  0.11..4999E 03
   J42=  0.15360000E-06      L42=  0.27469999E 02      J63=  0.11660000E-07    L63= -0.17500000E 01
   J43=  0.57190000E-07      L43= -0.10999999E 01      J64=  0.22560000E-08    L64=  0.60739999E 02
   J44=  0.47809999E-08      L44=  0.36010000E 02      J65=  0.33279999E-09    L65= -0.17749999E 02
   J51=  0.15090000E-06      L51= -0.82859999E 02      J66=  0.51530000E-10    L66= -0.14340000E 02

THE FOLLOWING BODIES ARE USED FOR PLANETARY PERTURBATIONS

NONE
```

10

11

12

Figure 5-31. Sample TRACE-D Program Tracking Listing (Continued)

```
J44= 0.47809999E-08     L44= 0.36010000E 02     J65= 0.33279999E-09     L65= -0.17749999E 02
J51= 0.15090000E-06     L51= -0.81859999E 02    J66= 0.51530000E-10     L66= -0.14340000E 02

THE FOLLOWING BODIES ARE USED FOR PLANETARY PERTURBATIONS

NONE

MULTIPLE SATELLITE INITIAL CONDITIONS

SATELLITE NO. 2          2/23/1965
ALPHA=       0.83169469E 02
DELTA=       0.
BETA =       0.89999999E 02
AZ   =       0.49999999E 01
R    =       0.21533351E 08
V    =       0.25667858E 05
TO   =       0.72000000E 03
CDA/W=       0.15000000E-01                         (13)

SATELLITE NO. 3          3/ 1/1964
ALPHA=       0.15882923E 03
DELTA=       0.
BETA =       0.89999999E 02
AZ   =       0.34499999E 03
R    =       0.21533351E 08
V    =       0.25567859E 05
TO   =       0.
CDA/W=       0.20000000E-01

FORMULATION
             COWELL (EQS.OF MOTION)

DIFFERENTIAL EQUATION SUBROUTINE
             GAUSS-JACKSON        E BAR=  0.1000E-09     A= 1.000     RATIO OF COWELL STEP TO RUNGE KUTTA  4
STEP SIZE -      INITIAL=     0.1000E 01     MINIMUM=     0.1563E-01     MAXIMUM=     0.6400E 02
 DO NOT RECOMPUTE PERTURBATIONS FOR CORRECTOR

EQUATIONS OF MOTION    USE N1 = 9 N2= 6

VARIATIONAL EQUATIONS USE N1 = 9 N2= 6 FOR V MATRIX (14)
      USES T MATRIX (15)
                                          CONSTANTS
```

Figure 5-31. Sample TRACE-D Program Tracking Listing (Continued)

```
COMPUTED VALUES FOR ITERATION    1    3/ 1/64       0.      MIN.

 A,E,I,O,U,T          MEAN ANM= 173.84310        APOGEE= 3951.65891
 0.24010736E 08       TRUE ANM= 173.84310        HT    =  507.85029
 0.14526680E-06           CDOT=    1.59386       PERIGEE= 3951.65778
 0.10500000E 03           UDOT=   -2.04779       HT    =  507.84916      (16)
 0.15882923E 03                                  PERIOD(K)=  103.84523
 0.18615689E 03                                  PERIOD(A)=  103.71713
-0.30146602E 02                                  PERIOD(N)=  103.75690
                                                                                                          (17)
ENTERING LINK NO.  5 FIT A
3/ 1/64     0.        MM  601421724760  177650254365  000000000000  171651221774  173422... ..  175422532313
 ***THRUST START AT       10.0 SECONDS FROM MIDNIGHT OF EPOCH***(18)
3/ 1/64   51.87918 MM  201421611154  577646667363  151774300000  571650366020  573422716353  575422736454
THRUST RESULTANT =      0.94960182E-04   FT/SEC*SEC (19)
3/ 1/64  103.76264 MM  601422111433  177646122413  552737400000  171647064332  173422563440  175422525655
THRUST RESULTANT =      0.90158949E-04   FT/SEC*SEC
3/ 1/64  155.64578 MM  201421773630  577644540745  152751600000  571646211710  573423055244  575422734010
THRUST RESULTANT =      0.85600495E-04   FT/SEC*SEC
 ***THRUST END AT   10000.0 SECONDS FROM MIDNIGHT OF EPOCH***  (20)
3/ 1/64  207.53263 MM  601422272020  177643770103  553755454000  171644726523  173422723003  175422523224
3/ 1/64  259.41747 MM  201422150267  577642365674  154536310000  571644047473  573423216157  575422734245
3/ 1/64  311.30351 MM  601422445672  177641611167  554515100000  171642502765  173423067507  175422524911
3/ 1/64  363.18882 MM  201422325656  577640204152  154637260000  571641621156  573423364043  575422732632
3/ 1/64  415.07563 MM  601422621146  177637423163  554447540000  171640316430  173423233045  175422525757
3/ 1/64  466.96070 MM  201422502342  577636031463  155457100000  571637412175  573423540032  575422731053
3/ 1/64  518.84810 MM  601422772364  177635244217  555634720000  171636171600  173423412732  175422527026
3/ 1/64  570.73259 MM  201422655213  577633663144  155660314300  571635251701  573423722110  575422730030
3/ 1/64  622.61977 MM  601423143133  177633073161  555571234300  171634031322  173423576154  175422527513
3/ 1/64  674.50414 MM  201423026515  577631513562  154434424000  571633127720  573424104515  575422727424
3/ 1/64  726.39095 MM  601423313701  177630723356  555631434000  171631665327  173423761420  175422527343
3/ 1/64  778.27482 MM  201423175471  577627337737  155757300000  571631005362  573424265753  575422730620
3/ 1/64  830.16116 MM  601423465601  177626553451  555535660000  171627505456  173424136375  175422525727
3/ 1/64  882.04481 MM  201423344157  577625157746  156523714000  571626600054  573424431146  575422733050
3/ 1/64  933.93166 MM  601423640727  177624400504  556610614000  171625301653  173424271447  175422523710
3/ 1/64  985.81609 MM  201423514335  577623000610  156555322400  571624342556  573424562105  575422734355

THE FOLLOWING POINTS ARE FOR SATELLITE NUMBER 2.(21)

2/23/65  720.00000 MM  175765226775  201405440430  000000000000  571641307726  166617761401  175454234611
```

Figure 5-31 Sample TRACE-D Program Tracking Listing (Continued)

RESIDUALS	(22)	(23)						(24)
CC	6060.0000	-0.3543E 04(R)	-0.7768E-01(A)	-0.4495E-03(E)	0.5447E 01(RD)	0.	0.	1 14060.00
CC	6060.0000	-0.6960E 03(X)	-0.2166E 04(Y)	0.4227E 04(Z)	-0.1931E-01(TR)	-0.4218E-01(TD)	0.3536E-01(HA)	1 14060.00
CC	6060.0000	-0.2245E-01(GR)	0.6898E-01(GD)	-0.1237E-01(U)	0.3290E-01(V)	0.	0.	1 14060.00
CC	6075.0000	-0.3258E 04(R)	-0.9727E-01(A)	0.2368E-00(E)	0.6010E 01(RD)	0.	0.	1 14115.00
CC	6075.0000	-0.9118E 03(X)	-0.1049E 04(Y)	0.4056E 04(Z)	0.2129E-01(TR)	0.3345E-01(TD)	-0.[illegible]53E-01(HA)	1 14115.00
CC	6075.0000	-0.5587E-01(GR)	0.1933E-02(GD)	-0.1244E-01(U)	0.3481E-01(V)	0.	0.	1 14115.00
CC	6090.0000	-0.3140E 04(R)	-0.8837E-02(A)	0.1485E-00(E)	0.6433E 01(RD)	0.	0.	1 14130.00
CC	6090.0000	0.2211E 03(X)	-0.9722E 03(Y)	0.4077E 04(Z)	-0.1122E-00(TR)	0.2557E-01(TD)	-0.4140E-01(HA)	1 14130.00
CC	6090.0000	-0.6009E-02(GR)	0.9681E-02(GD)	-0.1249E-01(U)	0.3673E-01(V)	0.	0.	1 14130.00
CC	6105.0000	-0.3292E 04(R)	0.1818E-01(A)	0.1111E-00(E)	0.7177E 01(RD)	0.	0.	1 14145.00
CC	6105.0000	-0.8594E 03(X)	-0.2380E 03(Y)	0.4338E 04(Z)	0.2770E-01(TR)	0.[illegible]786E-01(TD)	0.7242E-01(HA)	1 14145.00
CC	6105.0000	0.1202E-00(GR)	0.7529E-02(GD)	-0.1257E-01(U)	0.3865E-01(V)	0.	0.	1 14145.00
CC	6120.0000	-0.3086E 04(R)	-0.1211E-00(A)	0.5207E-01(E)	0.7518E 01(RD)	0.	0.	1 14160.00
CC	6120.0000	-0.1938E 04(X)	-0.8902E 03(Y)	0.4715E 04(Z)	-0.8728E-01(TR)	0.8745E-01(TD)	0.3414E-01(HA)	1 14160.00
CC	6120.0000	-0.5512E-01(GR)	-0.2101E-01(GD)	-0.1264E-01(U)	0.4052E-01(V)	0.	0.	1 14160.00
CC	6135.0000	-0.3164E 04(R)	0.1858E-01(A)	-0.5609E-01(E)	0.8387E 01(RD)	0.	0.	1 14215.00
CC	6135.0000	-0.2257E 04(X)	-0.1134E 04(Y)	0.5247E 04(Z)	-0.1173E-00(TR)	0.1270E-01(TD)	0.1164E-01(HA)	1 14215.00
CC	6135.0000	-0.7664E-01(GR)	-0.2048E-02(GD)	-0.1270E-01(U)	0.4230E-01(V)	0.	0.	1 14215.00
CC	6150.0000	-0.2782E 04(R)	-0.2419E-00(A)	0.1404E-00(E)	0.9035E 01(RD)	0.	0.	1 14230.00
CC	6150.0000	0.2495E 01(X)	-0.1157E 04(Y)	0.6299E 04(Z)	-0.1180E-01(TR)	0.6635E-01(TD)	0.4781E-04(HA)	1 14230.00
CC	6150.0000	0.6336E-01(GR)	0.3617E-01(GD)	-0.1276E-01(U)	0.4395E-01(V)	0.	0.	1 14230.00
CC	6165.0000	-0.2661E 04(R)	-0.1423E-00(A)	-0.1034E-00(E)	0.9590E 01(RD)	0.	0.	1 14245.00
CC	6165.0000	-0.1092E 04(X)	-0.2213E 04(Y)	0.6219E 04(Z)	0.7489E-02(TR)	0.3865E-01(TD)	0.1602E-01(HA)	1 14245.00
CC	6165.0000	-0.5565E-01(GR)	-0.2394E-01(GD)	-0.1284E-01(U)	0.4543E-01(V)	0.	0.	1 14245.00
CC	6180.0000	-0.2426E 04(R)	-0.1039E-00(A)	0.3238E-02(E)	0.1031E 02(RD)	0.	0.	1 14260.00
CC	6180.0000	-0.1768E 04(X)	-0.3429E 03(Y)	0.5881E 04(Z)	-0.8335E-01(TR)	0.2490E-01(TD)	-0.2657E-01(HA)	1 14260.00
CC	6180.0000	0.6153E-01(GR)	-0.2418E-01(GD)	-0.1286E-01(U)	0.4669E-01(V)	0.	0.	1 14260.00
CC	6195.0000	-0.2396E 04(R)	-0.2042E-00(A)	-0.1644E-00(E)	0.1066E 02(RD)	0.	0.	1 14315.00
CC	6195.0000	-0.1452E 04(X)	-0.9640E 03(Y)	0.6090E 04(Z)	-0.4328E-01(TR)	0.1040E-00(TD)	0.2538E-01(HA)	1 14315.00
CC	6195.0000	-0.4900E-01(GR)	-0.2426E-01(GD)	-0.1296E-01(U)	0.4769E-01(V)	0.	0.	1 14315.00
CC	6210.0000	-0.[illegible]057E 04(R)	-0.3912E-01(A)	-0.1989E-00(E)	0.1147E 02(RD)	0.	0.	1 14330.00
CC	6210.0000	-0.[illegible]458E 04(X)	-0.152[illegible] 04(Y)	0.5549E 04(Z)	0.2046E-01(TR)	0.7481E-01(TD)	-0.1134E-01(HA)	1 14330.00
CC	6210.0000	0.2289E-01(GR)	-0.1212E-01(GD)	-0.1302E-01(U)	0.4844E-01(V)	0.	0.	1 14330.00
CC	6225.0000	-0.1878E 04(R)	-0.4058E-01(A)	-0.1300E-00(E)	0.1221E 02(RD)	0.	0.	1 14345.00
CC	6225.0000	-0.2231E 04(X)	-0.6665E 03(Y)	0.6985E 04(Z)	-0.9004E-01(TR)	-0.1476E-02(TD)	0.2277E-01(HA)	1 14345.00
CC	6225.0000	0.1675E-01(GR)	-0.7289E-02(GD)	-0.1374E-01(U)	0.4908E-01(V)	0.	0.	1 14345.00
CC	6240.0000	-0.1848E 04(R)	-0.1052E-00(A)	0.3837E-01(E)	0.1294E 02(RD)	0.	0.	1 14360.00
CC	6240.0000	-0.3079E 04(X)	0.4629E 03(Y)	0.5359E 04(Z)	-0.7087E-01(TR)	0.7636E-01(TD)	-0.2344E-01(HA)	1 14360.00
CC	6240.0000	0.3913E-01(GR)	-0.3694E-01(GD)	0.1309E-01(U)	-0.4894E-01(V)	0.	0.	1 14360.00
CC	62[illegible].0000	-0.1517E 04(R)	-0.2490E-00(A)	0.8322E-01(E)	0.1354E 02(RD)	0.	0.	1 14415.00

Figure 5-31. Sample TRACE-D Program Tracking Listing (Continued)

```
CC    6660.0000  0.5310E-01(GR) -0.2519E-01(GD)  0.1490E-01(U)  -0.1437E-01(V)   0.               0.               1 15060.00
CC    6675.0000  0.3418E 04(R)   0.2530E-01(A)   0.9983E-01(E)   0.6303E 01(RD)  0.               0.               1 15115.00
CC    6675.0000 -0.4344E 04(X)   0.1594E 04(Y)   0.4493E 04(Z)   0.4627E-01(TR) -0.5321E-01(TD)   0.3757E-01(HA)   1 15115.00
CC    6675.0000  0.4707E-01(GR) -0.3519E-02(GD)  0.1497E-01(U)  -0.1369E-01(V)   0.               0.               1 15115.00
CC    6690.0000  0.3597E 04(R)   0.2734E-00(A)   0.2739E-01(E)   0.5951E 01(RD)  0.               0.               1 15130.00
CC    6690.0000 -0.3673E 04(X)   0.8941E 03(Y)   0.5772E 04(Z)   0.3758E-01(TR) -0.1647E-02(TD)  -0.1007E-00(HA)   1 15130.00
CC    6690.0000 -0.1619E-01(GR)  0.4209E-01(GD)  0.1503E-01(U)  -0.1305E-01(V)   0.               0.               1 15130.00
CC    6705.0000  0.3698E 04(R)  -0.1576E-00(A)  -0.9145E-02(E)   0.5461E 01(RD)  0.               0.               1 15145.00
CC    6705.0000 -0.7000E 04(X)   0.9679E 03(Y)   0.4490E 04(Z)   0.3638E-01(TR) -0.6177E-01(TD)  -0.7133E-01(HA)   1 15145.00
CC    6705.0000  0.8738E-02(GR) -0.3702E-02(GD)  0.1509E-01(U)  -0.1243E-01(V)   0.               0.               1 15145.00
CC    6720.0000  0.3694E 04(R)   0.1381E-00(A)  -0.9759E-02(E)   0.5286E 01(RD)  0.               0.               1 15160.00
CC    6720.0000 -0.4011E 04(X)  -0.3502E 03(Y)   0.5583E 04(Z)   0.1049E-00(TR) -0.2823E-02(TD)  -0.7938E-01(HA)   1 15160.00
CC    6720.0000 -0.1309E-01(GR)  0.4328E-01(GD)  0.1515E-01(U)  -0.1187E-01(V)   0.               0.               1 15160.00
      FOLLOWING (R)    RESIDUAL EDITED  25
AA    7065.0000 -0.6666E 04(R)   0.1491E-00(A)  -0.9913E-01(E)  -0.7076E 00(RD)  0.               0.               1 15745.00
AA    7065.0000 -0.4808E 04(X)   0.4925E 03(Y)   0.1917E 04(Z)   0.1881E-01(TR)  0.6577E-01(TD)  -0.1610E-01(HA)   1 15745.00
AA    7065.0000 -0.7967E-01(GR)  0.4052E-01(GD)  0.1655E-01(U)  -0.4653E-02(V)   0.               0.               1 15745.00
      FOLLOWING (R)    RESIDUAL EDITED
AA    7080.0000 -0.6715E 04(R)   0.1017E-00(A)   0.3050E-00(E)  -0.5319E 00(RD)  0.               0.               1 15760.00
AA    7080.0000 -0.6170E 04(X)   0.1840E 02(Y)   0.2495E 04(Z)  -0.2135E-01(TR) -0.7361E-02(TD)  -0.3839E-02(HA)   1 15760.00
AA    7080.0000  0.8047E-01(GR)  0.7015E-01(GD)  0.1662E-01(U)  -0.6479E-02(V)   0.               0.               1 15760.00
      FOLLOWING (R)    RESIDUAL EDITED
AA    7095.0000 -0.6724E 04(R)   0.6352E-03(A)   0.8375E-01(E)  -0.2855E-00(RD)  0.               0.               1 15815.00
AA    7095.0000 -0.6838E 04(X)   0.2697E 04(Y)   0.9956E 03(Z)   0.8042E-01(TR)  0.5053E-01(TD)   0.3886E-01(HA)   1 15815.00
AA    7095.0000 -0.6196E-01(GR) -0.9521E-01(GD)  0.1668E-01(U)  -0.4324E-02(V)   0.               0.               1 15815.00
      FOLLOWING (R)    RESIDUAL EDITED
AA    7110.0000 -0.6519E 04(R)  -0.6379E-01(A)   0.1649E-00(E)  -0.2930E-00(RD)  0.               0.               1 15830.00
AA    7110.0000 -0.6385E 04(X)   0.2974E 04(Y)   0.4381E 04(Z)  -0.7272E-01(TR)  0.8073E-01(TD)   0.1977E-01(HA)   1 15830.00
AA    7110.0000  0.3793E-01(GR) -0.8696E-02(GD)  0.1674E-01(U)  -0.4159E-02(V)   0.               0.               1 15830.00
      FOLLOWING (R)    RESIDUAL EDITED
AA    7125.0000 -0.6394E 04(R)   0.1102E-00(A)  -0.1285E-00(E)  -0.4499E-01(RD)  0.               0.               1 15845.00
AA    7125.0000 -0.5876E 04(X)   0.2948E 04(Y)   0.4211E 04(Z)  -0.5855E-01(TR)  0.7116E-01(TD)  -0.2127E-01(HA)   1 15845.00
AA    7125.0000 -0.2109E-01(GR)  0.3199E-01(GD)  0.1680E-01(U)  -0.4012E-02(V)   0.               0.               1 15845.00
      FOLLOWING (R)    RESIDUAL EDITED
AA    7140.0000 -0.6547E 04(R)  -0.1633E-00(A)   0.6657E-01(E)   0.1855E-00(RD)  0.               0.               1 15860.00
AA    7140.0000 -0.6225E 04(X)   0.2332E 04(Y)   0.2717E 04(Z)   0.3751E-01(TR)  0.4641E-01(TD)   0.1353E-01(HA)   1 15860.00
AA    7140.0000  0.8526E-01(GR)  0.2856E-01(GD)  0.1686E-01(U)  -0.3868E-02(V)   0.               0.               1 15860.00
      FOLLOWING (R)    RESIDUAL EDITED
AA    7155.0000 -0.6537E 04(R)  -0.1007E-00(A)  -0.1212E-00(E)   0.4016E-00(RD)  0.               0.               1 15915.00
AA    7155.0000 -0.6661E 04(X)   0.2446E 03(Y)   0.4346E 04(Z)   0.2804E-01(TR)  0.4375E-01(TD)   0.1391E-00(HA)   1 15915.00
AA    7155.0000  0.1104E-01(GR)  0.3849E-01(GD)  0.1692E-01(U)  -0.3728E-02(V)   0.               0.               1 15915.00
      FOLLOWING (R)    RESIDUAL EDITED
```

Figure 5-31. Sample TRACE-D Program Tracking Listing (Continued)

STAT	N	RMS RMS/SIG	TYPE	N	RMS RMS/SIG	TYPE	N	RMS RMS/SIG	TYPE
AA	57	0.117125E 04 0.117125E 02	(R)	92	0.922048E 00 0.922048E 01	(A)	84	0.113480E-00 0.113480E 01	(E)
AA	42	0.212510E 02 0.212510E 02	(RD)	42	0.659534E 04 0.659534E 01	(X)			
BB	27	0.139140E 03 0.139140E 01	(R)	27	0.109016E-00 0.109016E 01	(A)	27	0.130768E-00 0.130768E 01	(E)
CC	46	0.239388E 04 0.239388E 02	(R)	85	0.601024E 00 0.601024E 01	(A)	85	0.1[illegible]2003E-00 0.1[illegible]2003E 01	(E)
CC	45	0.113003E 02 0.113003E 02	(RD)	74	0.171160E 05 0.171160E 02	(X)			
EE	1	0.404818E 03 0.404818E 01	(R)	$\boxed{23}$	$\boxed{0.750748E\ 00 \quad 0.750748E\ 01}$	$\boxed{(A)}$	23	0.202472E-00 0.202472E 01	(E)
EE	0	0. 0.	(RD)	23	0.163137E 05 0.163137E 02	(X)			
FF	58	0.492022E 03 0.492022E 01	(R)	89	0.480115E-00 0.480115E 01	(A)	90	0.430031E-00 0.430031E 01	(E)
FF	0	0. 0.	(RD)	33	0.225802E 05 0.225802E C2	(X)			

Figure 5-31. Sample TRACE-D Program Tracking Listing (Continued)

```
ITERATION      1 (27)
 2196 DATA POINTS WERE USED IN THE SOLUTION (28)
CURRENT SOLUTION IS BEST SO FAR (29)

CURRENT SOLUTION IS

        ALPHA            DELTA            BETA              A                R                V
(30) 0.15882923E 03   0.               0.89999999E 02   0.34499999E 03   0.24010739E 08   0.24212933E 05
       THRST1           THRST2           2 ALPH           3 ALPH
     0.09999999E-03   0.09999999E-02   0.83169469E 02   0.15882923E 03

CURRENT SOLUTION (OCTAL MACHINE UNITS)
(31) 202542647714     000000000000     201622077324     203601274613     201445573331     175434273344
     163643334273     167406111565     201563465506     202542647705

****  RMS=   0.92469950E 01  FOR THIS SOLUTION (32)

CORRECTIONS
(33)  0.34906341E-02  -0.19726520E-01   0.15147365E-02  -0.12188543E-02  -0.28262528E 01  -0.75102438E 00
     -0.11871029E-04   0.94213346E-05   0.10945629E-03   0.86714013E 00
BOUNDS
(34)  0.99999999E 00   0.99999999E 00   0.99999999E 00   0.99999999E 00   0.99999999E 02   0.99999998E 01
      0.50000000E-04   0.50000009E-03   0.99999999E 00   0.99999999E 00

HITTING BOUNDS (35)

NEXT SOLUTION IS
(36)  0.15883273E 03  -0.19726520E-01   0.90001513E 02   0.34499877E 03   0.24010736E 08   0.24212182E 05
      0.88128971E-04   0.10094213E-02   0.83169580E 02   0.15969636E 03

NEXT SOLUTION (OCTAL MACHINE UNITS)
(37) 202542653710     565551010601     201622102702     203601274046     201445573317     175434266733
     163561507414     167410472172     201563465706     202544607570

****  PREDICTED RMS=   0.19848783E 01  FOR NEXT SOLUTION (38)

SIGMA(PARAMETERS)/SIGMA(NORMALIZED DATA)
```

Figure 5-31. Sample TRACE-D Program Tracking Listing (Continued)

39

1	2	3	4	5	6
ALPHA	DELTA	BETA	A	R	V
0.80073286E-02	0.29544347E-01	0.17941354E-02	0.11892873E-03	0.32731147E 04	0.15826564E 01

7	8	9	10
THRST1	THRST2	2 ALPH	3 ALPH
0.4[illegible]473320E-03	0.29483604E-01	0.72348882E-04	0.45485458E-01

CORRELATION MATRIX

40

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	-1.000	1.000								
3	-0.997	0.997	1.000							
4	0.197	-0.195	-0.237	1.000						
5	-0.993	0.995	0.996	-0.214	1.000					
6	0.992	-0.993	-0.996	0.226	-1.000	1.000				
7	0.984	-0.987	-0.989	0.208	-0.998	0.998	1.000			
8	0.972	-0.976	-0.979	0.211	-0.993	0.993	0.998	1.000		
9	0.	0.	0.	0.	0.	0.	0.	0.	1.000	
10	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.000

SCATTER COEFFICIENT =0.46508E-23 41

ENTERING LINK NO. 3 INITA

ENTERING LINK NO. 4 INITB

Figure 5-31. Sample TRACE-D Program Tracking Listing (Concluded)

Table 5-14. Tracking Output Listing Description

Item	Description	Page Reference
1	Permanent-station information. The six columns from left to right contain station identification, weighting sigma index number, geodetic latitude of station, longitude of station measured east from Greenwich, and height of station in feet above mean sea level.	3-5 5-32/5-34 5-62
	If temporary stations are present, corresponding information is printed immediately preceeding the observations for the appropriate block (see Item 2).	5-34, 5-35 5-67, 5-68
2	Observation time. The month, day, hour, minute, and second of an observation are associated with the set of values appearing in the same line. The year is assumed to be the same as that of epoch day.	5-35/5-37
3	Station identification. If pass identification (observation-card columns 3 and 4) is used, the characters entered will appear to the immediate right of the indicated letters.	5-35/5-37
4	Observation set number. The integers in the indicated column designate which specific observation types are represented by the measurements given on that line (see Item 5).	5-37
5	Observations. The specific observation type is indicated by the set number (see Item 4). For example, the last line on the first page of Figure 5-31 contains range, azimuth, and elevation measurements.	3-13/3-20 5-35/5-37
6	End of flock. This break in the listing of observations indicates that a TF card has been encountered by the TRAIN link input routine. If temporary stations are present in any flock, the corresponding station-location and-identification information is printed immediately preceeding the observations in that flock.	5-35/5-38 5-63

Table 5-14. Tracking Output Listing Description (Continued)

Item	Description	Page Reference
7	Satellite identification. A message such as the one indicated is printed when the input routine encounters a TT card in the observation deck. Satellites are numbered sequentially as the corresponding observations are encountered in the input deck.	5-18, 5-19 5-22 5-66
8	Indication of the quantities (parameters) to be differentially corrected, which in this case are α, δ, β, A, r, and v for Satellite No. 1, thrust amplitude and time constant for Satellite No. 1, α for Satellite No. 2, and α for Satellite No. 3.	2-7 3-70 5-13, 5-14 5-22, 5-23
9	Flock count. If a binary data tape is used to input the observations, this count should be ignored.	
10	Card image of the END card following the observation cards in the input deck.	5-67
11	Differential correction bounds. This sequence corresponds to the previously described sequence of the parameter indications (see Item 8).	2-11 3-70, 3-73 5-24
12	Weighting sigmas. The observation type number and the value of the weighting value are shown. For example, observation Type 19 corresponds to range rate and the assigned weight is one foot per second. Sigmas are entered for observation Types 1, 2, 3, and 19 only. A weighting sigma of zero is automatically applied to the other types of observations entered, which has the effect of excluding them from the least squares process (i.e., they are given zero weight).	2-7 3-70, 3-71 5-26, 5-27
13	Initial conditions for additional satellites. If more than one satellite orbit is defined, the ADBARV quantities at epoch for the additional orbits will appear at this location.	5-18/5-22

Table 5-14. Tracking Output Listing Description (Continued)

Item	Description	Page Reference
14	Gravity field indices for partials computation. These integers define the highest-degree and -order terms of the gravity-field expression that are to be used in the variational equations (see Table 5-12, Item 23).	3-52/3-54 3-60/3-62 Appendix F
15	T-matrix indicator. The disposition of the variational equation T-matrix term option is indicated at this point.	3-53 5-6 Appendix C
16	Definition of current orbit. For each iteration the current values for the parameters are used to compute the quantities shown (see Table 5-13, Items 8 through 10).	3-4 3-11, 3-12 3-63/3-66 5-97, 5-98
17	Node print. Each time the integrated trajectory crosses the equator (as determined by interpolation between integration steps) the date, time in minutes from midnight of epoch, and the rectangular elements (x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$,) in units of earth radii and earth radii per minute are printed in the octal mode. All output by the FITA link is in sequence by time for each satellite. The numerical integration interval for each satellite begins at epoch and ends at the time of the latest observation for that satellite. A message is printed each time any of the events of equator crossing, start of thrusting, end of thrusting, or orbit adjust are detected during the integration interval	3-63 5-5
18	Thrust start message. This output indicates the beginning of a thrusting interval for the satellite whose motion is being integrated.	3-51 5-8, 5-9
19	Thrust resultant. During the thrusting interval the thrust magnitude at the time of node crossing is printed on the line following the nodal elements.	3-51 5-8, 5-9
20	End of thrust message. No thrusting will be included in the equations of motion after the time appearing at this point.	3-51 5-8, 5-9

Table 5-14. Tracking Output Listing Description (Continued)

Item	Description	Page Reference
21	Satellite-number message. Integration of the equations of motion for several satellites is performed serially, wherein the complete trajectory for Satellite No. 1 is integrated, then the complete trajectory for Satellite No. 2, etc. This message indicates that integration of the equations of motion for Satellite No. 2 is starting and that the node prints following apply to that satellite.	5-18/5-22
22	Station identification and system time of residuals. These are to be associated with all residuals which appear on the same line. It should be noted that system time is defined as seconds from midnight of the current day.	
23	Residuals. These are unnormalized differences between the input observations (as modified by bias or refraction corrections) and corresponding values for the same observation types computed from the integrated trajectory position at the observation times. Up to six residuals for the same time are printed on one line. The observation type is indicated in parentheses immediately following the residual value in each case. Note that residuals appear for the unweighted as well as for the weighted observations.	2-9 3-70 3-76, 3-77 5-8
	Identification of the foregoing observation-type indicators with observation descriptions or symbols defined elsewhere in this report is in accordance with the following: Indicator / Description or Symbol / Unit R Range ft A Azimuth deg E Elevation deg TR Topocentric right ascension deg	3-14/3-19

Table 5-14. Tracking Output Listing Description (Continued)

Item	Description	Page Reference
23 (cont)	(see indicator listing below)	

Indicator	Description or Symbol	Units
TD	Topocentric declination	deg
HA	Topocentric hour angle	deg
GR	Geocentric right ascension	deg
GD	Geocentric declination	deg
U	Horizon scanner in-plane angle	deg
V	Horizon scanner cross-plane angle	deg
H	Altitude(h)	ft
X	$\hat{x}$	ft
Y	$\hat{y}$	ft
Z	$\hat{z}$	ft
R	Range	ft
P	Range difference	ft
Q	Range difference	ft
RD	Range rate	ft/sec
PD	Range-rate difference	ft/sec
QD	Range-rate difference	ft/sec

Item	Description	Page Reference
24	Observation time. The time identified in Item 2 in terms of seconds from midnight is given in alternate form, wherein the day of the month and the hour and minute of the day are given as integers and seconds are given to two decimal places.	
25	Editor message. Whenever the residuals editor deletes an observation, a message such as the one shown is printed. The indicated residual is not included in the least-squares process for the iteration in progress but will be reconsidered on the next iteration.	5-28
26	RMS summary of residuals. The root-mean-square of the residuals for each type of observation from each station is computed by the residuals editor and the result is printed at this location. Included are the station identification, number of residuals included in the RMS (i.e., the total number for that station and type minus the number deleted by	5-28

Table 5-14. Tracking Output Listing Description (Continued)

Item	Description	Page Reference
26 (cont)	the editor on this iteration), the RMS, and the RMS divided by the input weighting sigma. The observation-type indicators are the same as those used in the residuals print output (see Item 25). Units are feet, degrees, and seconds. Interpretation of the boxed printout area of the Item-28 sample output should be that twenty-three azimuth observations from station EE passed the residuals editor and were included in the least-squares process. The root-mean-square of these residuals is 0.753981 degree, which is 7.53981 times the sigma input for weighting azimuth observations from station EE (input sigma = 0.1 degree). A restriction associated with the residuals editor is apparent if the summary information is compared to the detailed residuals print described in Item 25. Only five observation types for some stations are represented on the summary output, whereas more than five types were used in the fit. Because of storage capacity constraints, the residuals editor will accumulate residuals and perform editing checks for only the first five observation types encountered for each station.	
27	Iteration number. TRACE-D performs the tracking, or orbit determination function, by computing series of differential corrections to the parameters selected by the user. The iteration number is advanced each time the process of computing a set is repeated. The number may be interpreted as an indication of the number of times the trajectory-integration/least-squares process has been performed.	2-9 3-77 5-28
28	Observation count. The number of individual observations included in the current least-squares computation is indicated.	

Table 5-14. Tracking Output Listing Description (Continued)

Item	Description	Page Reference
29	Convergence indicator. If the weighted RMS for all residuals for the current iteration is less than that for any previous iteration, then the fitting process is converging and the message shown is printed. If the RMS obtained on the current iteration is greater than the smallest RMS obtained on previous iterations, the message CURRENT ITERATION IS NOT GOOD (RMS = C.xxxxxxxx xx) is printed in this position.	2-9 3-77
30	Current solution. If the iteration is successful (i.e., the overall RMS has been lowered), indicated values are the parameter values used in the trajectory, partial derivative, and residuals computations which have just been completed. In the case of Iteration No. 1, they are the input values of the parameters. If the iteration is bad, the words GO BACK TO are printed, and the values of the parameters which so far have produced the lowest RMS are recovered from memory and printed at this point. Parameter names are generally self-explanatory, with the possible exceptions of the multiple-vehicle elements. The satellite number is printed before the element name for Satellites 2 through 6. In the case of the boxed printing on the Item 30 sample output, the parameter is right ascension of Satellite No. 2, where units are feet, degrees, and seconds.	3-77
31	Current solution in octal digits and machine units. These numbers corresponding to those described in Item 30 are given both in the octal mode and in the units used for internal computations. Use of these quantities permits bypassing units and number-system conversions during input and output.	5-5

Table 5-14. Tracking Output Listing Description (Continued)

Item	Description	Page Reference
32	RMS. This is the quantity which is to be minimized in the tracking or orbit determination process and is the root-mean-square of the normalized residuals included in the least-squares calculations on the current iteration.	2-9 3-76, 3-77
33	Corrections. The result of solving the system of normal equations associated with the current iteration. Each correction is associated with the parameter which occupies the corresponding position in the current solution block (see Item 30). Units are feet, degrees, and seconds.	3-73/3-76
34	Bounds. Current values of the numbers used to limit the size of corrections. In general, these bounds are automatically increased on a good iteration and automatically decreased on a bad one.	2-11 3-73 5-24
35	Bounding indicator. This message will be either HITTING BOUNDS or NOT HITTING BOUNDS The first message indicates that the magnitudes of the corrections have been controlled by solving the system in such a way that the constraint implied by the bounds is satisfied, and the latter that the normal equations have been solved without applying the bounds.	3-72/3-76
36	Next solution. Each value is the sum of the parameter value given in the corresponding position under "current solution" and the associated correction. These are the parameter values which will be used for the next iteration.	
37	Next solution in octal mode and units of earth radii, radians, and minutes. (see Item 31).	5-5

Table 5-14. Tracking Output Listing Description (Concluded)

Item	Description	Page Reference
38	Predicted RMS. If the fitting process is converging in a completely linear fashion, this will be the RMS on the next iteration. The comparison of this number with the current RMS (see Item 32) may be used to measure the degree to which the process has already converged.	2-9 3-76
39	Sigma of parameters divided by sigma of the normalized observations. The numbers given are the square roots of the diagonal elements of the inverse normal matrix. If certain assumptions are made about the characteristics of the observation set, the numbers may then be taken as the variances on the parameter solutions.	2-17 3-76
40	Correlation matrix, correlation coefficients for the parameter set. These values are computed directly from the covariance matrix (i.e., the inverse normal matrix). Rows and columns are in the same sequence as in the Item-39 block.	3-78

5.5.4 Data Generation Output

Listed output produced by a typical data generation run is shown in Figure 5-32. Supplementary descriptive information relating to the indicated areas of this listing is annotated in Table 5-15.

```
                                                 +++++
                                             ++++     ++++
                                          +++             +++
    TTTTTTTTTTTT     RRRRRRRRR         +               AA     +     CCCCCCCCCCC     EEEEEEEEEEE
    TTTTTTTTTTTT     RRRRRRRRRR       +               AAAA     +    CCCCCCCCCCC     EEEEEEEEEEE
         TT          RR       RR      +             AA  AA     +    CC              EE
         TT          RRRRRRRRRR      +            AA    AA      +   CC              EEEEEE
         TT          RRRRRRRRR       +          AAAAAAAAAA      +   CC              EEEEEE
         TT          RR      RR      +        AAAAAAAAAAAA      +   CC              EE
         TT          RR       RR      +   AA               AA    +  CCCCCCCCCCC     EEEEEEEEEEE
         TT          RR        RR     + AA                 AA    +  CCCCCCCCCCC     EEEEEEEEEEE
                                        +                       +
                                          +++              +++
                                             ++++     ++++         A0014D
                                                 +++++             REFERENCE

ORBIT I DATA GENERATION    2-23-65

EPOCH         YEAR     MONTH     DAY    TZONE    HOUR    MIN     SEC
              1965.    2.        23.    0.       0.      0.      0.

 ENTERING LINK NO.  3 INITA
```

Figure 5-32. Sample TRACE-D Program Data-Generation Output Listing

```
LRAPAR05,99        001,0188  RBIAS     03,011000.   1              FINP INPUT CARD   1
END                                                                FINP INPUT CARD   2

ORBIT I DATA GENERATION     2-23-65

BOUNDS
0.

                           SIGMA TABLE
 1 .1C00000E 03  2 .1000000E-00  3 .1000000E-00  4 .5000000E-01  5 .5000000E-01  6 .5000000E-01   2
 7 .50C0000E-01  8 .5000000E-01 13 .1000000E 04 14 .1000000E 04 15 .1000000E 04 19 .100C000E-00
                                  INITIAL CONDITIONS

X    =     -0.10055931E 07        ALPHA=    0.92676648E 02        A=      0.21533351E 08
Y    =      0.21509859E 08        DELTA=    0.                    E=      0.42269762E-07
Z    =      0.                    BETA =    0.89999999E 02        I=      0.65000000E 02
XDOT=      -0.10793655E 05        AZ   =    0.24999999E 02        O=      0.92676648E 02
YDOT=      -0.50460658E 03        R    =    0.21533352E 08        U=      0.20245775E 03
ZDOT=       0.23172350E 05        V    =    0.25567859E 05        T=     -0.38595824E 02

  ATMOSPHERE - LOCKHEED        CDA/W = 0.09999999E-01     W/CDA = 0.99999999E 02
                               D1    = 0.68299999E 01     D2    =-0.15684000E 02

                                    EARTH MODEL
GM=  0.55304505E-02   J2=  0.10827599E-02   J3= -0.26930000E-05   J4= -0.15600000E-05   J5= -0.59999999E-08
J6=  0.38999999E-06   J7= -0.63299999E-06   J8=  0.                J9=  0.20999999E-06   J10=  0.
       J21=  0.                 L21=  0.                    J52=  0.53290000E-07     L52= -0.26000000E 01
       J22=  0.17230000E-05     L22= -0.13180000E 02        J53=  0.58419999E-08     L53= -0.34499999E 01
       J31=  0.18560000E-05     L31=  0.53100000E 01        J54=  0.40929999E-08     L54=  0.58250000E 02
       J32=  0.44819999E-06     L32= -0.13330000E 02        J55=  0.84620000E-09     L55= -0.15320000E 02
       J33=  0.16429999E-06     L33=  0.16000000E 02        J61=  0.66110000E-07     L61=  0.15720999E 03
       J41=  0.72780000E-06     L41= -0.13368000E 03        J62=  0.37459999E-07     L62=  0.11314999E 03
       J42=  0.15360000E-06     L42=  0.27469999E 02        J63=  0.11660000E-07     L63= -0.17500000E 01
       J43=  0.57190000E-07     L43= -0.10999999E 01        J64=  0.22560000E-08     L64=  0.60739999E 02
       J44=  0.47809999E-08     L44=  0.36010000E 02        J65=  0.33279999E-09     L65= -0.17749999E 02
```

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

```
 J51=  0.15090000E-06    L51= -0.81859999E 02    J66=  0.51530000E-10    L66= -0.14340000E 02
THE FOLLOWING BODIES ARE USED FOR PLANETARY PERTURBATIONS

NONE

FORMULATION
            COWELL (EQS.OF MOTION)

DIFFERENTIAL EQUATION SUBROUTINE
            GAUSS-JACKSON         E BAR=  0.1000E-09    A= 1.000   RATIO OF COWELL STEP TO RUNGE KUTTA  4
STEP SIZE -     INITIAL=    0.1000E 01    MINIMUM=    0.156 E-01    MAXIMUM=    0.6400E 02
 DO NOT RECOMPUTE PERTURBATIONS FOR CORRECTOR

EQUATIONS OF MOTION   USE N1 = 9 N2= 6

                                              CONSTANTS
                         DECIMAL          OCTAL                               DECIMAL          OCTAL
OMEGA E              0.43752695E-02   171436571536   GM                   0.55304505E-02   171552343100
ALPHA G              0.26647102E 01   202525052346
FT/KM                0.32808398E 04   214632065602   EARTH RADIUS - FT    0.20925741E 08   231477232264
E.R./A.U.            0.23454864E 05   217556366727   N.M./E.R.            0.34439336E 04   214656373600
I-O. DISTANCE        0.20925741E 08   231477232264   I-O. VELOCITY        0.34876230E 06   223524455115
G                    0.32173999E 02   206401310550   FT/SEC//E.R./N.M.    0.34876230E 06   223524455115
DEG/SEC//RAD/MIN     0.10471976E 01   201414052221   1/EPS.               0.29830000E 03   211452231463
RELATIVE MASS(SUN)   0.33295130E 06   223505113515   (MOON)               0.12299900E-01   172623026050
(VENUS)              0.81497900E 00   200641211666   (MARS)               0.10782100E-00   175671505015
(JUPITER)            0.31788700E 03   211475706112   (SATURN)             0.95129000E 02   207574410142

 ENTERING LINK NO.  4 INIT8
```

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

STATIONS

	SIG	REF	LATITUDE	LONGITUDE	HEIGHT
AA	-0.	-0.	75.00000000	300.00000000	1000.0000
BB	-0.	-0.	45.00000000	240.00000000	100.0000
CC	-0.	-0.	0.	315.00000000	500.0000
DD	-0.	-0.	0.	15.00000000	200.0000
EE	-0.	-0.	-15.00000000	135.00000000	100.0000
FF	-0.	-0.	-65.00000000	75.00000000	5000.0000

DATA

	INTERVAL SEC.	MIN.EL DEG.	MAX.EL DEG.	MAX.RANGE N.M	MDT MIN.	START DA	START HR	START MN	STOP DA	STOP HR	STOP MN
AA	15.0	-0.	-0.	-0.	70.0	-0.	-0.	-0.	-0.	24.	-0.
BB	15.0	-0.	-0.	-0.	70.0	-0.	-0.	-0.	-0.	24.	-0.
CC	15.0	-0.	-0.	-0.	70.0	-0.	-0.	-0.	-0.	24.	-0.
DD	15.0	-0.	-0.	-0.	70.0	-0.	-0.	-0.	-0.	24.	-0.
EE	15.0	-0.	-0.	-0.	70.0	-0.	-0.	-0.	-0.	24.	-0.
FF	15.0	-0.	-0.	-0.	70.0	-0.	-0.	-0.	-0.	24.	-0.

PARAMETERS TO BE CORRECTED
BB
RBIAS

DATA TYPES

STAT	RANGE SUR.R	AZMTH HIGHT	ELEV. DOPLR	R.DOT LOOK	P.DOT VARI.	Q.DOT KAPPA	P ASPCT	Q ATTEN	AZ.DT X,Y,Z	E.DOT T-R,D	R.DOT G-R,D	MU.VIS HR.ANG.	LAT. U,V	LONG.
AA	X	X	X	X										
		Y							Y	Y	Y	Y	Y	
BB	X	X	X	X										
									Y	Y	Y	Y	Y	
CC	X	X	X	X										
	Y								Y	Y	Y	Y	Y	
DD	X	X	X	X										
									Y	Y	Y	Y	Y	
EE	X	X	X	X										

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

```
                                          Y     Y          Y     Y     Y     Y     Y
 FF    X     X     X     X                                                           } 6
                                                           Y     Y     Y     Y     Y
                                                  7
 ENTERING GAINA.  LINK  9

 **NODE AT  0.                MME. 574611526466 201407112565 000000000000 573773036372 567573220671 175420112222
 **NODE AT  0.44033574E 02 MME. 174567664743 601406633670 151712176600 173773657164 167553060360 575420412242
 **NODE AT  0.88066886E 02 MME. 574547047443 201407141317 551660216000 573773147040 567532201346 175420110011
 **NODE AT  0.13209847E 03 MME. 174525144775 601406657224 153700624000 173773752375 167511414627 575420412232
 **NODE AT  0.17613067E 03 MME. 574504322737 201407164645 553635460000 573773230066 567471017206 175420110235
 **NODE AT  0.22015995E 03 MME. 174462534356 601406676753 154625564000 173774025250 167450214037 575420416305
 **NODE AT  0.26418976E 03 MME. 574441727002 201407205175 555411354000 573773262016 567427565021 175420113567
 **NODE AT  0.30821671E 03 MME. 174420301210 601406713447 154566520000 173774060436 167407115646 575420425332
 **NODE AT  0.35224350E 03 MME. 573777125014 201407222672 555474500000 573773302144 566754754233 175420120723
 **NODE AT  0.39626819E 03 MME. 173734233026 601406726233 155431604000 173774104203 166714230135 575420434045
 **NODE AT  0.44029197E 03 MME. 573672415133 201407236105 555427110000 573773325516 566652422337 175420125077
 **NODE AT  0.48431518E 03 MME. 173627746377 601406740575 154440040000 173774141256 166612612044 575420436540
 **NODE AT  0.52833630E 03 MME. 573565752216 201407247564 554641320000 573773372130 566547776454 175420126403
 **NODE AT  0.57235882E 03 MME. 173523455321 601406753042 155415100000 173774204625 166510622634 575420434760
 **NODE AT  0.61637844E 03 MME. 573461307465 201407256003 555726350000 573773443751 566445776605 175420130012
 **NODE AT  0.66039868E 03 MME. 173416710427 601406762664 155727030000 173774233553 166405535163 575420434325
 **NODE AT  0.70441744E 03 MME. 572731261126 201407260246 555606630000 573773474437 565711410161 175420134266
 **NODE AT  0.74843368E 03 MME. 172623700362 601406767454 155546710000 173774255155 165604326316 575420435511
 **NODE AT  0.79245120E 03 MME. 572517416005 201407260607 555713640000 573773522262 565505632423 175420137127
 **NODE AT  0.83646527E 03 MME. 172411771700 601406773313 155727350000 173774305450 164776206340 575420433457
 **NODE AT  0.88048101E 03 MME. 571612671145 201407257514 555753560000 573773561330 564600427554 175420140017
 **NODE AT  0.92449455E 03 MME. 171400623043 601406775634 156403250000 173774322161 163755176150 575420430755
 **NODE AT  0.96850783E 03 MME. 567734661454 201407253216 555656400000 573773603247 562725474460 175420144364
 **NODE AT  0.10125195E 04 MME. 565434041210 601406775031 154733060000 173774312163 560444276000 575420431703
 **NODE AT  0.10565304E 04 MME. 170467514705 201407244136 556501014000 573773602530 163450471320 175420153217
 **NODE AT  0.11005396E 04 MME. 571444463510 601406771171 156404530000 173774274116 564434260420 575420434122
 **NODE AT  0.11445484E 04 MME. 171657116056 201407233556 555655150000 573773574067 164635277234 175420160712
 **NODE AT  0.11885553E 04 MME. 572433522537 601406762012 156520160000 173774254576 565422372711 575420437331
 **NODE AT  0.12325617E 04 MME. 172540776502 201407222073 556506410000 573773555070 165523562113 175420165114
 **NODE AT  0.12765661E 04 MME. 572644345302 601406746755 156547654000 173774242532 565626441375 575420443462
 **NODE AT  0.13205701E 04 MME. 172751730535 201407207751 556643204000 573773534452 165730217260 175420166307
 **NODE AT  0.13645724E 04 MME. 573426555023 601406731571 155707740000 173774247301 566415510070 575420445340
 **NODE AT  0.14085749E 04 MME. 173471510004 201407174706 556474440000 573773534147 166456034251 175420164300

 ENTERING LINK NO. 10    GAIN 8
```

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

```
                                    FEBRUARY  23,  1965

ST HR   MINS   T-ST       RANGE    AZIMUTH  ELEVATION RANGE RATE    HEIGHT          X          Y          Z
                     NAUT MILES    DEGREES    DEGREES     FT/SEC NAUT MILES NAUT MILES NAUT MILES NAUT MILES

                     TOP RT.ASC TOP DECLIN GEO RT.ASC GEO DECLIN TOP HR.ANG          U          V
                        DEGREES    DEGREES    DEGREES    DEGREES    DEGREES    DEGREES    DEGREES

AA  9  RISE (   -0.    DEGREES ELEV.)        3. HOURS   14.57 MINUTES     AZIMUTH 162.149 DEGREES 8

AA  3.  14.75  194.75     846.228    159.302      0.951  -7043.192    105.438   1089.054  -1287.258   3113.294
                          162.863    -13.821    151.773     61.598    338.601     76.039     64.300
AA  3.  15.00  195.00     830.888    155.082      1.067  -5391.049    105.459   1114.292  -1232.523   3126.459
                          167.008    -13.118    153.703     61.936    334.471     77.061     64.395
AA  3.  15.25  195.25     819.686    151.011      1.038  -3656.266    105.476   1138.418  -1177.469   3138.609
                          171.393    -12.394    155.599     62.396    330.298     78.083     64.483
AA  3.  15.50  195.50     812.851    146.749      1.177  -1861.778    105.487   1163.051  -1122.465   3149.817
                          175.703    -11.744    157.702     62.831    325.834     79.105     64.563
AA  3.  15.75  195.75     810.491    142.548      1.164    -36.002    105.493   1187.225  -1066.650   3160.057
                          180.264    -10.915    159.883     63.184    321.602     80.126     64.635
AA  MAX EL (    1.73 DEGREES ELEV.)          3. HOURS   15.76 MINUTES     AZIMUTH 142.251 DEGREES 10
AA  3.  16.00  196.00     812.689    137.950      1.300   1790.380    105.495   1210.846  -1010.934   3169.426
                          184.615    -10.253    161.928     63.523    317.122     81.148     64.700
AA  3.  16.25  196.25     819.345    133.679      1.027   3586.948    105.492   1234.634   -954.779   3177.545
                          188.964     -9.675    164.182     63.896    312.917     82.170     64.758
AA  3.  16.50  196.50     830.334    129.585      0.899   5325.288    105.483   1257.861   -898.134   3184.642
                          193.181     -9.068    166.377     64.095    308.647     83.192     64.808
AA  3.  16.75  196.75     845.543    125.491      0.962   6982.133    105.470   1280.764   -841.684   3190.900
                          197.448     -8.466    168.699     64.459    304.597     84.214     64.851

AA     SET  (  -0.    DEGREES ELEV.)         3. HOURS   16.93 MINUTES     AZIMUTH 122.587 DEGREES 11
 ELAPSED VISIBLE TIME =       2.37  MINUTES DURING REVOLUTION NUMBER      0. 12

AA     RISE (  -0.    DEGREES ELEV.)         4. HOURS   42.80 MINUTES     AZIMUTH 204.539 DEGREES

AA  4.  43.00  283.00     827.770    201.835      1.262 -15084.742    105.342    552.374  -1572.874   3123.122
                          141.006    -13.393    152.890     61.940     22.457     76.822     64.376
```

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

FEBRUARY 23, 1965

ST	HR	MINS	T-ST	RANGE NAUT MILES / TOP DECLIN DEGREES	AZIMUTH DEGREES / GEO RT.ASC DEGREES	ELEVATION DEGREES / GEO DECLIN DEGREES	RANGE RATE FT/SEC / TOP HR.ANG DEGREES	X NAUT MILES / U DEGREES	Y NAUT MILES / V DEGREES	Z NAUT MILES	TOP RT.ASC DEGREES
88	RISE (	-0.	DEGREES ELEV.)		4. HOURS	33.11 MINUTES		AZIMUTH 149.915 DEGREES			
88	4.	33.25	273.25	824.778	147.958	0.878	-14120.582	-1090.609	-2762.254	1931.364	142.342
				-36.553	109.567	32.996	318.748	36.990	52.207		
88	4.	33.50	273.50	791.448	144.488	1.240	-12858.077	-1052.710	-2744.715	1976.949	146.039
				-34.396	110.293	33.885	315.226	38.011	52.849		
88	4.	33.75	273.75	761.435	140.601	1.757	-11429.840	-1014.736	-2726.279	2021.856	149.712
				-31.784	110.878	34.806	311.546	39.032	53.463		
88	4.	34.00	274.00	735.159	136.423	2.244	-9830.403	-976.582	-2707.254	2065.779	153.531
				-29.057	111.515	35.634	307.744	40.053	54.052		
88	4.	34.25	274.25	713.038	131.768	2.728	-8061.617	-937.736	-2687.162	2109.373	157.550
				-26.041	112.203	36.574	303.944	41.074	54.617		
88	4.	34.50	274.50	695.475	127.080	2.982	-6135.888	-898.363	-2666.542	2151.769	161.553
				-22.942	112.821	37.362	299.907	42.095	55.157		
88	4.	34.75	274.75	682.832	122.086	3.155	-4078.350	-858.966	-2645.050	2193.960	165.588
				-19.547	113.634	38.289	295.871	43.116	55.675		
88	4.	35.00	275.00	675.414	116.837	3.387	-1926.767	-819.127	-2622.875	2235.303	169.681
				-16.126	114.256	39.105	291.893	44.137	56.172		
88	MAX EL (		3.63 DEGREES ELEV.)		4. HOURS	35.24 MINUTES		AZIMUTH 111.684 DEGREES			
88	4.	35.25	275.25	673.379	111.505	3.363	271.301	-778.805	-2599.860	2276.566	173.831
				-12.566	114.974	39.995	287.887	45.158	56.648		
88	4.	35.50	275.50	676.738	106.419	3.512	2463.264	-738.553	-2576.315	2315.695	177.831
				-9.071	115.756	40.781	283.874	46.179	57.103		
88	4.	35.75	275.75	685.470	101.171	3.388	4597.012	-698.169	-2551.969	2355.062	181.673
				-5.699	116.521	41.701	280.090	47.200	57.540		
88	4.	36.00	276.00	699.353	96.345	3.038	6626.619	-657.132	-2526.806	2393.204	185.578
				-2.391	117.274	42.532	276.338	48.221	57.959		
88	4.	36.25	276.25	718.085	91.364	2.768	8517.982	-615.740	-2501.454	2430.938	189.235
				0.611	118.068	43.380	272.793	49.242	58.360		

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

```
BB  4. 36.50  276.50     741.304     86.916      2.2.9  10248.757   -574.469  -2474.607    2468.008     192.675
                           3.420    118.915     44.081    269.272     50.264     58.744
BB  4. 36.75  276.75     768.564     82.759      1.561  11809.252   -532.545  -2447.505    2504.130     196.048
                           6.093    119.763     45.035    265.908     51.285     59.112
BB  4. 37.00  277.00     799.476     79.114      1.338  13198.923   -490.637  -2419.195    2539.463     199.385
                           8.340    120.589     45.805    262.812     52.306     59.464
BB  4. 37.25  277.25     833.586     75.326      0.952  14425.557   -448.587  -2390.945    2574.058     202.273
                          10.527    121.565     46.637    259.844     53.327     59.801

SB    SET  ( -0.   DEGREES ELEV.)        4. HOURS  37.38 MINUTES     AZIMUTH  73.682 DEGREES
 ELAPSED VISIBLE TIME =      4.27  MINUTES DURING REVOLUTION NUMBER    0.

BB    RISE ( -0.   DEGREES ELEV.)        6. HOURS   1.85 MINUTES     AZIMUTH 228.700 DEGREES

BB  6.  2.00  362.00     808.543    229.047      1.222 -23616.276  -1940.195  -2134.562    2056.507      65.148
                         -26.871    111.162     35.517     58.305     39.835     53.932
BB  6.  2.25  362.25     750.361    230.198      1.910 -23518.696  -1896.704  -2131.219    2099.878      65.689
                         -25.509    111.813     36.269     58.405     40.856     54.501
BB  6.  2.50  362.50     692.458    231.288      3.125 -23387.964  -1852.581  -2126.563    2142.765      64.767
                         -23.882    112.511     37.204     58.617     41.877     55.047
BB  6.  2.75  362.75     634.918    233.001      4.269 -23212.219  -1808.205  -2121.919    2184.885      64.580
                         -21.912    113.196     38.086     59.095     42.898     55.570
BB  6.  3.00  363.00     577.864    234.325      5.657 -22974.130  -1763.305  -2116.581    2226.701      64.145
                         -19.827    113.826     38.883     59.579     43.919     56.071
BB  6.  3.25  363.25     521.563    236.636      7.268 -22646.232  -1717.303  -2110.768    2267.245      63.511
                         -17.339    114.654     39.799     60.133     44.941     56.551
BB  6.  3.50  363.50     466.164    239.109      9.371 -22186.667  -1671.124  -2104.604    2307.555      62.733
                         -14.441    115.385     40.657     61.102     45.962     57.011
BB  6.  3.75  363.75     412.174    242.313     11.220 -21523.695  -1624.229  -2097.271    2346.718      61.820
                         -10.644    116.044     41.463     62.160     46.983     57.451
BB  6.  4.00  364.00     360.171    246.467     13.813 -20539.792  -1577.206  -2090.079    2385.442      60.365
                          -5.983    116.944     42.267     63.529     48.004     57.874
BB  6.  4.25  364.25     311.206    252.205     16.997 -19030.549  -1529.401  -2082.100    2423.405      58.373
                          -0.040    117.895     43.115     65.673     49.025     58.278
BB  6.  4.50  364.50     266.941    260.208     20.865 -16652.285  -1480.860  -2073.328    2459.927      55.432
                           8.077    118.479     43.978     68.531     50.046     58.666
BB  6.  4.75  364.75     230.108    271.659     24.917 -12891.343  -1432.090  -2064.055    2496.491      51.186
                          18.594    119.278     44.840     72.956     51.068     59.037
BB  6.  5.00  365.00     204.817    287.694     28.972  -7261.500  -1383.130  -2054.404    2532.809      44.492
                          32.103    120.226     45.641     79.673     52.089     59.393
```

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

```
88  6.  5.25  365.25    195.613   307.535    30.737    -41.163  -1333.156  -2043.572  2566.496     33.090
                         47.170   121.276    46.496     91.167     53.110     59.733
88  MAX EL (  30.65 DEGREES ELEV.)       6. HOURS   5.25 MINUTES     AZIMUTH 309.858 DEGREES
88  6.  5.50  365.50    204.608   323.842    29.163   7188.565  -1282.869  -2032.853  2600.568     12.884
                         60.257   122.124    47.261    111.367     54.131     60.059
88  6.  5.75  365.75    224.751   344.126    25.267  12837.076  -1232.707  -2021.729  2633.702    342.405
                         62.462   123.026    48.061    142.046     55.153     60.371
88  6.  6.00  366.00    266.443   355.855    20.866  16613.275  -1181.411  -2009.994  2666.162    313.967
                         65.691   123.901    48.904    170.372     56.274     60.669
88  6.  6.25  366.25    310.649     3.723    17.266  19000.539  -1129.816  -1997.472  2697.055    296.513
                         61.923   125.070    49.580    187.951     57.195     60.956
88  6.  6.50  366.50    359.544     9.637    13.959  20514.468  -1078.106  -1984.393  2727.391    286.812
                         57.899   126.085    50.388    197.861     58.217     61.227
88  6.  6.75  366.75    411.501    13.945    11.385  21500.205  -1026.111  -1970.844  2757.201    280.774
                         54.350   127.154    51.106    203.825     59.238     61.487
88  6.  7.00  367.00    469.459    17.102     9.363  22163.221   -973.334  -1956.876  2786.040    276.945
                         51.305   128.321    51.932    207.708     60.260     61.736
88  6.  7.25  367.25    520.747    19.520     7.522  22622.261   -920.423  -1942.182  2814.193    274.322
                         48.685   129.352    52.637    210.385     61.281     61.973
88  6.  7.50  367.50    577.019    21.873     5.913  22948.695   -867.342  -1927.823  2841.234    272.449
                         46.507   130.536    53.389    212.243     62.303     62.199
88  6.  7.75  367.75    633.938    23.263     4.676  23185.049   -813.706  -1912.102  2866.801    271.145
                         44.501   131.832    53.958    213.651     63.324     62.415
88  6.  8.00  368.00    691.452    24.725     3.263  23358.677   -759.953  -1896.294  2891.965    270.098
                         42.782   133.141    54.745    214.736     64.346     62.620
88  6.  8.25  368.25    749.265    26.147     2.335  23487.674   -705.910  -1879.709  2916.576    269.422
                         41.217   134.476    55.447    215.597     65.367     62.814
88  6.  8.50  368.50    807.374    27.088     1.278  23583.079   -651.898  -1862.961  2939.793    268.805
                         39.772   135.751    56.037    216.242     66.389     62.999

88    SET  (  -0.   DEGREES ELEV.)        6. HOURS   8.71 MINUTES    AZIMUTH  27.798 DEGREES
 ELAPSED VISIBLE TIME =     6.86  MINUTES DURING REVOLUTION NUMBER    0.
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Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

	ST HR	MINS	T-ST	RANGE NAUT MILES / TOP DECLIN DEGREES	AZIMUTH DEGREES / GEO RT.ASC DEGREES	ELEVATION DEGREES / GEO DECLIN DEGREES	RANGE RATE FT/SEC / TOP HR.ANG DEGREES	X NAUT MILES / U DEGREES	Y NAUT MILES / V DEGREES	Z NAUT MILES	TOP RT.ASC DEGREES
DD	RISE (-0. DEGREES ELEV.)				6. HOURS	32.59 MINUTES		AZIMUTH 356.612 DEGREES			
DD	6.	32.75	392.75	796.087	356.522	0.936	-22929.000	3340.423	851.014	797.708	180.284
				86.937	265.483	13.066	85.873	165.596	28.078		
DD	6.	33.00	393.00	742.548	358.396	1.497	-22702.767	3346.767	877.196	742.258	215.756
				88.172	265.892	12.145	50.400	166.619	26.394		
DD	6.	33.25	393.25	686.880	0.397	2.439	-22410.595	3351.481	902.497	686.099	275.556
				87.713	266.395	11.271	350.846	167.643	24.652		
DD	6.	33.50	393.50	631.988	2.652	3.559	-22030.087	3355.897	928.186	630.073	302.710
				85.754	266.702	10.412	323.553	168.666	22.853		
DD	6.	33.75	393.75	578.222	4.894	4.812	-21529.219	3358.535	953.179	573.941	313.771
				83.105	267.201	9.374	312.637	169.690	20.998		
DD	6.	34.00	394.00	525.825	8.162	6.274	-20860.770	3361.101	977.686	517.523	319.728
				79.832	267.630	8.405	306.599	170.713	19.090		
DD	6.	34.25	394.25	475.365	11.865	7.726	-19955.083	3362.193	1002.196	460.850	323.724
				75.737	268.143	7.511	302.779	171.737	17.130		
DD	6.	34.50	394.50	427.575	16.622	9.483	-18711.763	3361.922	1026.254	404.135	326.944
				71.005	268.598	6.628	299.645	172.760	15.123		
DD	6.	34.75	394.75	383.376	22.677	11.246	-16985.163	3361.070	1050.231	347.310	329.460
				64.884	268.982	5.634	297.294	173.784	13.073		
DD	6.	35.00	395.00	344.260	30.042	13.367	-14585.379	3359.171	1073.829	290.250	331.558
				57.537	269.402	4.615	295.034	174.808	10.984		
DD	6.	35.25	395.25	312.112	39.354	15.213	-11306.146	3356.018	1096.677	233.126	333.843
				48.390	269.890	3.804	293.010	175.831	8.861		
DD	6.	35.50	395.50	289.274	50.571	16.785	-7034.212	3352.220	1119.783	175.881	335.439
				37.539	270.260	2.818	291.293	176.855	6.711		
DD	6.	35.75	395.75	278.071	63.363	17.801	-1941.985	3346.926	1142.020	118.659	337.277
				25.299	270.794	1.942	289.673	177.878	4.540		
DD	MAX EL (17.77 DEGREES ELEV.)				6. HOURS	35.84 MINUTES		AZIMUTH 68.376 DEGREES			

Table title: FEBRUARY 22, 1965

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

DD	6.	36.00	396.00	279.869	76.672	17.672	34[illegible]2.870	3341.062	1164.086	61.206	339.034
				12.680	271.164	0.989	[illegible]8.083	178.902	2.354		
DD	6.	36.25	396.25	294.537	89.108	16.368	8[illegible]19.827	3333.874	1185.390	4.177	340.674
				0.849	271.557	0.091	[illegible]6.321	179.926	0.160		
DD	6.	36.50	396.50	320.249	99.976	14.660	12319.934	3325.788	1207.074	-53.190	342.218
				-9.528	272.048	-0.841	284.759	179.051	2.034		
DD	6.	36.75	396.75	354.552	108.555	12.825	15340.090	3316.587	1228.102	-110.435	343.804
				-18.179	272.416	-1.871	283.364	178.027	4.222		
DD	6.	37.00	397.00	395.291	115.586	10.764	17534.603	3306.555	1248.747	-167.626	345.347
				-25.092	272.910	-2.659	281.788	177.004	6.396		
DD	6.	37.25	397.25	440.627	121.002	8.840	19112.583	3295.547	1269.034	-224.959	346.938
				-30.678	273.262	-3.591	280.210	175.980	8.550		
DD	6.	37.50	397.50	489.288	125.585	7.296	20252.646	3283.296	1288.934	-281.682	348.510
				-35.266	273.768	-4.512	278.798	174.957	10.677		
DD	6.	37.75	397.75	540.347	129.055	5.979	21086.524	3269.945	1308.211	-339.031	350.070
				-38.866	274.172	-5.521	277.215	173.933	12.771		
DD	6.	38.00	398.00	593.202	131.896	4.343	21705.681	3256.065	1327.596	-395.665	351.665
				-41.924	274.548	-6.427	275.797	172.910	14.828		
DD	6.	38.25	398.25	647.417	134.227	3.229	22171.948	3240.830	1348.355	-452.557	353.179
				-44.319	275.096	-7.348	274.317	171.886	16.841		
DD	6.	38.50	398.50	702.594	136.451	2.134	22527.675	3224.782	1364.581	-509.176	354.710
				-46.483	275.567	-8.352	272.769	170.862	18.808		
DD	6.	38.75	398.75	758.570	138.377	1.222	22802.253	3207.654	1382.239	-565.750	356.241
				-48.222	276.021	-9.278	271.322	169.839	20.724		

DD SET (-0. DEGREES ELEV.) 6. HOURS 38.99 MINUTES AZIMUTH 139.668 DEGREES
ELAPSED VISIBLE TIME = 6.29 MINUTES DURING REVOLUTION NUMBER 0.

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

FEBRUARY 23, 1965

	ST HR	MINS	T-ST	RANGE NAUT MILES / TOP DECLIN DEGREES	AZIMUTH DEGREES / GEO RT.ASC DEGREES	ELEVATION DEGREES / GEO DECLIN DEGREES	RANGE RATE FT/SEC / TOP HR.ANG DEGREES	X NAUT MILES / U DEGREES	Y NAUT MILES / V DEGREES	Z NAUT MILES	TOP RT.ASC DEGREES
FF	RISE (	-0.	DEGREES ELEV.)		5. HOURS	22.69 MINUTES		AZIMUTH 356.435 DEGREES			
FF	5.	22.75	322.75	828.780	356.980	0.891	-18811.820	612.506	2125.230	-2762.505	305.428
				24.732	307.557	-51.307	3.128	120.514	61.548		
FF	5.	23.00	323.00	783.326	359.962	1.482	-18024.201	563.819	2101.013	-2791.083	308.697
				23.953	308.619	-52.089	0.003	119.491	61.794		
FF	5.	23.25	323.25	739.953	3.169	2.059	-17084.302	514.705	2078.576	-2818.732	312.081
				23.144	309.858	-52.790	356.610	118.468	62.029		
FF	5.	23.50	323.50	699.126	6.924	2.833	-15963.971	465.419	2051.624	-2845.801	316.009
				22.120	311.023	-53.627	352.785	117.445	62.253		
FF	5.	23.75	323.75	661.336	10.709	3.736	-14631.942	415.693	2026.179	-2871.696	320.357
				21.091	312.195	-54.254	348.537	116.422	62.466		
FF	5.	24.00	324.00	627.057	15.217	4.465	-13059.093	366.300	1999.884	-2896.513	325.165
				19.881	313.486	-54.912	343.829	115.399	62.668		
FF	5.	24.25	324.25	597.059	20.066	5.187	-11220.254	316.147	1973.163	-2920.850	330.231
				18.370	314.802	-55.574	338.778	114.377	62.861		
FF	5.	24.50	324.50	571.925	25.720	5.813	-9105.192	266.142	1945.860	-2943.678	335.822
				16.859	316.282	-56.288	333.102	113.354	63.044		
FF	5.	24.75	324.75	552.309	31.689	6.252	-6724.233	215.906	1918.155	-2966.023	341.778
				15.086	317.693	-56.971	327.218	112.331	63.217		
FF	5.	25.00	325.00	538.845	38.043	6.707	-4118.402	165.563	1898.014	-2987.715	348.048
				13.152	319.160	-57.636	321.141	111.309	63.381		
FF	5.	25.25	325.25	532.097	44.587	6.989	-1360.905	114.684	1861.513	-3008.019	354.424
				11.032	320.719	-58.248	314.796	110.286	63.536		
FF	MAX EL (	6.83 DEGREES ELEV.)			5. HOURS	25.39 MINUTES		AZIMUTH	48.307 DEGREES		
FF	5.	25.50	325.50	532.230	51.043	7.000	1449.130	64.509	1832.226	-3026.913	0.917
				8.885	322.306	-58.792	308.364	109.263	63.682		
FF	5.	25.75	325.75	539.212	57.939	6.803	4202.964	13.687	1802.487	-3045.584	7.225
				6.869	323.946	-59.375	302.161	108.241	63.819		

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

FF	5.	26.00	326.00	552.842	64.123	6.548	6802.539	-37.304	1772.525	-3062.853	13.219
				4.841	325.648	-59.963	296.097	107.218	63.948		
FF	5.	26.25	326.25	572.610	69.983	5.910	9175.303	-87.717	1741.870	-3079.319	18.948
				3.055	327.307	-60.499	290.496	106.195	64.068		
FF	5.	26.50	326.50	597.906	75.685	5.113	11281.579	-138.648	1711.013	-3094.731	24.306
				1.467	329.109	-61.125	285.214	105.173	64.181		
FF	5.	26.75	326.75	628.096	80.597	4.658	13111.176	-189.549	1679.722	-3109.470	29.116
				0.040	331.078	-61.404	280.503	104.150	64.285		
FF	5.	27.00	327.00	662.441	85.045	3.784	14676.093	-240.108	1647.861	-3122.930	33.606
				-1.119	332.891	-61.931	276.035	103.128	64.382		
FF	5.	27.25	327.25	700.343	89.185	3.166	16000.550	-291.373	1615.639	-3135.484	37.629
				-2.141	334.883	-62.265	272.121	102.105	64.471		
FF	5.	27.50	327.50	741.240	92.706	2.256	17114.684	-342.018	1582.789	-3147.333	41.287
				-2.865	336.956	-62.860	268.505	101.083	64.552		
FF	5.	27.75	327.75	784.697	95.617	1.548	18049.040	-392.745	1549.982	-3157.404	44.622
				-3.518	339.062	-63.129	265.212	100.061	64.626		
FF	5.	28.00	328.00	830.222	98.483	0.842	18832.215	-443.598	1516.892	-3166.899	47.458
				-3.957	341.186	-63.492	262.333	99.038	64.692		

FF SET (-0. DEGREES ELEV.) 5 HOURS 28.12 MINUTES AZIMUTH 99.867 DEGREES
ELAPSED VISIBLE TIME = 5.43 MINUTES DURING REVOLUTION NUMBER 0.

FF RISE (-0. DEGREES ELEV.) 6. HOURS 51.90 MINUTES AZIMUTH 309.732 DEGREES

FF	6.	52.00	412.00	820.479	310.063	0.993	-23710.095	1108.924	1712.216	-2891.573	278.429
				15.437	313.025	-54.814	52.506	115.605	62.630		
FF	6.	52.25	412.25	762.057	311.026	1.611	-23613.551	1052.516	1706.611	-2915.696	279.890
				14.700	314.456	-55.460	51.384	114.582	62.825		
FF	6.	52.50	412.50	703.928	312.130	2.758	-23483.446	995.665	1700.065	-2939.225	281.154
				14.055	315.848	-56.151	49.756	113.559	63.009		
FF	6.	52.75	412.75	646.145	313.266	3.862	-23309.529	939.241	1693.919	-2961.459	282.975
				13.253	317.156	-56.714	48.129	112.537	63.185		
FF	6.	53.00	413.00	588.904	314.851	5.361	-23073.913	881.848	1687.175	-2983.175	285.026
				12.425	318.639	-57.448	46.155	111.514	63.350		
FF	6.	53.25	413.25	532.299	316.907	6.995	-22750.818	824.555	1679.708	-3003.725	287.347
				11.506	320.161	-58.085	43.853	110.491	63.507		
FF	6.	53.50	413.50	476.663	319.211	8.785	-22299.288	766.633	1671.934	-3023.170	290.268
				10.490	321.744	-58.670	40.972	109.468	63.655		
FF	6.	53.75	413.75	422.368	322.342	10.677	-21652.098	708.303	1664.084	-3041.742	293.913
				9.281	323.356	-59.218	37.485	108.446	63.794		

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

FF	6.	54.00	414.00	370.022	326.321	13.413	-20697.109	650.192	1655.276	-3059.415	298.552
				7.815	325.030	-59.808	32.948	107.423	63.924		
FF	6.	54.25	414.25	320.595	331.797	16.165	-19243.800	591.460	1646.396	-3076.087	304.488
				5.941	326.785	-60.300	27.040	106.400	64.047		
FF	6.	54.50	414.50	275.660	339.435	19.877	-16970.704	532.485	1637.283	-3091.664	312.281
				3.713	328.534	-60.827	19.317	105.378	64.161		
FF	6.	54.75	414.75	237.880	350.400	23.839	-13395.160	474.094	1627.654	-3106.625	322.751
				0.811	330.429	-61.381	8.827	104.355	64.267		
FF	6.	55.00	415.00	211.023	5.331	27.851	-8023.742	415.059	1617.212	-3120.156	336.574
				-2.815	332.362	-61.863	355.138	103.332	64.355		
FF	6.	55.25	415.25	199.675	24.631	29.658	-1000.993	356.041	1607.187	-3132.846	353.300
				-6.680	334.370	-62.215	338.606	102.310	64.455		
FF	MAX EL (	29.78 DEGREES ELEV.)			6. HOURS	55.29 MINUTES		AZIMUTH	28.871 DEGREES		
FF	6.	55.50	415.50	206.256	44.570	28.783	6270.080	296.308	1596.082	-3144.525	10.472
				-9.740	336.301	-62.720	321.296	101.287	64.538		
FF	6.	55.75	415.75	229.418	60.937	25.183	12145.378	236.793	1585.418	-3155.699	25.847
				-11.457	338.344	-63.089	306.172	100.265	64.613		
FF	6.	56.00	416.00	264.729	72.907	21.174	16161.866	176.819	1573.771	-3165.227	37.863
				-12.073	340.463	-63.342	294.194	99.242	64.681		
FF	6.	56.25	416.25	307.995	81.294	17.417	18728.531	117.326	1561.478	-3173.837	46.824
				-12.046	342.595	-63.718	285.221	98.220	64.742		
FF	6.	56.50	416.50	356.410	87.261	14.226	20361.923	57.449	1549.163	-3181.463	53.850
				-11.625	344.910	-63.938	278.457	97.197	64.795		
FF	6.	56.75	416.75	408.096	91.756	11.537	21426.018	-2.222	1536.590	-3188.102	58.760
				-11.104	347.231	-64.205	273.410	96.175	64.841		
FF	6.	57.00	417.00	461.906	94.952	9.426	22140.733	-61.879	1523.545	-3193.808	62.765
				-10.505	349.564	-64.495	269.443	95.153	64.880		
FF	6.	57.25	417.25	517.237	97.475	7.491	22635.575	-121.781	1510.619	-3198.447	65.909
				-9.913	351.874	-64.771	266.374	94.130	64.911		
FF	6.	57.50	417.50	573.579	99.497	6.047	22986.833	-181.462	1496.773	-3202.165	68.592
				-9.224	354.254	-64.747	263.750	93.108	64.936		
FF	6.	57.75	417.75	630.640	101.303	4.556	23241.778	-241.362	1482.583	-3204.674	70.693
				-8.764	356.593	-64.871	261.715	92.086	64.954		
FF	6.	58.00	418.00	688.274	102.611	3.116	23429.082	-300.855	1467.942	-3206.533	72.679
				-8.078	359.015	-64.839	259.927	91.063	64.965		
FF	6.	58.25	418.25	746.263	103.596	2.124	23568.433	-360.715	1453.280	-3207.076	74.193
				-7.523	1.372	-65.000	258.365	90.041	64.968		
FF	6.	58.50	418.50	804.600	104.681	1.211	23672.035	-420.359	1438.394	-3206.483	75.465
				-6.848	3.867	-65.006	256.952	89.019	64.965		

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

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                         FEBRUARY  23,  1965

FF    SFT  (  -0.   DEGREES ELEV.)        6. HOURS  58.72 MINUTES    AZIMUTH 105.361 DEGREES
 ELAPSED VISIBLE TIME =     6.81  MINUTES DURING REVOLUTION NUMBER    0.

 CHAIN.    LINK NO.  1
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Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

FEBRUARY 23, 1965

ST	HR	MINS	T-ST	RANGE	AZIMUTH	ELEVATION	RANGE RATE	SUR RANGE	X	Y	Z
				NAUT MILES	DEGREES	DEGREES	FT/SEC	NAUT MILES	NAUT MILES	NAUT MILES	NAUT MILES
				TOP RT.ASC	TOP DECLIN	GEO RT.ASC	GEO DECLIN	TOP HR.ANG	U	V	
				DEGREES	DEGREES	DEGREES	DEGREES	DEGREES	DEGREES	DEGREES	
CC	RISE (	-0.	DEGREES ELEV.)		10. HOURS	57.34 MINUTES		AZIMUTH 329.068 DEGREES			
CC	10.	57.50	657.50	772.883	328.405	1.096	-23962.695	758.364	2156.431	-2726.426	659.755
				183.734	58.494	265.738	10.719	88.714	168.131	23.800	
CC	10.	57.75	657.75	713.816	327.884	2.056	-23890.711	699.279	2181.314	-2719.911	603.453
				185.828	57.696	256.298	9.760	86.654	169.155	21.974	
CC	10.	58.00	658.00	654.950	326.977	3.134	-23789.750	640.370	2205.075	-2712.484	547.155
				187.923	56.684	266.732	8.899	84.737	170.178	20.093	
CC	10.	58.25	658.25	596.371	325.579	4.323	-23649.512	581.692	2228.451	-2704.222	490.295
				190.154	55.385	267.171	8.019	82.495	171.202	18.160	
CC	10.	58.50	658.50	538.249	324.088	[illegible]	-23452.568	523.321	2251.037	-2695.310	433.774
				192.342	53.770	267.637	7.060	80.415	172.226	16.177	
CC	10.	58.75	658.75	460.662	322.030	7.623	-23172.149	465.372	2272.601	-2685.429	376.915
				194.792	51.611	268.083	6.103	78.056	173.249	14.149	
CC	10.	59.00	659.00	423.908	319.950	[illegible]	-22762.260	408.026	2293.865	-2674.613	320.429
				197.388	49.002	268.539	5.282	75.585	174.273	12.079	
CC	10.	59.25	659.25	368.451	316.820	11.780	-22142.496	351.577	2314.574	-2663.441	262.788
				200.117	45.481	268.906	4.348	72.979	175.297	9.973	
CC	10.	59.50	659.50	314.903	312.490	14.844	-21164.683	296.538	2334.015	-2651.273	205.617
				202.867	40.812	269.286	3.404	70.179	176.320	7.037	
CC	10.	59.75	659.75	264.447	306.470	18.989	-19544.806	243.864	2353.170	-2637.805	148.723
				206.046	34.200	269.786	2.376	67.016	177.344	5.676	
CC	11.	-0.	660.00	219.367	297.109	23.795	-16739.407	195.475	2371.248	-2624.310	91.643
				209.563	24.571	270.258	1.503	63.682	178.367	3.496	
CC	11.	0.25	660.25	183.545	282.426	29.713	-11843.980	155.426	2389.217	-2610.148	34.107
				213.395	10.700	270.614	0.560	59.802	179.391	1.306	
CC	11.	0.50	660.50	163.274	260.230	34.054	-4135.683	131.563	2406.049	-2594.044	-23.514
				217.854	-8.280	271.113	-0.410	55.488	179.585	0.889	
CC	MAX EL (	34.72	DEGREES ELEV.)		11. HOURS	0.61 MINUTES		AZIMUTH 247.504 DEGREES			

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

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CC 11.  0.75  660.75      164.398    233.187     33.925   5010.111     132.914   2421.982  -2578.218     -80.360
                          222.990    -29.347    271.538     -1.284      50.268    178.562      3.081
CC 11.  1.00  661.00      186.519    212.460     28.962  12449.368     158.829   2437.378  -2560.920    -137.957
                          229.419    -47.663    271.947     -2.160      43.975    177.538      5.263
CC 11.  1.25  661.25      223.491    198.388     23.345  17090.323     199.978   2452.210  -2543.198    -195.092
                          237.317    -60.704    272.418     -3.222      36.164    176.514      7.429
CC 11.  1.50  661.50      269.262    189.307     18.319  19742.044     248.914   2466.117  -2524.869    -252.176
                          247.493    -69.509    272.777     -4.114      25.968    175.491      9.570
CC 11.  1.75  661.75      320.060    183.506     14.643  21278.034     301.873   2479.300  -2505.657    -309.320
                          260.634    -75.122    273.251     -4.974      12.932    174.467     11.682
CC 11.  2.00  662.00      373.846    179.523     11.753  22209.126     337.068   2491.473  -2485.456    -365.911
                          277.140    -78.344    273.802     -5.934     356.642    173.443     13.759
CC 11.  2.25  662.25      429.416    176.255      9.224  22801.744     413.604   2503.132  -2464.177    -422.918
                          295.398    -80.058    274.252     -6.850     338.217    172.420     15.796
CC 11.  2.50  662.50      486.217    173.973      7.362  23194.646     470.998   2514.130  -2442.780    -479.696
                          313.287    -80.605    274.572     -7.854     320.347    171.396     17.787
CC 11.  2.75  662.75      543.882    172.123      5.652  23463.897     528.968   2524.735  -2420.545    -536.097
                          328.263    -80.484    275.155     -8.777     305.565    170.372     19.730
CC 11.  3.00  663.00      602.008    170.871      4.397  23652.753     587.344   2533.722  -2397.059    -592.721
                          339.992    -79.924    275.503     -9.664     293.848    169.349     21.621
CC 11.  3.25  663.25      660.580    169.717      3.018  23787.115     646.015   2542.619  -2373.143    -648.502
                          348.809    -79.175    275.785    -10.528     284.924    168.325     23.458
CC 11.  3.50  663.50      719.459    168.479      1.852  23883.575     704.907   2550.579  -2349.005    -704.825
                          355.816    -78.469    276.359    -11.536     278.070    167.302     25.238
CC 11.  3.75  663.75      778.486    167.737      1.039  23952.030     763.967   2557.713  -2323.477    -760.465
                            1.431    -77.747    276.806    -12.381     272.580    166.278     26.961

CC    SET  ( -0.   DEGREES ELEV.)       11. HOURS    3.89 MINUTES     AZIMUTH 167.308 DEGREES
 ELAPSED VISIBLE TIME =     6.54  MINUTES DURING REVOLUTION NUMBER    0.

CC    RISE ( -0.   DEGREES ELEV.)       21. HOURS   57.22 MINUTES     AZIMUTH 156.800 DEGREES

CC 21. 57.25 1317.25      821.585    156.457      0.982 -16654.001     806.220   2669.068  -2203.617    -752.567
                          167.697    -66.387     83.332    -12.231     270.224     13.564     26.700
CC 21. 57.50 1317.50      781.704    153.083      1.188 -15651.093     766.308   2692.589  -2192.756    -697.314
                          166.233    -63.118     83.795    -11.319     271.729     12.543     24.972
CC 21. 57.75 1317.75      744.469    149.547      1.646 -14482.317     729.066   2715.795  -2181.424    -641.289
                          165.131    -59.423     84.213    -10.396     272.834     11.521     23.187
CC 21. 58.00 1318.00      710.337    145.394      2.380 -13127.241     694.926   2738.165  -2169.375    -585.257
                          164.318    -55.457     84.668     -9.433     273.775     10.500     21.346
```

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

```
CC 21. 58.25 1318.25      679.804    141.053      3.068 -11569.098    664.369   2759.341  -2156.756    -528.744
                          163.791    -51.016     85.114     -8.620    274.383      9.478     19.451
CC 21. 58.50 1318.50      653.397    136.141      3.627  -9797.554    637.912   2780.202  -2143.432    -472.276
                          163.465    -46.297     85.617     -7.686    274.820      8.457     17.506
CC 21. 58.75 1318.75      631.597    131.059      3.860  -7815.269    616.086   2799.691  -2129.580    -415.684
                          163.202    -41.116     85.985     -6.786    275.098      7.436     15.512
CC 21. 59.00 1319.00      614.981    125.896      4.288  -5640.215    599.400   2818.509  -2114.930    -358.355
                          163.159    -35.711     86.456     -5.790    275.119      6.415     13.474
CC 21. 59.25 1319.25      603.890    120.232      4.594  -3310.740    588.291   2836.489  -2099.685    -301.852
                          163.232    -30.041     86.929     -4.881    274.989      5.393     11.397
CC 21. 59.50 1319.50      598.741    114.208      4.786   -883.370    583.084   2853.718  -2083.658    -244.937
                          163.489    -24.193     87.314     -3.970    274.983      4.372      9.285
CC  MAX EL (    4.80 DEGREES ELEV.)       21. HOURS  59.59 MINUTES    AZIMUTH 112.040 DEGREES
CC 21. 59.75 1319.75      599.565    108.248      4.630   1571.328    583.936   2869.974  -2067.440    -187.631
                          163.771    -18.223     87.708     -2.999    274.726      3.351      7.144
CC 22. -0.   1320.00      606.408    102.407      4.492   3980.006    390.824   2885.369  -2050.502    -130.619
                          164.105    -12.490     88.163     -2.054    274.541      2.329      4.982
CC 22.  0.25 1320.25      619.127     96.743      4.438   6274.739    603.545   2899.851  -2032.629     -73.325
                          164.582     -6.789     88.562     -1.195    274.031      1.308      2.803
CC 22.  0.50 1320.50      637.286     91.465      3.952   8403.376    621.741   2913.674  -2014.314     -16.240
                          164.953     -1.445     89.043     -0.262    273.599      0.287      0.615
CC 22.  0.75 1320.75      660.414     86.467      3.431  10333.619    644.952   2926.251  -1995.281      41.022
                          165.657      3.593     89.601      0.661    273.211      0.735      1.575
CC 22.  1.00 1321.00      688.103     81.681      2.826  12050.920    672.661   2937.942  -1975.872      98.290
                          166.162      8.277     89.858      1.558    272.615      1.756      3.760
CC 22.  1.25 1321.25      719.723     77.561      2.145  13557.752    704.337   2948.871  -1955.793     155.553
                          166.803     12.463     90.268      2.503    271.985      2.777      5.933
CC 22.  1.50 1321.50      754.886     73.422      1.764  14865.483    739.475   2959.333  -1935.081     212.852
                          167.625     16.398     90.841      3.466    271.320      3.799      8.087
CC 22.  1.75 1321.75      793.007     70.005      1.003  15992.374    777.605   2968.352  -1913.947     269.896
                          168.403     19.891     91.156      4.331    270.639      4.820     10.215

CC      SET  (  -0.   DEGREES ELEV.)       22. HOURS    1.96 MINUTES    AZIMUTH  67.384 DEGREES
 ELAPSED VISIBLE TIME =      4.74  MINUTES DURING REVOLUTION NUMBER    0.

CC      RISE (  -0.   DEGREES ELEV.)       23. HOURS   27.57 MINUTES    AZIMUTH 252.252 DEGREES

CC 23. 28.50 1408.50      734.575    268.609      2.095  -9493.661    719.189   1930.989  -2969.473     -16.960
                           12.478     -1.284     88.804     -0.287     88.388      0.307      0.658
```

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

```
CC 23. 28.75 1408.75        713.347     273.044       2.450  -7726.235     697.892    1950.506   -2956.220      39.746
                             12.836       3.185      89.283      0.686      87.984       0.715       1.532
CC 23. 29.00 1409.00        696.570     278.079       2.770  -5810.257     681.143    1968.436   -2942.970      97.123
                             13.230       8.075      89.540      1.648      87.590       1.736       3.717
CC 23. 29.25 1409.25        684.727     283.000       2.758  -3771.789     669.284    1986.093   -2928.653     154.518
                             13.552      13.002      90.145      2.443      87.173       2.737       5.890
CC 23. 29.50 1409.50        678.015     288.189       3.054  -1648.730     662.574    2003.356   -2913.272     211.598
                             13.940      18.138      90.528      3.424      87.087       3.779       8.044
CC  MAX EL (    3.17 DEGREES ELEV.)          23. HOURS   29.69 MINUTES      AZIMUTH 292.146 DEGREES
CC 23. 29.75 1409.75        676.613     293.377       2.958    512.333     661.169    2020.317   -2896.848     268.542
                             14.165      23.396      90.999      4.328      86.967       4.800      10.173
CC 23. 30.00 1410.00        680.563     298.458       3.023   2660.094     665.100    2035.736   -2879.799     325.732
                             14.270      28.622      91.467      5.194      86.846       5.821      12.271
CC 23. 30.25 1410.25        689.697     303.708       2.822   4745.537     674.270    2051.432   -2862.337     382.434
                             14.137      33.577      91.841      6.163      86.914       6.843      14.332
CC 23. 30.50 1410.50        703.872     308.751       2.512   6725.358     688.469    2066.520   -2843.083     438.995
                             14.154      38.529      92.332      7.084      87.151       7.864      16.352
CC 23. 30.75 1410.75        722.787     313.387       2.242   8567.851     707.393    2080.300   -2823.223     495.739
                             13.889      43.235      92.765      8.029      87.524       8.885      18.326
CC 23. 31.00 1411.00        746.070     318.065       1.775  10252.785     730.670    2093.971   -2802.759     552.406
                             13.494      47.728      93.144      8.998      87.835       9.907      20.251
CC 23. 31.25 1411.25        773.297     321.773       1.310  11771.277     757.899    2106.367   -2781.307     608.475
                             12.935      51.820      93.612      9.875      88.485      10.928      22.123
CC 23. 31.50 1411.50        804.054     325.909       0.864  13124.404     788.668    2119.183   -2759.231     664.741
                             12.247      55.729      94.037     10.844      89.296      11.949      23.940

CC    SET (  -0.   DEGREES ELEV.)         23. HOURS   31.67 MINUTES      AZIMUTH 328.215 DEGREES
 ELAPSED VISIBLE TIME =      4.10  MINUTES DURING REVOLUTION NUMBER    0.
```

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

FEBRUARY 23, 1965

ST HR	MINS	T-ST	RANGE NAUT MILES	AZIMUTH DEGREES	ELEVATION DEGREES	RANGE RATE FT/SEC	KAPPA DEGREES	ASPECT ANG DEGREES	1 AND 2 DEGREES	X NAUT MILES
			Y NAUT MILES	Z NAUT MILES	TOP RT.ASC DEGREES	TOP DECLIN DEGREES	GEO RT.ASC DEGREES	GEO DECLIN DEGREES	TOP HR.ANG DEGREES	U DEGREES
			V DEGREES							
EE	RISE (	-0.	DEGREES ELEV.)	10. HOURS	8.69 MINUTES		AZIMUTH 193.856 DEGREES			
EE 10.	8.75	608.75	824.893	193.825	0.666	-23799.836	76.401	224.203	19.146	-2070.369
			2346.064	-1660.726	303.939	-69.952	76.781	-27.804	136.345	31.150
			47.955							
EE 10.	9.00	609.00	766.224	192.843	1.626	-23731.551	76.350	225.714	19.753	-2107.458
			2347.713	-1611.680	304.208	-71.287	77.286	-27.009	136.205	30.129
			47.097							
EE 10.	9.25	609.25	707.768	191.733	2.752	-23636.979	76.211	227.193	20.534	-2143.762
			2348.340	-1561.481	304.178	-72.730	77.694	-26.171	136.229	29.108
			46.201							
EE 10.	9.50	609.50	649.565	190.668	3.676	-23507.535	75.959	228.621	21.532	-2179.589
			2348.770	-1511.586	304.099	-74.499	78.290	-25.297	136.395	28.087
			45.264							
EE 10.	9.75	609.75	591.696	189.274	5.043	-23329.216	75.566	229.976	22.805	-2214.396
			2347.695	-1460.806	303.557	-76.408	78.779	-24.308	136.897	27.066
			44.289							
EE 10.	10.00	610.00	534.427	187.531	6.821	-23081.344	74.988	231.235	24.437	-2248.895
			2346.746	-1409.659	302.336	-78.670	79.335	-23.474	138.318	26.045
			43.269							
EE 10.	10.25	610.25	477.848	185.448	8.222	-22730.896	74.165	232.372	26.546	-2281.974
			2344.910	-1357.991	299.296	-81.445	79.933	-22.607	141.259	25.024
			42.205							
EE 10.	10.50	610.50	422.287	182.675	10.687	-22222.637	73.015	233.362	29.309	-2314.844
			2342.494	-1306.047	290.466	-84.581	80.427	-21.635	150.213	24.003
			41.093							

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

```
EE 10. 10.75  610.75     368.316    178.964     12.782 -21461.998     71.416    234.181     32.994  -2347.023
                        2339.273  -1253.400    239.213    -87.784     80.862    -20.729    201.690     22.982
                          39.934
EE 10. 11.00  611.00     316.683    174.196     16.026 -20279.660     69.207    234.803     38.017  -2378.312
                        2335.506  -1200.612    160.329    -84.399     81.335    -19.822    280.514     21.961
                          38.724
EE 10. 11.25  611.25     268.795    167.240     19.679 -18369.889     66.193    235.205     45.023  -2408.836
                        2331.013  -1147.407    147.300    -77.194     81.890    -18.811    293.538     20.940
                          37.463
EE 10. 11.50  611.50     227.022    157.257     24.628 -15201.952     62.277    235.366     54.937  -2438.816
                        2325.766  -1094.055    143.596    -66.610     82.326    -17.976    297.460     19.920
                          36.149
EE 10. 11.75  611.75     195.331    141.870     29.211 -10049.319     57.923    235.265     68.754  -2467.920
                        2319.844  -1040.121    142.190    -52.128     82.824    -17.082    298.936     18.899
                          34.780
EE 10. 12.00  612.00     179.329    120.988     32.704  -2647.976     54.966    234.888     86.433  -2495.664
                        2313.229   -985.423    141.604    -33.908     83.241    -16.099    299.488     17.878
                          33.355
EE  MAX EL (  32.86 DEGREES ELEV.)      10. HOURS  12.08 MINUTES    AZIMUTH 112.547 DEGREES
EE 10. 12.25  612.25     183.040     97.685     31.746   5542.089     55.687    234.224    105.369  -2523.859
                        2305.673   -930.910    141.514    -14.373     83.657    -15.233    299.752     16.857
                          31.874
EE 10. 12.50  612.50     205.364     79.019     27.712  12189.359     59.441    233.267    121.778  -2550.107
                        2297.619   -876.277    141.792      2.521     84.195    -14.308    299.521     15.836
                          30.334
EE 10. 12.75  612.75     241.246     65.829     22.782  16544.801     63.752    232.019    134.058  -2576.336
                        2289.247   -821.350    142.123     15.543     84.550    -13.427    299.162     14.815
                          28.737
EE 10. 13.00  613.00     285.615     57.035     18.421  17131.752     67.345    230.488    142.743  -2601.928
                        2280.137   -765.438    142.615     24.836     85.111    -12.481    298.669     13.795
                          27.081
EE 10. 13.25  613.25     335.122     50.885     14.597  20783.045     70.044    228.692    148.867  -2626.296
                        2270.299   -710.010    143.267     31.496     85.527    -11.594    298.153     12.774
                          25.367
EE 10. 13.50  613.50     387.759     46.711     11.811  21788.605     72.009    226.657    153.256  -2649.413
                        2259.561   -654.198    143.903     36.591     86.007    -10.648    297.534     11.753
                          23.596
EE 10. 13.75  613.75     442.374     43.468      9.388  22445.765     73.428    224.418    156.464  -2672.352
                        2248.366   -598.077    144.500     40.506     86.530     -9.740    296.891     10.732
                          21.768
```

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

```
EE 10. 14.00  614.00      498.398     40.884       7.610  22891.073      74.447     222.017     158.845  -2694.295
                         2236.506   -541.732     145.428     43.663      86.946      -8.749     296.232      9.711
                           19.887
EE 10. 14.25  614.25      555.298     39.042       5.969  23201.917      75.172     219.498     160.631  -2715.631
                         2223.560   -485.045     146.288     46.081      87.392      -7.899     295.476      8.691
                           17.954
EE 10. 14.50  614.50      612.870     37.317       4.305  23423.689      75.677     216.913     161.977  -2735.629
                         2210.535   -428.842     147.175     48.230      87.720      -6.941     294.587      7.570
                           15.972
EE 10. 14.75  614.75      670.926     36.251       3.203  23584.898      76.015     214.307     162.988  -2755.165
                         2196.609   -371.895     147.906     50.003      88.281      -6.061     293.600      6.649
                           13.945
EE 10. 15.00  615.00      729.293     35.038       2.020  23703.041      76.222     211.726     163.740  -2773.797
                         2182.388   -315.108     148.984     51.513      88.612      -5.153     292.978      5.629
                           11.878
EE 10. 15.25  615.25      787.946     33.950       1.276  23789.554      76.327     209.208     164.287  -2791.532
                         2167.269   -257.879     149.843     52.751      89.121      -4.208     292.086      4.608
                            9.774

EE      SET  (  -0.   DEGREES ELEV.)       10. HOURS  15.43 MINUTES      AZIMUTH  33.480 DEGREES
 ELAPSED VISIBLE TIME =      6.74  MINUTES DURING REVOLUTION NUMBER     01

EE      RISE (  -0.   DEGREES ELEV.)       22. HOURS  45.04 MINUTES      AZIMUTH  -0.098 DEGREES

EE 22. 45.25 1365.25      759.308      1.332       1.266 -22362.174      76.704     118.007      29.326  -2512.184
                         2485.971   -155.121     274.971     74.123     270.290      -2.551     354.843    177.225
                            5.928
EE 22. 45.50 1365.50      704.490      3.205       1.948 -22017.376      76.590     116.674      31.086  -2526.043
                         2467.455   -212.288     281.443     72.915     270.613      -3.479     348.525    176.201
                            8.087
EE 22. 45.75 1365.75      650.688      5.676       2.925 -21574.111      76.402     115.430      33.197  -2539.357
                         2448.193   -269.511     287.940     71.259     271.120      -4.395     342.118    175.177
                           10.221
EE 22. 46.00 1366.00      598.106      8.396       4.126 -20999.495      76.098     114.287      35.742  -2551.702
                         2428.414   -326.321     294.201     69.128     271.549      -5.317     335.843    174.153
                           12.323
EE 22. 46.25 1366.25      547.126     11.581       5.546 -20246.259      75.667     113.258      38.825  -2563.437
                         2407.932   -383.531     300.471     66.614     272.009      -6.274     329.609    173.129
                           14.388
```

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

```
EE 22. 46.50 1366.50    498.339     15.356      7.016 -19247.960     75.088    112.353     42.581  -2574.103
                       2386.359   -440.396    306.660     63.366    272.466     -7.062    323.669    172.106
                         16.412
EE 22. 46.75 1366.75    452.374     20.376      8.347 -17912.150     74.347    111.583     47.180  -2584.239
                       2363.958   -496.873    312.417     59.172    272.806     -8.093    317.765    171.082
                         18.389
EE 22. 47.00 1367.00    410.259     26.294     10.150 -16115.370     73.442    110.958     52.821  -2593.607
                       2341.303   -553.600    318.200     53.989    273.342     -9.088    312.183    170.058
                         20.317
EE 22. 47.25 1367.25    373.330     33.260     11.510 -13709.603     72.408    110.488     59.710  -2602.224
                       2317.914   -609.974    323.492     47.675    273.695     -9.963    306.933    169.034
                         22.192
EE 22. 47.50 1367.50    343.221     41.987     13.236 -10555.447     71.346    110.181     67.996  -2609.501
                       2293.393   -665.925    328.524     39.746    274.162    -10.865    302.009    168.010
                         24.012
EE 22. 47.75 1367.75    321.854     52.125     14.509  -6605.204     70.438    110.045     77.651  -2616.869
                       2266.389   -721.828    333.351     30.494    274.653    -11.746    297.223    166.986
                         25.775
EE 22. 48.00 1368.00    311.111     63.495     15.351  -2020.094     69.923    110.086     88.332  -2623.085
                       2242.671   -777.728    337.763     20.218    275.085    -12.688    292.847    165.962
                         27.481
EE  MAX EL (  15.24 DEGREES ELEV.)       22. HOURS  48.11 MINUTES      AZIMUTH  68.884 DEGREES
EE 22. 48.25 1368.25    312.065     75.398     15.201   2785.679     69.971    110.311     99.357  -2628.396
                       2216.322   -833.068    341.988      9.796    275.549    -13.684    288.642    164.938
                         29.127
EE 22. 48.50 1368.50    324.625     86.663     14.451   7291.792     70.566    110.724    109.906  -2633.047
                       2189.286   -888.358    345.953     -0.589    275.911    -14.504    284.711    163.915
                         30.715
EE 22. 48.75 1368.75    347.528     96.725     12.943  11119.716     71.515    111.329    119.342  -2636.885
                       2161.263   -943.125    349.807     -9.606    276.393    -15.461    281.024    162.891
                         32.245
EE 22. 49.00 1369.00    378.862    105.107     11.548  14148.407     72.584    112.127    127.379  -2639.842
                       2132.621   -998.046    353.293    -17.096    276.930    -16.340    277.565    161.867
                         33.717
EE 22. 49.25 1369.25    416.761    111.974     10.068  16447.630     73.602    113.119    134.029  -2642.579
                       2103.829  -1052.029    356.646    -23.510    277.470    -17.295    274.336    160.843
                         35.132
EE 22. 49.50 1369.50    459.609    117.680      8.337  18162.432     74.482    114.303    139.454  -2643.949
                       2073.801  -1105.984    359.668    -28.702    277.893    -18.291    271.150    159.819
                         36.490
```

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Continued)

```
EE 22. 49.75 1369.75        506.062    122.033      6.863  19438.317     75.197    115.673    143.862  -2644.979
                           2043.326  -1159.214      2.755    -32.898    278.414    -19.131    268.405    158.795
                             37.795
EE 22. 50.00 137C.00        555.305    126.024      5.586  20393.591     75.751    117.223    147.449  -2644.549
                           2012.409  -1212.674      5.541    -36.179    278.918    -20.029    265.618    157.772
                             39.046
EE 22. 50.25 1370.25        606.564    129.088      4.096  21115.633     76.160    118.942    150.376  -2644.105
                           1980.400  -1265.505      8.102    -38.843    279.341    -20.077    262.986    156.748
                             40.245
EE 22. 50.50 1370.50        659.425    131.813      3.242  21667.927     76.445    120.817    152.774  -2642.506
                           1947.814  -1318.027     10.668    -41.045    279.774    -21.826    260.585    155.724
                             41.395
EE 22. 50.75 1370.75        713.469    133.988      1.945  22095.104     76.626    122.830    154.744  -2640.156
                           1914.817  -1370.206     13.078    -42.839    280.404    -22.814    258.219    154.700
                             42.497
EE 22. 51.0G 1371.00        768.451    135.838      1.372  22428.469     76.718    124.962    156.364  -2637.244
                           1881.154  -1421.396     15.342    -44.297    280.858    -23.672    256.079    153.676
                             43.552

EE    SET  (  -0.   DEGREES ELEV.)      22. HOURS  51.21 MINUTES    AZIMUTH 137.349 DEGREES
 ELAPSED VISIBLE TIME =      6.17  MINUTES DURING REVOLUTION NUMBER   0.
```

Figure 5-32. Sample TRACE-D Program Data-Generation Listing (Concluded)

Table 5-15. Data Generation Output Listing Description

Item	Description	Page Reference
1	Group-2 FINP input. The entries shown indicate that a bias of 1000 feet is to be applied to all range observations generated for Station BB.	5-46/5-48
2	Sigmas. In the case of a data generation run, the sigma table is used for the purpose of specifying the standard deviation of random noise which is to be applied to the generated data. Interpretation of the printed output shown is otherwise the same as described under Item 12, Table 5-14.	3-39 5-46/5-48
3	Specification-I input data. The eight columns from left to right contain station identification, data interval, minimum elevation angle, maximum elevation angle, maximum range in nautical miles, and data-generation start and stop times in days, hours, and minutes from midnight of epoch day. The information shown indicates that the following specifications have been given to the program: a. Observations are to be generated for Station AA at 15-second intervals whenever the computed local elevation angle is above zero degrees. b. No maximum elevation angle is assumed and no maximum range is to be considered (data generated for all ranges and for all elevation angles greater than zero). c. Station AA is assumed to be active during the time interval from epoch until a time 24 hours after midnight of epoch day. In the case of this particular data generation run, the corresponding Specification-I input items for Stations BB through FF are identical to those for Station AA. However, this need not be true in general, inasmuch as each station is independent of the others with respect to these input items.	3-15 5-47/5-50

Table 5-15. Data Generation Output Listing Description (Continued

Item	Description	Page Reference
4	Parameter indication. The message indicates that the range bias parameter has been selected for Station BB. In the case of a data generation run, radar parameters are selected only for the purpose of applying biases to the generated data, in this case a range bias on Station BB data. The message alluding to the correction of parameters therefore should be ignored.	3-39, 3-40 5-48
5	Specification-II data headings. These headings define the format of the table appearing beneath them. The purpose of this table is to define the types of data which are to be generated for each station. Except for the station-identification column header, which is printed vertically, the headings appear in two horizontal rows, with the top row displaying the symbols RANGE on the left and LONG. on the right and the second row the symbols SUR. R on the left and U, V on the right. The individual headings are interpreted in accordance with the following:	3-13/3-19 5-50/5-54

Heading	Description or Symbol	Units
RANGE	Range	n mi
AZMTH	Local azimuth angle	deg
ELEV.	Local elevation angle	deg
R. DOT	Range rate	ft/sec
P. DOT	Range-rate difference	ft/sec
Q. DOT	Range-rate difference	ft/sec
P	Range difference	ft
Q	Range difference	ft
AZ. DT	Rate of change of local azimuth	deg/sec
E. DOT	Rate of change of local elevation	deg/sec
R. DDT	Second time derivative of range	ft/sec^2
MU. VIS	Mutual visibility	(indicator)
LAT	Latitude of sub-vehicle point	deg
LONG.	Longitude of sub-vehicle point	deg
SUR. R	Surface range, station to sub-vehicle point	n mi

Table 5-15. Data Generation Output Listing Description (Continued)

<table>
<tr><th>Item</th><th colspan="3">Description</th><th>Page Reference</th></tr>
<tr><td>5 (cont)</td><td><u>Heading</u></td><td><u>Description or Symbol</u></td><td><u>Units</u></td><td></td></tr>
<tr><td></td><td>HIGHT</td><td>Altitude above oblate earth</td><td>n mi</td><td></td></tr>
<tr><td></td><td>DOPLR</td><td>Doppler frequency shift</td><td>cps</td><td></td></tr>
<tr><td></td><td>LOOK</td><td>Look angle</td><td>deg</td><td></td></tr>
<tr><td></td><td>VARI</td><td>Variances</td><td>Same as corresp. observations</td><td></td></tr>
<tr><td></td><td>KAPPA</td><td>Angle between radius vector and local vertical</td><td>deg</td><td></td></tr>
<tr><td></td><td>ASPCT</td><td>Aspect angles</td><td>deg</td><td></td></tr>
<tr><td></td><td>ATTEN</td><td>Signal attenuation</td><td>db</td><td></td></tr>
<tr><td></td><td>X, Y, Z</td><td>$\hat{x}, \hat{y}, \hat{z}$</td><td>n mi</td><td></td></tr>
<tr><td></td><td>T-R, D</td><td>Topocentric right ascension and declination</td><td>deg</td><td></td></tr>
<tr><td></td><td>G-R, D</td><td>Geocentric right ascension and declination</td><td>deg</td><td></td></tr>
<tr><td></td><td>HR. ANG</td><td>Topocentric hour angle</td><td>deg</td><td></td></tr>
<tr><td></td><td>U, V</td><td>Horizon-scanner angles u, v</td><td>deg</td><td></td></tr>
<tr><td>6</td><td colspan="3">Data-type specification table (Specification-II information). The two horizontal lines associated with each station are the line containing the station-identification symbol and the line immediately below it. An X on the first line or a Y on the second line indicate that the corresponding quantity a defined by the table header is to be generated. In the present example the indicated data types for Station AA are range, azimuth, elevation, range rate, height (altitude), $\hat{x}$, $\hat{y}$, $\hat{z}$, topocentric right ascension and declination, topocentric hour angle, and horizon-sensor angles u and v.</td><td>5-50/5-54</td></tr>
<tr><td>7</td><td colspan="3">Node prints. Each time the integrated position of the satellite crosses the equator, the time in minutes from midnight of epoch day and the satellite position and velocity in the basic coordinate system are printed. Position and velocity are given in the octal mode and in units of earth radii and earth radii per minute.</td><td>3-63
5-5</td></tr>
</table>

Table 5-15. Data Generation Output Listing Description (Concluded)

Item	Description	Page Reference
8	Rise message. The time when the satellite becomes visible from a particular station (at the specified minimum elevation angle) is obtained by interpolation and printed in the manner shown, along with the local azimuth angle for the corresponding time.	3-38, 3-39 5-39
9	Time corresponding to generated data. The time shown is to be associated with the data quantities appearing on the line to the right of the time printout and on the line following. The time is given both in hours (0 through 24) and minutes of the day as identified at the top of the output page, as well as in minutes from start (i. e., minutes from epoch). In the case of the particular output shown, time from start corresponds to the time of day because the epoch chosen for the run happened to be midnight.	5-48
10	Maximum elevation point. The time when the elevation angle reaches its maximum is obtained by interpolation and printed along with the corresponding values for the elevation and azimuth angles in the manner shown.	5-39
11	Set message. When the time at which the elevation angle reaches the specified minimum from above is obtained by interpolation, the print shown at this position occurs.	3-38, 3-39 5-39
12	Duration message. After each pass, a message is printed giving the time in minutes during which the elevation angle was above the input minimum value and the range was below the input maximum value.	5-39

5.5.5 Residuals Analysis Output Description

The listed output created by a typical residuals analysis run is shown in Figure 5-33. Supplementary descriptive information relating to the indicated areas of this listing is annotated in Table 5-16.

```
                                                    +++++
                                               ++++       ++++
                                           +++                 +++
     TTTTTTTTTTT     RRRRRRRRR           +                AA      +        CCCCCCCCCCC     EEEEEEEEEEE
     TTTTTTTTTTT     RRRRRRRRR          +                AAAA      +       CCCCCCCCCCC     EEEEEEEEEEE
         TT          RR       RR        +              AA  AA      +       CC              EE
         TT          RRRRRRRRR         +             AA    AA       +      CC              EEEEEE
         TT          RRRRRRRRR         +           AAAAAAAAAA       +      CC              EEEEEE
         TT          RR      RR        +         AAAAAAAAAAAA       +      CC              EE
         TT          RR       RR        +    AA            AA      +       CCCCCCCCCCC     EEEEEEEEEEE
         TT          RR        RR       +  AA              AA      +       CCCCCCCCCCC     EEEEEEEEEEE
                                         +                        +
                                           +++                 +++
                                               ++++       ++++        AUDI40
                                                    +++++             REFERENCE
RESIDUALS ANALYSIS EXAMPLE.

EPOCH          YEAR      MONTH      DAY     TZONE      HOUR     MIN       SEC
               1964.       2.       30.       0.         0.      0.        0.

 ENTERING LINK NO.  2 TRAIN

PERMANENT STATIONS
      SIG    REF      LATITUDE           LONGITUDE          HEIGHT

AA     0.     0.    0.75000000E 02     0.30000000E 03     0.09999999E 04
BB     0.     0.    0.45000000E 02     0.23999999E 03     0.09999999E 03
CC     0.     0.    0.                 0.31300000E 03     0.50000000E 03
DD     0.     0.    0.                 0.15000000E 02     0.20000000E 03
EE     0.     0.   -0.15000000E 02     0.13500000E 03     0.09999999E 03
FF     0.     0.   -0.65000000E 02     0.75000000E 02     0.50000000E 04
OBSERVATIONS
 MO DY HR MN  SEC         STA  TYP
  3  1  1 41  0.           CC   10   0.98321319E 07   0.11314791E 03   0.60135999E 01
  3  1  1 41  0.           CC   50   0.21882530E 08  -0.91494808E 07  -0.38542949E 07
  3  1  1 41 15.0000       CC   10   0.96271659E 07   0.11143903E 03   0.69271099E 01
```

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Output Listing

3	1	1	41	15.0000	CC	50	0.21864933E	08	-0.92808560E	07	-0.35073360E	07
3	1	1	41	30.0000	CC	10	0.94291389E	07	0.10976144E	03	0.75158699E	01
3	1	1	41	30.0000	CC	50	0.21863415E	08	-0.94111580E	07	-0.31609789E	07
3	1	1	41	45.0000	CC	10	0.92384560E	07	0.10794291E	03	0.81498498E	01
3	1	1	41	45.0000	CC	50	0.21854419E	08	-0.95386470E	07	-0.28128549E	07
3	1	1	42	0.	CC	10	0.90562890E	07	0.10587610E	03	0.87525899E	01
3	1	1	42	0.	CC	50	0.21840169E	08	-0.96653250E	07	-0.24639679E	07
3	1	1	42	15.0000	CC	10	0.88825469E	07	0.10400422E	03	0.92930599E	01
3	1	1	42	15.0000	CC	50	0.21821429E	08	-0.97893589E	07	-0.21143589E	07
3	1	1	42	30.0000	CC	10	0.87185479E	07	0.10164676E	03	0.10117289E	02
3	1	1	42	30.0000	CC	50	0.21800026E	08	-0.99108949E	07	-0.17637429E	07
3	1	1	42	45.0000	CC	10	0.85641560E	07	0.99563129E	02	0.10483950E	02
3	1	1	42	45.0000	CC	50	0.21770038E	08	-0.10031149E	08	-0.14138529E	07
3	1	1	43	0.	CC	10	0.84203320E	07	0.97332370E	02	0.11171219E	02
3	1	1	43	0.	CC	50	0.21735252E	08	-0.10146124E	08	-0.10638950E	07
3	1	1	43	15.0000	CC	10	0.82873520E	07	0.94877850E	02	0.11555240E	02
3	1	1	43	15.0000	CC	50	0.21696250E	08	-0.10261197E	08	-0.71314499E	06
3	1	1	43	30.0000	CC	10	0.81663059E	07	0.92606051E	02	0.12034769E	02
3	1	1	43	30.0000	CC	50	0.21650736E	08	-0.10373781E	08	-0.36297899E	06
3	1	1	43	45.0000	CC	10	0.80572929E	07	0.90088639E	02	0.12575369E	02
3	1	1	43	45.0000	CC	50	0.21601278E	08	-0.10482479E	08	-0.10750999E	05
3	1	1	44	0.	CC	10	0.79608650E	07	0.87433948E	02	0.13168079E	02
3	1	1	44	0.	CC	50	0.21545587E	08	-0.10588404E	08	0.33841900E	06
3	1	1	44	15.0000	CC	10	0.78779669E	07	0.84632299E	02	0.13586140E	02
3	1	1	44	15.0000	CC	50	0.21486814E	08	-0.10693917E	08	0.68885899E	06
3	1	1	44	30.0000	CC	10	0.78085350E	07	0.82162429E	02	0.13745189E	02
3	1	1	44	30.0000	CC	50	0.21421335E	08	-0.10797413E	08	0.10406379E	07
3	1	1	44	45.0000	CC	10	0.77532610E	07	0.79464659E	02	0.14204849E	02
3	1	1	44	45.0000	CC	50	0.21350726E	08	-0.10895403E	08	0.13896960E	07
3	1	1	45	0.	CC	10	0.77125780E	07	0.76455370E	02	0.14104610E	02
3	1	1	45	0.	CC	50	0.21274888E	08	-0.10990801E	08	0.17395010E	07
3	1	1	45	15.0000	CC	10	0.76864460E	07	0.73666049E	02	0.14379250E	02
3	1	1	45	15.0000	CC	50	0.21197424E	08	-0.11086029E	08	0.20902880E	07
3	1	1	45	30.0000	CC	10	0.76749189E	07	0.70750369E	02	0.14425959E	02
3	1	1	45	30.0000	CC	50	0.21109664E	08	-0.11177154E	08	0.24395909E	07
3	1	1	45	45.0000	CC	10	0.76784709E	07	0.67895159E	02	0.14538459E	02
3	1	1	45	45.0000	CC	50	0.21019173E	08	-0.11265577E	08	0.27869489E	07

OBSERVATIONS

MO	DY	HR	MN	SEC	STA	TYP						
3	1	1	46	0.	CC	10	0.76989180E	07	0.65183309E	02	0.14347190E	02
3	1	1	46	0.	CC	50	0.20924477E	08	-0.11350807E	08	0.31355510E	07

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

3	1	1	46	15.0000	CC	10	0.77302170E	07	0.62396419E	02	0.14229389E	02
3	1	1	46	15.0000	CC	50	0.20824111E	08	-0.11433195E	08	0.34821200E	07
3	1	1	46	30.0000	CC	10	0.77781550E	07	0.59694660E	02	0.13815550E	02
3	1	1	46	30.0000	CC	50	0.20719042E	08	-0.11514574E	08	0.38293350E	07
3	1	1	46	45.0000	CC	10	0.78402229E	07	0.56899290E	02	0.13712530E	02
3	1	1	46	45.0000	CC	50	0.20609713E	08	-0.11592726E	08	0.41729020E	07
3	1	1	47	0.	CC	10	0.79163630E	07	0.54135890E	02	0.13520490E	02
3	1	1	47	0.	CC	50	0.20492141E	08	-0.11664816E	08	0.45192739E	07
3	1	1	47	15.0000	CC	10	0.80062460E	07	0.51558549E	02	0.12961940E	02
3	1	1	47	15.0000	CC	50	0.20373330E	08	-0.11737457E	08	0.48627230E	07
3	1	1	47	30.0000	CC	10	0.81088810E	07	0.48741280E	02	0.12497450E	02
3	1	1	47	30.0000	CC	50	0.20247084E	08	-0.11805889E	08	0.52073160E	07
3	1	1	47	45.0000	CC	10	0.82244769E	07	0.46461780E	02	0.12126270E	02
3	1	1	47	45.0000	CC	50	0.20120246E	08	-0.11870093E	08	0.55470549E	07
3	1	1	48	0.	CC	10	0.83519020E	07	0.43984099E	02	0.11330250E	02
3	1	1	48	0.	CC	50	0.19984081E	08	-0.11934046E	08	0.58869839E	07
3	1	1	48	15.0000	CC	10	0.84910059E	07	0.41760810E	02	0.11021250E	02
3	1	1	48	15.0000	CC	50	0.19845566E	08	-0.11994646E	08	0.62249779E	07
3	1	1	48	30.0000	CC	10	0.86410429E	07	0.39477800E	02	0.10121160E	02
3	1	1	48	30.0000	CC	50	0.19702294E	08	-0.12050914E	08	0.65610650E	07
3	1	1	48	45.0000	CC	10	0.88013799E	07	0.37280410E	02	0.97606999E	01
3	1	1	48	45.0000	CC	50	0.19553179E	08	-0.12104429E	08	0.68989920E	07
3	1	1	49	0.	CC	10	0.89711229E	07	0.35273589E	02	0.91562498E	01
3	1	1	49	0.	CC	50	0.19401298E	08	-0.12155575E	08	0.72328540E	07
3	1	1	49	15.0000	CC	10	0.91506990E	07	0.33185620E	02	0.84869499E	01
3	1	1	49	15.0000	CC	50	0.19247[illegible]6E	08	-0.12199187E	08	0.75622640E	07
3	1	1	49	30.0000	CC	10	0.93382869E	07	0.31529979E	02	0.78336299E	01
3	1	1	49	30.0000	CC	50	0.19082852E	08	-0.12245121E	08	0.78939760E	07
3	1	1	49	45.0000	CC	10	0.95342118E	07	0.29640260E	02	0.69194799E	01
3	1	1	49	45.0000	CC	50	0.18915865E	08	-0.12286843E	08	0.82241669E	07
3	1	1	50	0.	CC	10	0.97375929E	07	0.27957650E	02	0.64043099E	01
3	1	1	50	0.	CC	50	0.18745715E	08	-0.12324352E	08	0.85508609E	07
3	1	1	50	15.0000	CC	10	0.99480788E	07	0.26290020E	02	0.55319299E	01
3	1	1	50	15.0000	CC	50	0.18573032E	08	-0.12357724E	08	0.88751750E	07
3	1	1	50	30.0000	CC	10	0.10165258E	08	0.24608099E	02	0.49819399E	01
3	1	1	50	30.0000	CC	50	0.18390345E	08	-0.12389456E	08	0.91994659E	07
3	1	1	50	45.0000	CC	10	0.10388227E	08	0.23265090E	02	0.42675199E	01
3	1	1	50	45.0000	CC	50	0.18208781E	08	-0.12420805E	08	0.95193148E	07
3	1	1	51	0.	CC	10	0.10616960E	08	0.21953849E	02	0.35398299E	01
3	1	1	51	0.	CC	50	0.18023016E	08	-0.12443338E	08	0.98368640E	07
3	1	1	51	15.0000	CC	10	0.10851057E	08	0.20527969E	02	0.29359495E	01
3	1	1	51	15.0000	CC	50	0.17828877E	08	-0.12464226E	08	0.10154117E	08

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

```
 3  1  1 51 30.0000          CC  10  0.11090098E 08  0.19472930E 02  0.22065400E 01
 3  1  1 51 30.0000          CC  50  0.17633715E 08 -0.12493431E 08  0.10469531E 08

OBSERVATIONS
MO DY HR MN  SEC        STA  TYP
 3  1  1 51 45.0000          CC  10  0.11333594E 08  0.17796630E 02  0.15377700E 01
 3  .  1 51 45.0000          CC  50  0.17430504E 08 -0.12498587E 08  0.10779984E 08
 3  1  1 52  0.              CC  10  0.11581186E 08  0.16901939E 02  0.94261999E 00
 3  1  1 52  0.              CC  50  0.17229609E 08 -0.12511849E 08  0.11090338E 08
 3  1  1 57 45.0000          AA  10  0.11717547E 08  0.15971985E 03  0.70477000E 00
 3  1  1 57 45.0000          AA  50  0.11570190E 08 -0.11863647E 08  0.17362802E 08
 3  1  1 58  0.              AA  10  0.11410024E 08  0.16021693E 03  0.17808400E 01
 3  1  1 58  0.              AA  50  0.11287018E 08 -0.11795828E 08  0.17593618E 08
 3  1  1 58 15.0000          AA  10  0.11103345E 08  0.16068786E 03  0.23914800E 01
 3  1  1 58 15.0000          AA  50  0.11002123E 08 -0.11721557E 08  0.17818327E 08
 3  1  1 58 30.0000          AA  10  0.10797819E 08  0.16122548E 03  0.33310699E 01
 3  1  1 58 30.0000          AA  50  0.10716010E 08 -0.11646397E 08  0.18043838E 08
 3  1  1 58 45.0000          AA  10  0.10493302E 08  0.16203438E 03  0.39515600E 01
 3  1  1 58 45.0000          AA  50  0.10427681E 08 -0.11568262E 08  0.18261661E 08
 3  1  1 59  0.              AA  10  0.10189763E 08  0.16243205E 03  0.50818600E 01
 3  1  1 59  0.              AA  50  0.10136297E 08 -0.11487453E 08  0.18473978E 08
 3  1  1 59 15.0000          AA  10  0.98878300E 07  0.16320541E 03  0.59329899E 01
 3  1  1 59 15.0000          AA  50  0.98427029E 07 -0.11404864E 08  0.18685186E 08
 3  1  1 59 30.0000          AA  10  0.95875900E 07  0.16405807E 03  0.71234299E 01
 3  1  1 59 30.0000          AA  50  0.95490730E 07 -0.11314785E 08  0.18887617E 08
 3  1  1 59 45.0000          AA  10  0.92890790E 07  0.16495316E 03  0.80794999E 01
 3  1  1 59 45.0000          AA  50  0.92506488E 07 -0.11222759E 08  0.19090694E 08
 3  1  2 -0  0.              AA  10  0.89927840E 07  0.16573819E 03  0.91624599E 01
 3  1  2 -0  0.              AA  50  0.89505649E 07 -0.11127768E 08  0.19285378E 08
 3  1  2 -0 15.0000          AA  10  0.86992729E 07  0.16644253E 03  0.10394990E 02
 3  1  2 -0 15.0000          AA  50  0.86512668E 07 -0.11031011E 08  0.19480997E 08
 3  1  2 -0 30.0000          AA  10  0.84083088E 07  0.16773359E 03  0.11723410E 02
 3  1  2 -0 30.0000          AA  50  0.83478019E 07 -0.10930445E 08  0.19666374E 08
 3  1  2 -0 45.0000          AA  10  0.81200540E 07  0.16851755E 03  0.12852490E 02
 3  1  2 -0 45.0000          AA  50  0.80421680E 07 -0.10827199E 08  0.19849823E 08
 3  1  2  1  0.              AA  10  0.78358750E 07  0.16986194E 03  0.14019679E 02
 3  1  2  1  0.              AA  50  0.77410129E 07 -0.10718500E 08  0.20029015E 08
 3  1  2  1 15.0000          AA  10  0.75552320E 07  0.17033922E 03  0.15411229E 02
 3  1  2  1 15.0000          AA  50  0.74304320E 07 -0.10610124E 08  0.20201900E 08
 3  1  2  1 30.0000          AA  10  0.72795640E 07  0.17242993E 03  0.16639969E 02
 3  1  2  1 30.0000          AA  50  0.71207329E 07 -0.10494956E 08  0.20371526E 08
 3  1  2  1 45.0000          AA  10  0.70087429E 07  0.17379969E 03  0.18190210E 02
```

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

3	1	2	1	45.0000	AA	50	0.68096679E 07	-0.10381257E 08	0.20536461E 08
3	1	2	2	0.	AA	10	0.67440160E 07	0.17550372E 03	0.19783240E 02
3	1	2	2	0.	AA	50	0.64991840E 07	-0.10259937E 08	0.20698682E 08
3	1	2	2	15.0000	AA	10	0.64857240E 07	0.17723697E 03	0.21398280E 02
3	1	2	2	15.0000	AA	50	0.61871359E 07	-0.10138491E 08	0.20853349E 08
3	1	2	2	30.0000	AA	10	0.62348289E 07	0.17910665E 03	0.23002119E 02
3	1	2	2	30.0000	AA	50	0.58719080E 07	-0.10013583E 08	0.21001804E 08

OBSERVATIONS

NO	DY	HR	MN	SEC	STA	TYP			
3	1	2	2	45.0000	AA	10	0.59928240E 07	0.18161342E 03	0.24571630E 02
3	1	2	2	45.0000	AA	50	0.55557550E 07	-0.98838140E 07	0.21147589E 08
3	1	2	3	0.	AA	10	0.57600300E 07	0.18372463E 03	0.26620530E 02
3	1	2	3	0.	AA	50	0.52426049E 07	-0.97539310E 07	0.21289126E 08
3	1	2	3	15.0000	AA	10	0.55382700E 07	0.18651590E 03	0.28266789E 02
3	1	2	3	15.0000	AA	50	0.49247009E 07	-0.96185820E 07	0.21424872E 08
3	1	2	3	30.0000	AA	10	0.53288160E 07	0.18931605E 03	0.30330110E 02
3	1	2	3	30.0000	AA	50	0.46066019E 07	-0.94830608E 07	0.21556039E 08
3	1	2	3	45.0000	AA	10	0.51330650E 07	0.19281007E 03	0.32295460E 02
3	1	2	3	45.0000	AA	50	0.42902379E 07	-0.93426818E 07	0.21681213E 08
3	1	2	4	0.	AA	10	0.49531939E 07	0.19645271E 03	0.34120909E 02
3	1	2	4	0.	AA	50	0.39695830E 07	-0.91994069E 07	0.21802921E 08
3	1	2	4	15.0000	AA	10	0.47902320E 07	0.20064579E 03	0.36215920E 02
3	1	2	4	15.0000	AA	50	0.36501840E 07	-0.90557949E 07	0.21917184E 08
3	1	2	4	30.0000	AA	10	0.46465690E 07	0.20528434E 03	0.37746470E 02
3	1	2	4	30.0000	AA	50	0.33293920E 07	-0.89064180E 07	0.22029565E 08
3	1	2	4	45.0000	AA	10	0.45242250E 07	0.21024834E 03	0.39512980E 02
3	1	2	4	45.0000	AA	50	0.30090230E 07	-0.87538460E 07	0.22136236E 08
3	1	2	5	0.	AA	10	0.44243889E 07	0.21521650E 03	0.40737379E 02
3	1	2	5	0.	AA	50	0.26885579E 07	-0.86020440E 07	0.22238749E 08
3	1	2	5	15.0000	AA	10	0.43489989E 07	0.22234725E 03	0.41965660E 02
3	1	2	5	15.0000	AA	50	0.23666359E 07	-0.84434830E 07	0.22333542E 08
3	1	2	5	30.0000	AA	10	0.42995880E 07	0.22865429E 03	0.42806160E 02
3	1	2	5	30.0000	AA	50	0.20454159E 07	-0.82887440E 07	0.22425885E 08
3	1	2	5	45.0000	AA	10	0.42766320E 07	0.23548123E 03	0.43166330E 02
3	1	2	5	45.0000	AA	50	0.17234340E 07	-0.81244069E 07	0.22508897E 08
3	1	2	6	0.	AA	10	0.42807080E 07	0.24231999E 03	0.43081579E 02
3	1	2	6	0.	AA	50	0.14003240E 07	-0.79611000E 07	0.22589833E 08
3	1	2	6	15.0000	AA	10	0.43118840E 07	0.24887969E 03	0.42704249E 02
3	1	2	6	15.0000	AA	50	0.10802980E 07	-0.77951230E 07	0.22665829E 08
3	1	2	6	30.0000	AA	10	0.43693409E 07	0.25529926E 03	0.41813470E 02
3	1	2	6	30.0000	AA	50	0.75929199E 06	-0.76244030E 07	0.22736628E 08

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

3	1	2	6	45.0000	AA	10	0.44521059E 07	0.26132669E 03	0.40685610E 02
3	1	2	6	45.0000	AA	50	0.43926200E 06	-0.74541280E 07	0.22802050E 08
3	1	2	7	0.	AA	10	0.45586060E 07	0.26683030E 03	0.39170659E 02
3	1	2	7	0.	AA	50	0.11705600E 06	-0.72811989E 07	0.22860585E 08
3	1	2	7	15.0000	AA	10	0.46874880E 07	0.27188690E 03	0.37309589E 02
3	1	2	7	15.0000	AA	50	-0.20319100E 06	-0.71053839E 07	0.22915304E 08
3	1	2	7	30.0000	AA	10	0.48371969E 07	0.27619137E 03	0.35579970E 02
3	1	2	7	30.0000	AA	50	-0.52344999E 06	-0.69258339E 07	0.22966627E 08
3	1	2	7	45.0000	AA	10	0.50053079E 07	0.28050916E 03	0.33608639E 02
3	1	2	7	45.0000	AA	50	-0.84457500E 06	-0.67442549E 07	0.23009976E 08
3	1	2	8	0.	AA	10	0.51900440E 07	0.28417168E 03	0.31645949E 02
3	1	2	8	0.	AA	50	-0.11629080E 07	-0.65623309E 07	0.23047504E 08
3	1	16	19	45.0000	CC	50	0.11636721E 08	-0.19308977E 08	0.81987140E 07
3	1	16	20	0.	CC	10	0.96978320E 07	0.32450624E 03	0.62844299E 01
3	1	16	20	0.	CC	50	0.11588509E 08	-0.19473395E 08	0.78682979E 07
3	1	16	20	15.0000	CC	10	0.95283960E 07	0.32276943E 03	0.66149199E 01
3	1	16	20	15.0000	CC	50	0.11540005E 08	-0.19634192E 08	0.75400320E 07

OBSERVATIONS

MO	DY	HR	MN	SEC	STA	TYP			
3	1	16	20	30.0000	CC	10	0.93676889E 07	0.32095750E 03	0.73811699E 01
3	1	16	20	30.0000	CC	50	0.11487950E 08	-0.19789366E 08	0.72043310E 07
3	1	16	20	45.0000	CC	10	0.92167079E 07	0.31867152E 03	0.79318599E 01
3	1	16	20	45.0000	CC	50	0.11431418E 08	-0.19938693E 08	0.68680320E 07
3	1	16	21	0.	CC	10	0.90754160E 07	0.31671560E 03	0.86020699E 01
3	1	16	21	0.	CC	50	0.11373356E 08	-0.20085092E 08	0.65310990E 07
3	1	16	21	15.0000	CC	10	0.89443590E 07	0.31457587E 03	0.86383498E 01
3	1	16	21	15.0000	CC	50	0.11311962E 08	-0.20227427E 08	0.61950960E 07
3	1	16	21	30.0000	CC	10	0.88244010E 07	0.31230375E 03	0.93243898E 01
3	1	16	21	30.0000	CC	50	0.11246680E 08	-0.20362257E 08	0.58556990E 07
3	1	16	21	45.0000	CC	10	0.87157940E 07	0.30995499E 03	0.97845999E 01
3	1	16	21	45.0000	CC	50	0.11177643E 08	-0.20493323E 08	0.55153260E 07
3	1	16	22	0.	CC	10	0.86189209E 07	0.30763445E 03	0.10211830E 02
3	1	16	22	0.	CC	50	0.11107693E 08	-0.20621544E 08	0.51737040E 07
3	1	16	22	15.0000	CC	10	0.85346120E 07	0.30516189E 03	0.10486279E 02
3	1	16	22	15.0000	CC	50	0.11036535E 08	-0.20743886E 08	0.48317870E 07
3	1	16	22	30.0000	CC	10	0.84624040E 07	0.30273311E 03	0.10806569E 02
3	1	16	22	30.0000	CC	50	0.10961318E 08	-0.20860074E 08	0.44854769E 07
3	1	16	22	45.0000	CC	10	0.84035000E 07	0.30039778E 03	0.10882930E 02
3	1	16	22	45.0000	CC	50	0.10883936E 08	-0.20971065E 08	0.41436110E 07
3	1	16	23	0.	CC	10	0.83576270E 07	0.29743589E 03	0.11319390E 02

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

```
  3  1 16 23  0.              CC  50  0.10802460E 08 -0.21081577E 08  0.37964520E 07
  3  1 16 23 15.0000          CC  10  0.83250460E 07  0.29484776E 03  0.11135620E 02
  3  1 16 23 15.0000          CC  50  0.10714953E 08 -0.21182210E 08  0.34494260E 07
  3  1 16 23 30.0000          CC  10  0.83062289E 07  0.29231963E 03  0.11604050E 02
  3  1 16 23 30.0000          CC  50  0.10629988E 08 -0.21279008E 08  0.30998839E 07
  3  1 16 23 45.0000          CC  10  0.83010560E 07  0.28974903E 03  0.11731389E 02
  3  1 16 23 45.0000          CC  50  0.10538916E 08 -0.21369562E 08  0.27532890E 07
  3  1 16 24  0.              CC  10  0.83096389E 07  0.28697113E 03  0.11589859E 02
  3  1 16 24  0.              CC  50  0.10449780E 08 -0.21457098E 08  0.24039980E 07
  3  1 16 24 15.0000          CC  10  0.83316170E 07  0.28461847E 03  0.11226100E 02
  3  1 16 24 15.0000          CC  50  0.10356621E 08 -0.21539813E 08  0.20539000E 07
  3  1 16 24 30.0000          CC  10  0.83673619E 07  0.28199329E 03  0.10986770E 02
  3  1 16 24 30.0000          CC  50  0.10258131E 08 -0.21616668E 08  0.17045350E 07
  3  1 16 24 45.0000          CC  10  0.84164650E 07  0.27944347E 03  0.11274379E 02
  3  1 16 24 45.0000          CC  50  0.10158322E 08 -0.21687373E 08  0.13548540E 07
  3  1 16 25  0.              CC  10  0.84783188E 07  0.27690309E 03  0.10793170E 02
  3  1 16 25  0.              CC  50  0.10055370E 08 -0.21755313E 08  0.10045899E 07
  3  1 16 25 15.0000          CC  10  0.85533600E 07  0.27443108E 03  0.10387450E 02
  3  1 16 25 15.0000          CC  50  0.99536340E 07 -0.21814833E 08  0.65421399E 06
  3  1 16 25 30.0000          CC  10  0.86404729E 07  0.27210242E 03  0.10120380E 02
  3  1 16 25 30.0000          CC  50  0.98460299E 07 -0.21873774E 08  0.30200300E 06
  3  1 16 25 45.0000          CC  10  0.87397648E 07  0.26987718E 03  0.96085798E 01
  3  1 16 25 45.0000          CC  50  0.97374018E 07 -0.21923011E 08 -0.49061000E 05
  3  1 16 26  0.              CC  10  0.88507790E 07  0.26752786E 03  0.94605300E 01
  3  1 16 26  0.              CC  50  0.96247170E 07 -0.21970341E 08 -0.40161099E 06
  3  1 16 26 15.0000          CC  10  0.89724180E 07  0.26504107E 03  0.90192599E 01
  3  1 16 26 15.0000          CC  50  0.95121180E 07 -0.22008537E 08 -0.75384700E 06

OBSERVATIONS
 MO DY HR MN  SEC      STA  TYP
  3  1 16 26 30.0000          CC  10  0.91051228E 07  0.26307444E 03  0.83806098E 01
  3  1 16 26 30.0000          CC  50  0.93961140E 07 -0.22042498E 08 -0.11026379E 07
  3  1 16 26 45.0000          CC  10  0.92477770E 07  0.26076229E 03  0.78175199E 01
  3  1 16 26 45.0000          CC  50  0.92793940E 07 -0.22073445E 08 -0.14545050E 07
  3  1 16 27  0.              CC  10  0.94001579E 07  0.25881436E 03  0.73505699E 01
  3  1 16 27  0.              CC  50  0.91573730E 07 -0.22097253E 08 -0.18029810E 07
  3  1 16 27 15.0000          CC  10  0.95612188E 07  0.25680999E 03  0.68123199E 01
  3  1 16 27 15.0000          CC  50  0.90360098E 07 -0.22118917E 08 -0.21540699E 07
  3  1 16 27 30.0000          CC  10  0.97314630E 07  0.25494622E 03  0.62952699E 01
  3  1 16 27 30.0000          CC  50  0.89110320E 07 -0.22132085E 08 -0.25027130E 07
  3  1 16 27 45.0000          CC  10  0.99096020E 07  0.25321196E 03  0.55394799E 01
  3  1 16 27 45.0000          CC  50  0.87820939E 07 -0.22138870E 08 -0.28503370E 07
```

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

```
 3  1 16 28  0.          CC  10  0.10095143E 08  0.25147051E 03  0.50525799E 01
 3  1 16 28  0.          CC  50  0.86549020E 07 -0.22142012E 08 -0.31979480E 07
 3  1 16 28 15.0000      CC  10  0.10288278E 08  0.24972907E 03  0.43353099E 01
 3  1 16 28 15.0000      CC  50  0.85244498E 07 -0.22142044E 08 -0.35478400E 07
 3  1 16 28 30.0000      CC  10  0.10487918E 08  0.24822015E 03  0.38247599E 01
 3  1 16 28 30.0000      CC  50  0.83907100E 07 -0.22132224E 08 -0.38938329E 07
 3  1 16 28 45.0000      CC  10  0.10693694E 08  0.24669360E 03  0.31274199E 01
 3  1 16 28 45.0000      CC  50  0.82578560E 07 -0.22119981E 08 -0.42390790E 07
 3  1 16 29  0.          CC  10  0.10905723E 08  0.24510150E 03  0.25162700E 01
 3  1 16 29  0.          CC  50  0.81210320E 07 -0.22100075E 08 -0.45824099E 07
 3  1 16 29 15.0000      CC  10  0.11122876E 08  0.24372969E 03  0.20374899E 01
 3  1 16 29 15.0000      CC  50  0.79862540E 07 -0.22077250E 08 -0.49261770E 07
 3  1 16 29 30.0000      CC  10  0.11345667E 08  0.24221850E 03  0.14126199E 01
 3  1 16 29 30.0000      CC  50  0.78418040E 07 -0.22048592E 08 -0.52694570E 07
 3  1 16 29 45.0000      CC  10  0.11572734E 08  0.24103080E 03  0.83920000E 00
 3  1 16 29 45.0000      CC  50  0.76985070E 07 -0.22015622E 08 -0.56100609E 07
 3  1 16 54 45.0000      FF  10  0.11846369E 08  0.14052901E 03  0.79513000E 00
 3  1 16 54 45.0000      FF  50 -0.71407880E 07  0.25428589E 07 -0.22763252E 08
 3  1 16 55  0.          FF  10  0.11714226E 08  0.13861485E 03  0.11195999E 01
 3  1 16 55  0.          FF  50 -0.72230080E 07  0.28964180E 07 -0.22695605E 08
 3  1 16 55 15.0000      FF  10  0.11588953E 08  0.13706320E 03  0.11978299E 01
 3  1 16 55 15.0000      FF  50 -0.73039070E 07  0.32493179E 07 -0.22623130E 08
 3  1 16 55 30.0000      FF  10  0.11470847E 08  0.13544410E 03  0.13358299E 01
 3  1 16 55 30.0000      FF  50 -0.73800780E 07  0.36039440E 07 -0.22544499E 08
 3  1 16 55 45.0000      FF  10  0.11360148E 08  0.13358006E 03  0.15644100E 01
 3  1 16 55 45.0000      FF  50 -0.74544689E 07  0.39560340E 07 -0.22459226E 08
 3  1 16 56  0.          FF  10  0.11257105E 08  0.13173418E 03  0.20241399E 01
 3  1 16 56  0.          FF  50 -0.75254590E 07  0.43081489E 07 -0.22371063E 08
 3  1 16 56 15.0000      FF  10  0.11161847E 08  0.13008589E 03  0.23097799E 01
 3  1 16 56 15.0000      FF  50 -0.75957859E 07  0.46593290E 07 -0.22278128E 08
 3  1 16 56 30.0000      FF  10  0.11074729E 08  0.12828288E 03  0.25525799E 01
 3  1 16 56 30.0000      FF  50 -0.76640510E 07  0.50096220E 07 -0.22179049E 08

OBSERVATIONS
MO DY HR MN  SEC     STA  TYP
 3  1 16 56 45.0000      FF  10  0.10995676E 08  0.12622979E 03  0.27248000E 01
 3  1 16 56 45.0000      FF  50 -0.77269460E 07  0.53589809E 07 -0.22075123E 08
 3  1 16 57  0.          FF  10  0.10925236E 08  0.12440733E 03  0.27818999E 01
 3  1 16 57  0.          FF  50 -0.77905240E 07  0.57071440E 07 -0.21966015E 08
 3  1 16 57 15.0000      FF  10  0.10863597E 08  0.12237718E 03  0.30765899E 01
 3  1 16 57 15.0000      FF  50 -0.78503530E 07  0.60534990E 07 -0.21853118E 08
 3  1 16 57 30.0000      FF  10  0.10810743E 08  0.12041485E 03  0.30450200E 01
```

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

```
3  1 16 57 30.0000      FF  50 -0.79075440E 07  0.64001179E 07 -0.21733818E 08
3  1 16 57 45.0000      FF  10  0.10766657E 08  0.11877514E 03  0.34582300E 01
3  1 16 57 45.0000      FF  50 -0.79607320E 07  0.67413569E 07 -0.21608109E 08
3  1 16 58  0.          FF  10  0.10731950E 08  0.11664797E 03  0.33559299E 01
3  1 16 58  0.          FF  50 -0.80142840E 07  0.70847320E 07 -0.21480990E 08
3  1 16 58 15.0000      FF  10  0.10706554E 08  0.11488955E 03  0.35006300E 01
3  1 16 58 15.0000      FF  50 -0.80638760E 07  0.74269200E 07 -0.21347561E 08
3  1 16 58 30.0000      FF  10  0.10690071E 08  0.11273795E 03  0.34078197E 01
3  1 16 58 30.0000      FF  50 -0.81096960E 07  0.77659520E 07 -0.21207023E 08
3  1 16 58 45.0000      FF  10  0.10683165E 08  0.11079045E 03  0.35931700E 01
3  1 16 58 45.0000      FF  50 -0.81533270E 07  0.81045660E 07 -0.21062546E 08
3  1 16 59  0.          FF  10  0.10685775E 08  0.10869715E 03  0.37136000E 01
3  1 16 59  0.          FF  50 -0.81947699E 07  0.84425458E 07 -0.20915537E 08
3  1 16 59 15.0000      FF  10  0.10697578E 08  0.10665406E 03  0.35945699E 01
3  1 16 59 15.0000      FF  50 -0.82338380E 07  0.87750370E 07 -0.20761803E 08
3  1 16 59 30.0000      FF  10  0.10718857E 08  0.10488074E 03  0.34858400E 01
3  1 16 59 30.0000      FF  50 -0.82714019E 07  0.91079228E 07 -0.20603471E 08
3  1 16 59 45.0000      FF  10  0.10749312E 08  0.10259884E 03  0.34116999E 01
3  1 16 59 45.0000      FF  50 -0.83069420E 07  0.94384490E 07 -0.20441667E 08
3  1 17 -0  0.          FF  10  0.10788938E 08  0.10086722E 03  0.33831099E 01
3  1 17 -0  0.          FF  50 -0.83383200E 07  0.97675069E 07 -0.20274554E 08
3  1 17 -0 15.0000      FF  10  0.10837789E 08  0.98768629E 02  0.30154700E 01
3  1 17 -0 15.0000      FF  50 -0.83661150E 07  0.10092148E 08 -0.20102349E 08
3  1 17 -0 30.0000      FF  10  0.10895590E 08  0.96924419E 02  0.29319900E 01
3  1 17 -0 30.0000      FF  50 -0.83907408E 07  0.10417742E 08 -0.19924382E 08
3  1 17 -0 45.0000      FF  10  0.10962172E 08  0.95218398E 02  0.23938700E 01
3  1 17 -0 45.0000      FF  50 -0.84148200E 07  0.10739630E 08 -0.19744237E 08
3  1 17  1  0.          FF  10  0.11037286E 08  0.93312789E 02  0.25593200E 01
3  1 17  1  0.          FF  50 -0.84380490E 07  0.11058372E 08 -0.19558313E 08
3  1 17  1 15.0000      FF  10  0.11121137E 08  0.91345809E 02  0.22064000E 01
3  1 17  1 15.0000      FF  50 -0.84523909E 07  0.11375476E 08 -0.19368389E 08
3  1 17  1 30.0000      FF  10  0.11213119E 08  0.89411869E 02  0.19716500E 01
3  1 17  1 30.0000      FF  50 -0.84705909E 07  0.11689585E 08 -0.19173736E 08
3  1 17  1 45.0000      FF  10  0.11313207E 08  0.87598749E 02  0.19262099E 01
3  1 17  1 45.0000      FF  50 -0.84841488E 07  0.12001903E 08 -0.18972143E 08
3  1 17  2  0.          FF  10  0.11421078E 08  0.85907938E 02  0.14549900E 01
3  1 17  2  0.          FF  50 -0.84936010E 07  0.12310637E 08 -0.18769769E 08
3  1 17  2 15.0000      FF  10  0.11536587E 08  0.84279999E 02  0.11737499E 01
3  1 17  2 15.0000      FF  50 -0.85015120E 07  0.12615631E 08 -0.18563436E 08
3  1 17  2 30.0000      FF  10  0.11659139E 08  0.82664429E 02  0.12119599E 01
3  1 17  2 30.0000      FF  50 -0.85079630E 07  0.12922007E 08 -0.18350556E 08
3  1 17  2 45.0000      FF  10  0.11788971E 08  0.81099600E 02  0.89399999E 00
```

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

```
  3  1 17  2 45.0000         EE  50 -0.85079939E 07  0.13220190E 08 -0.18133456E 08
  3  1 17  5 45.0000         EE  10  0.11733196E 08  0.20957030E 03  0.10064200E 01
  3  1 17  5 45.0000         EE  50 -0.83428040E 07  0.16571406E 08 -0.15229718E 08

OBSERVATIONS
 NO DY HR MN  SEC       STA  TYP
  3  1 17  6  0.             EE  10  0.11579624E 08  0.21128590E 03  0.12257800E 01
  3  1 17  6  0.             EE  50 -0.83104970E 07  0.16825931E 08 -0.14965133E 08
  3  1 17  6 15.0000         EE  10  0.11433715E 08  0.21281937E 03  0.14753900E 01
  3  1 17  6 15.0000         EE  50 -0.82791770E 07  0.17081486E 08 -0.14695815E 08
  3  1 17  6 30.0000         EE  10  0.11295734E 08  0.21463316E 03  0.15800200E 01
  3  1 17  6 30.0000         EE  50 -0.82439590E 07  0.17327311E 08 -0.14424123E 08
  3  1 17  6 45.0000         EE  10  0.11165529E 08  0.21632013E 03  0.21611100E 01
  3  1 17  6 45.0000         EE  50 -0.82043299E 07  0.17573050E 08 -0.14147092E 08
  3  1 17  7  0.             EE  10  0.11044125E 08  0.21802647E 03  0.23245800E 01
  3  1 17  7  0.             EE  50 -0.81655880E 07  0.17812911E 08 -0.13867300E 08
  3  1 17  7 15.0000         EE  10  0.10931194E 08  0.21978930E 03  0.25599649E 01
  3  1 17  7 15.0000         EE  50 -0.81232289E 07  0.18048301E 08 -0.13585152E 08
  3  1 17  7 30.0000         EE  10  0.10827442E 08  0.22177371E 03  0.29248200E 01
  3  1 17  7 30.0000         EE  50 -0.80780240E 07  0.18279584E 08 -0.13300446E 08
  3  1 17  7 45.0000         EE  10  0.10732955E 08  0.22376884E 03  0.31013700E 01
  3  1 17  7 45.0000         EE  50 -0.80317290E 07  0.18507739E 08 -0.13010798E 08
  3  1 17  8  0.             EE  10  0.10648017E 08  0.22551045E 03  0.34753200E 01
  3  1 17  8  0.             EE  50 -0.79803760E 07  0.18730204E 08 -0.12719373E 08
  3  1 17  8 15.0000         EE  10  0.10572820E 08  0.22764165E 03  0.37946900E 01
  3  1 17  8 15.0000         EE  50 -0.79294199E 07  0.18948969E 08 -0.12423497E 08
  3  1 17  8 30.0000         EE  10  0.10507835E 08  0.22935767E 03  0.39275300E 01
  3  1 17  8 30.0000         EE  50 -0.78769980E 07  0.19163440E 08 -0.12126642E 08
  3  1 17  8 45.0000         EE  10  0.10452825E 08  0.23160023E 03  0.40430699E 01
  3  1 17  8 45.0000         EE  50 -0.78209520E 07  0.19374534E 08 -0.11825907E 08
  3  1 17  9  0.             EE  10  0.10408758E 08  0.23346875E 03  0.42264199E 01
  3  1 17  9  0.             EE  50 -0.77600159E 07  0.19581135E 08 -0.11525248E 08
  3  1 17  9 15.0000         EE  10  0.10374714E 08  0.23569038E 03  0.42372099E 01
  3  1 17  9 15.0000         EE  50 -0.77023699E 07  0.19780812E 08 -0.11218929E 08
  3  1 17  9 30.0000         EE  10  0.10351495E 08  0.23757470E 03  0.41818599E 01
  3  1 17  9 30.0000         EE  50 -0.76377060E 07  0.19975794E 08 -0.10911227E 08
  3  1 17  9 45.0000         EE  10  0.10338978E 08  0.23965615E 03  0.45533799E 01
  3  1 17  9 45.0000         EE  50 -0.75728449E 07  0.20166916E 08 -0.10597820E 08
  3  1 17 10  0.             EE  10  0.10336944E 08  0.24178915E 03  0.45184899E 01
  3  1 17 10  0.             EE  50 -0.75055480E 07  0.20352647E 08 -0.10286964E 08
  3  1 17 10 15.0000         EE  10  0.10346130E 08  0.24399946E 03  0.44196099E 01
  3  1 17 10 15.0000         EE  50 -0.74369539E 07  0.20530982E 08 -0.99715862E 07
```

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

```
3  1 17 10 30.0000      EE  10  0.10365885E 08  0.24585570E 03  0.44[illegible]75199E 01
3  1 17 10 30.0000      EE  50 -0.73651569E 07  0.20712515E 08 -0.[illegible]25369E 07
3  1 17 10 45.0000      EE  10  0.10396121E 08  0.24805012E 03  0.42028200E 01
3  1 17 10 45.0000      EE  50 -0.72929390E 07  0.20883453E 08 -0.93314240E 07
3  1 17 11  0.          EE  10  0.10437164E 08  0.24996119E 03  0.40536299E 01
3  1 17 11  0.          EE  50 -0.72165290E 07  0.21051491E 08 -0.90104569E 07
3  1 17 11 15.0000      EE  10  0.10488293E 08  0.25189796E 03  0.38566599E 01
3  1 17 11 15.0000      EE  50 -0.71399729E 07  0.21214408E 08 -0.86866589E 07

PARAMETERS TO BE CORRECTED

000000

   0 ITERATIONS (MAXIMUM),     0 PARAMETERS

184 OBSERVATION TIMES, 1101 OBSERVATIONS,    6 STATIONS, 1653 CELLS IN COMPACTED DATA LIST
   8 BLOCKS (IF BCD INPUT)

ENTERING LINK NO.  3 INITA
```

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

```
END                                                                          FINP INPUT CARD   1

RESIDUALS ANALYSIS EXAMPLE.

BOUNDS
0.

                                 SIGMA TABLE
  0.

                                        INITIAL CONDITIONS

X    =      -0.22390211E [illegible]     ALPHA=    0.15882923E 03        A=     0.24010736E 08
Y    =       0.86714486E 07              DELTA=    0.                    E=     0.14526680E-06
Z    =       0.                          BETA =    0.89999999E 02        I=     0.20500000E 03
XDOT =       0.22632358E 04              AZ   =    0.34499999E 03        O=     0.15882923E 03
YDOT =       0.58438147E 04              R    =    0.24010736E 08        U=     0.18615689E 03
ZDOT =       0.23387897E 05              V    =    0.24212933E 05        T=    -0.50146602E 02

  ATMOSPHERE - LOCKHEED         CDA/W = 0.20000000E-01      W/CDA = 0.49999999E 02
                                D1    = 0.68299999E 01      D2    =-0.15684000E 02

                                            EARTH MODEL
GM=  0.55304505E-02  J2=  0.10827599E-02  J3= -0.26930000E-05  J4= -0.15600000E-05  J5= -0.59999999E-08
J6=  0.38999999E-06  J7= -0.63299999E-06  J8=  0.                 J9=  0.20999999E-06 J10=  0.
   J21=  0.                  L21=  0.                  J52=  0.53290000E-07    L52= -0.26000000E 01
   J22=  0.17230000E-05      L22= -0.13180000E 02      J53=  0.58419999E-08    L53= -0.34499999E 01
   J31=  0.18500000E-05      L31=  0.53100000E 01      J54=  0.40929999E-08    L54=  0.58250000E 02
   J32=  0.44819999E-06      L32= -0.13330000E 02      J55=  0.84620000E-09    L55= -0.15320000E 02
   J33=  0.16429999E-06      L33=  0.16000000E 02      J61=  0.66110000E-07    L61=  0.15720999E 03
   J41=  0.72780000E-06      L41= -0.13668000E 03      J62=  0.37459999E-07    L62=  0.11314999E 03
   J42=  0.15350000E-06      L42=  0.27469999E 02      J63=  0.11660000E-07    L63= -0.17500000E 01
   J43=  0.57190000E-07      L43= -0.10999999E 01      J64=  0.22560000E-08    L64=  0.60739999E 02
   J44=  0.47809999E-09      L44=  0.36010000E 02      J65=  0.33279999E-09    L65= -0.17749999E 02
   J51=  0.15090000E-06      L51= -0.81859999E 02      J66=  0.51530000E-10    L66= -0.14340000E 02

THE FOLLOWING BODIES ARE USED FOR PLANETARY PERTURBATIONS

NONE
```

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

```
FORMULATION
          COWELL (EQS.OF MOTION)

DIFFERENTIAL EQUATION SUBROUTINE
          GAUSS-JACKSON          E BAR=  C.1000E-C9      A= 1.000      RATIO OF COWELL STEP TO RUNGE KUTTA  4
STEP SIZE -    INITIAL=    0.1000E 01    MINIMUM=     0.1563E-01      MAXIMUM=    0.6400E 02
 DO NOT RECOMPUTE PERTURBATIONS FOR CORRECTOR

EQUATIONS OF MOTION    USE N1 = 9 N2= 6
```

CONSTANTS

	DECIMAL	OCTAL		DECIMAL	OCTAL
OMEGA E	C.437526695E-02	171436571536	GM	0.55304505E-02	171552343100
ALPHA G	0.27720932E 01	202542647715			
FT/KM	0.32808398E 04	214632065602	EARTH RADIUS - FT	0.20925741E 08	231477232264
E.R./A.U.	0.23454864E 05	217556366727	N.M./E.R.	0.34439336E 04	214656373600
I-O, DISTANCE	0.20925741E 08	231477232264	I-O, VELOCITY	0.34876230E 06	223524455115
G	0.32173999E 02	206401310550	FT/SEC//E.R./N.M.	0.34876230E 06	223524455115
DEG/SEC//RAD/MIN	0.10471976E 01	201414052221	1/EPS.	0.29830000E 03	211452231463
RELATIVE MASS(SUN)	0.33295130E 06	223505113515	(MOON)	0.12299900E-01	172623026050
(VENUS)	0.81497900E 00	200641211666	(MARS)	0.10782100E-00	175671505015
(JUPITER)	0.31788700E 03	211475706112	(SATURN)	0.95129000E 02	207574410142

```
ENTERING LINK NO.  4 INITB
```

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

```
COMPUTED VALUES FOR ITERATION   1    3/ 1/64      0.      MIN.

 A,E,I,O,U,T          MEAN ANM= 173.84310         APOGEE= 3951.65891
 0.24010736E 03       TRUE ANM= 173.84310           HT  =  507.85029
 0.14526680E-06           ODOT=   1.59386        PERIGEE= 3951.65778
 0.10500000E 03           UDOT=  -2.04779           HT  =  507.34916
 0.15882923E 03                                PERIOD(K)=  103.84523
 0.18615689E 03                                PERIOD(A)=  103.71713
-0.50146602E 02                                PERIOD(N)=  103.75690

ENTERING LINK NO.  5 FIT A
 3/ 1/64     0.       MM  601421724760  177650254365  000000000000  171651221774  173422416315  175422532313
  ***THRUST START AT       10.0 SECONDS FROM MIDNIGHT OF EPOCH***
 3/ 1/64   51.87918 PM  201421511154  577646667363  151774300000  571650366020  573422716353  575422736454
THRUST RESULTANT =      0.94960182E-04  FT/SEC*SEC
 3/ 1/64  103.76264 MM  601422111433  177646122413  552737400000  171647064332  173422563440  175422525655
THRUST RESULTANT =      0.90158949E-04  FT/SEC*SEC
 3/ 1/64  155.66578 MM  201421773630  577644540745  152751600000  571646211710  573423055244  575422734010
THRUST RESULTANT =      0.85600495E-04  FT/SEC*SEC
  ***THRUST END AT   10000.0 SECONDS FROM MIDNIGHT OF EPOCH***
 3/ 1/64  207.53263 MM  601422272020  177643770103  553755454000  171644726523  173422723003  175422523224
 3/ 1/64  259.41747 MM  201422150267  577642365674  154536310000  571644047473  573423216157  575422734246
 3/ 1/64  311.30351 MM  601422445672  177641611167  554515100000  171642502765  173423067507  175422524511
 3/ 1/64  363.18882 MM  201422325656  577640204152  154637260000  571641621156  573423364043  575422732632
 3/ 1/64  415.07563 MM  601422621146  177637423163  554447540000  171640316430  173423233045  175422525757
 3/ 1/64  466.96070 MM  201422502342  577636031463  155457100000  571637412175  573423540032  575422731053
 3/ 1/64  518.84810 MM  601422772364  177635244217  555634720000  171636171600  173423412732  175422527026
 3/ 1/64  570.73259 MM  201422655213  577633663144  155660314300  571635251701  573423722110  575422730030
 3/ 1/64  622.61977 MM  601423143133  177633073161  555571234300  171634031322  173423576154  175422527513
 3/ 1/64  674.50414 MM  201423026515  577631513562  154434424000  571633127720  573424104515  575422727424
 3/ 1/64  726.39095 MM  601423313701  177630723356  555631434000  171631665327  173423761420  175422527343
 3/ 1/64  778.27482 MM  201423175471  577627337737  155757300000  571631005362  573424265753  575422730620
 3/ 1/64  830.16116 MM  601423465601  177626553451  555535660000  171627505456  173424136375  175422525727
 3/ 1/64  882.04481 MM  201423344157  577625157746  156523714000  571626600054  573424431146  575422733050
 3/ 1/64  933.93166 MM  601423640727  177624400504  556610614000  171625301653  173424271447  175422523710
 3/ 1/64  985.81609 MM  201423514335  577623000610  156555322400  571624342556  573424562105  575422734355

ENTERING LINK NO.  3 INITA

ENTERING LINK NO.  4 INITB

ENTERING LINK NO. 11  RESIDUAL ANALYSIS
```

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

```
                             CALCULATED                             TIME          RESIDUAL VECTOR       DIFFERENCES
ST      TIME      TYPE      OBSERVATION      RESIDUAL             RESIDUAL       RESOLVED INTO              FROM
                                                                                  ORBIT PLANE              THE MEAN

3/ 1/64    HR= 1    MIN=40    SEC=59.9999    SYSTM=  0.60600000E 04
CC     101.00000   R      9.835873E 06   -3.740401E 03   -1.264157E-01   RADIAL      = -4.576779E 02
       101.00000   A      1.132256E 06   -7.767889E-04   -7.053574E-01   IN TRACK    = -1.325666E 03
       101.00000   E      5.844359E 00   -4.495118E-04   -9.644994E-03   CROSS TRACK= -1.365414E 04
                                                                          RSUBT       =  1.378326E 04

3/ 1/64    HR= 1    MIN=40    SEC=59.9999    SYSTM=  0.60600000E 04
CC     101.00000   X      2.186323E 07   -6.959774E 02   -1.986769E 00   RADIAL      = -1.366209E 03
       101.00000   Y     -9.147315E 06   -2.165887E 03    2.432930E-01   IN TRACK    =  4.486258E 03
       101.00000   Z     -3.858522E 06    4.227158E 03    1.833111E-01   CROSS TRACK=  1.025319E 03
                                                                          RSUBT       =  4.800450E 03

3/ 1/64    HR= 1    MIN=41    SEC=15.0000    SYSTM=  0.60749999E 04
CC     101.25000   R      9.630629E 06   -3.463439E 03   -1.184926E-01   RADIAL      =  3.948287E 04
       101.25000   A      1.115363E 02   -9.727299E-02   -8.447873E-01   IN TRACK    = -6.161978E 03
       101.25000   E      6.541174E 00    2.387984E-01    5.160069E 00   CROSS TRACK= -1.703249E 04
                                                                          RSUBT       =  4.343929E 04

3/ 1/64    HR= 1    MIN=41    SEC=15.0000    SYSTM=  0.60749999E 04
CC     101.25000   X      2.186584E 07   -9.117553E 02    8.178835E 02   RADIAL      = -1.612260E 03
       101.25000   Y     -9.279807E 06   -1.048955E 03    1.197053E-01   IN TRACK    =  4.782333E 03
       101.25000   Z     -3.512192E 06    4.856173E 03    2.100745E-01   CROSS TRACK= -2.095889E 02
                                                                          RSUBT       =  5.051141E 03

3/ 1/64    HR= 1    MIN=41    SEC=30.0000    SYSTM=  0.60900000E 04
CC     101.50000   R      9.432490E 06   -3.351730E 03   -1.165610E-01   RADIAL      =  2.382763E 04
       101.50000   A      1.097703E 02   -8.837411E-03   -7.342668E-02   IN TRACK    = -6.264651E 03
       101.50000   E      7.231896E 00    1.484803E-01    3.242608E 00   CROSS TRACK= -1.955923E 03
                                                                          RSUBT       =  2.471492E 04
```

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

************** ACCUMULATED RESULTS FOR RESIDUAL ANALYSIS **************

ST	TYPE (RESOLVED)		SOS	N	RMS	MEAN
AA	R	RADIAL	9.309044E 09	42	1.488771E 04	4.035640E 03
	A	IN TRACK	5.754371E 09	42	1.170508E 04	-6.076568E 03
	E	CROSS TRACK	4.093779E 09	42	9.872737E 03	-4.651292E 03
		(RSUBT)	1.915719E 10		2.135705E 04	1.786597E 04
	X	RADIAL	1.283739E 08	42	1.748291E 03	-1.368309E 03
	Y	IN TRACK	6.038412E 08	42	3.791724E 03	3.178267E 03
	Z	CROSS TRACK	1.852422E 09	42	6.641180E 03	-6.459186E 03
		(RSUBT)	2.584637E 09		7.844678E 03	7.787115E 03
CC	R	RADIAL	4.262205E 10	85	2.239276E 04	-1.842275E 03
	A	IN TRACK	2.306513E 11	85	5.209169E 04	3.243828E 04
	E	CROSS TRACK	6.295146E 11	85	8.605843E 04	-6.202710E 04
		(RSUBT)	9.027879E 11		1.030584E 05	8.126018E 04
	X	RADIAL	6.881529E 10	86	2.828741E 04	-1.953763E 04
	Y	IN TRACK	7.851672E 11	86	9.561108E 04	6.889135E 04
	Z	CROSS TRACK	3.439786E 10	86	1.999938E 04	6.345263E 03
		(RSUBT)	8.893803E 11		1.016938E 05	7.339431E 04
CE	R	RADIAL	3.640534E 10	23	3.978493E 04	2.031317E 04
	A	IN TRACK	2.357219E 11	23	1.012363E 05	-1.004414E 05
	E	CROSS TRACK	2.026510E 11	23	1.048025E 05	-1.040601E 05
		(RSUBT)	5.247782E 11		1.510511E 05	1.500179E 05
	X	RADIAL	6.321196E 10	23	5.242467E 04	-5.102299E 04
	Y	IN TRACK	1.778465E 11	23	8.793439E 04	8.567109E 04
	Z	CROSS TRACK	3.153937E 11	23	1.171015E 05	-1.167869E 05
		(RSUBT)	5.564522E 11		1.555428E 05	1.555396E 05
FF	R	RADIAL	4.178481E 10	33	3.558379E 04	9.993897E 02
	A	IN TRACK	4.870995E 10	33	3.841951E 04	-3.548063E 04
	E	CROSS TRACK	7.339624E 11	33	1.491351E 05	-1.476168E 05

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Continued)

```
          (RSUBT)        8.244571E 11        1.580619E 05  1.567814E 05
   X      RADIAL         1.328875E 11    33  6.345782E 04 -6.246407E 04
   Y      IN TRACK       2.019611E 11    33  7.823064E 04 -6.609761E 04
   Z      CROSS TRACK    4.457719E 11    33  1.162250E 05 -1.141853E 05
          (RSUBT)        7.806205E 11        1.538024E 05  1.538002E 05

CHAIN.    LINK NO.  1
```

Figure 5-33. Sample TRACE-D Program Residuals-Analysis Listing (Concluded)

Table 5-16. Residuals Analysis Output Listing Description

Item	Description	Page Reference
1	Observation time. This is the time point associated with the information which follows on the next four lines of printed output. The month, day, and year are shown at the left side of the page on the line indicated, followed by the time of day in hours, minutes, and seconds. This in turn is followed by system time (SYSTM), which is the time of day given in terms of seconds from midnight.	
2	Station identification.	5-32/5-34
3	Observation time in minutes from midnight of epoch.	5-35
4	Observation type. Each of these letters indicates the observation type which produced the three quantities shown on the same line of output immediately to the right of that letter (see Items 5, 6, and 7). In this case the types are range, azimuth, and elevation.	5-37
5	Calculated observation values. The indicated numbers are values of range, azimuth, and elevation which have been computed from the integrated position of the satellite at the observation time and the location of the station associated with the observation. Units are feet and degrees.	3-13/3-19
6	Residuals. The quantities indicated represent the difference between the input observation values and the calculated values (see Item 5) for range, azimuth, and elevation measurements reported by Station CC at the time noted (see Items 1 and 3). Units are identical to observation input units for all residuals.	
7	Time residuals (i.e., the observation residual divided by the calculated time rate of change of the observation). Units are seconds.	3-86

Table 5-15. Residuals Analysis Output Listing Description (Continued)

Item	Description	Page Reference
8	Orbit-plane residuals. These are the components of the residual vector, as determined by the R, A, E residuals, in the orbit-plane coordinate system. Units are feet.	3-84/3-86 5-58
9	R_T, or the square root of the sum of the squares of the radial, in-track, and cross-track components of the residual vector.	3-87
10	Accumulated SOS. The three quantities indicated are the square root of the sum of the squares for the radial, in-track, and cross-track components, respectively, for all R, A, E residuals sets from Station AA.	
11	R_T of accumulated SOS components. The square root of the sum of squares of the radial, in-track, and cross-track SOS components (Item-10 quantities).	3-87
12	Number of points. This indicates the number of R, A, E residual vectors, resolved into orbit-plane components, that are represented in the accumulated results. This number is printed three times only for programming convenience.	3-84/3-86
13	RMS of components. The root-mean-square of the radial, in-track, and cross-track components, respectively, for all R, A, E residual vectors for Station AA. Units are feet.	
14	RMS vector magnitude. The square root of the sum of squares of the components specified in Item 13. Units are feet.	
15	Component mean values. The arithmetic average of all radial, in-track, and cross-track components, respectively, for residual vectors computed from R, A, E sets from Station AA. Units are feet.	

Table 5-15. Residuals Analysis Output Listing Description (Concluded)

Item	Description	Page Reference
16	Mean-vector magnitude. Square root of the sum of the squares of the quantities specified in Item 15. Units are feet.	
17	Accumulated results for $\hat{x}$, $\hat{y}$, $\hat{z}$ data from Station AA. The quantities in this block correspond directly to those in similar positions in the R, A, E block described in Items 1 through 16.	
18	Accumulated results for Station CC.	

APPENDIX A

STANDARD VALUES AND STORAGE LOCATION OF CONSTANTS

APPENDIX A

STANDARD VALUES AND STORAGE LOCATIONS OF CONSTANTS

Figures A-1 through A-3 illustrate tab listings of standard values and of storage locations of constants applicable to TRACE-D program runs. The listings given in Figures A-1 and A-2 tabulate numerical values and descriptions for the standard entries to the INTEG (integration constants) and C (physical constants) regions. Corresponding entry locations within each of these regions are indicated by the numbers in the left-hand columns of these tabulations. The listing presented in Figure A-3, delineating the "basic running deck" (except for the REIN binary cards), itemizes the standard values assigned to those FINP entries which are input on all TRACE-D runs unless special instructions to the contrary are given.

One particular TRACE-D characteristic associated with input of physical constants which should be noted is that all physical constants are input on each run as part of the FINP data and that none are built into the program. Care must therefore be exercised by the user whenever any of the standard values are altered due to the fact that they are interrelated in some cases. For example, if the number of feet per earth radii were to be changed, it might be necessary to change the entries at C(2), C(3), C(15), C(23), C(24), C(25), and C(30).

Description of entries to locations other than the INTEG or C regions has been given in the basic TRACE-D Program report (see Section 4.3).

INTEG		PARAMETERS OF THE TRAJECTORY INTEGRATION	
1	1	FORMULATION. 1 - COWELL (EQS. OF MOTION), 2 - ENCKE-T (NOTE. THE ENCKE-T FORMULATION IS PRESENTLY NOT AVAILABLE.)	
2	2	DIFFERENTIAL EQUATION SUBROUTINE. 1 - AMRK, 2 - COW	
3		ANGLE IN DEGREES FOR CHANGING MEAN TO TRUE EQUINOX	
4	1	SUN	(4-9) ARE SELECTORS FOR OTHER-BODY PERTURBATIONS
5	1	MOON	IF TAPE UNIT NUMBER AT NUMB(18) IS NOT ZERO, THEN
6	0	VENUS	PERTURBATIONS ARE INCLUDED OR OMITTED ACCORDING
7	0	MARS	AS THE SELECTOR IS NON-ZERO OR ZERO.
8	0	JUPITER	
9	0	SATURN	
10	C(22)	SUN	(10-15) ARE INTERPOLATION SCALE FACTORS FOR USE
11	1.00002516	MOON	
12	C(22)	VENUS	USING CONTENTS OF C(22), THE NUMBER OF EARTH-
13	C(22)	MARS	RADII IN AN ASTRONOMICAL UNIT.
14	C(22)	JUPITER	
15	C(22)	SATURN	
16 THRU 21	0	SIMILAR VELOCITY SCALE FACTORS. (NOT NOW USED, AND ALL ZERO).	
22		(USED INTERNALLY - ENCKE)	
23	1E-10	E BAR, COW SUBROUTINE TRUNCATION ERROR CONTROL PARAMETER.	
24		(USED INTERNALLY - ENCKE)	
25		(USED INTERNALLY - ENCKE)	
26	1.	A, SEE COW WRITEUP.	
27-29		(NOT USED)	
30	1.	INITIAL TIME STEP SIZE.	
31	.015625	MINIMUM TIME STEP SIZE (1/64)	
32	64.	MAXIMUM TIME STEP SIZE	
33	1E-7	KEPLER EQUATION CONVERGENCE CRITERION.	
34	4	RATIO OF COWELL STEP SIZE TO RUNGE-KUTTA STEP SIZE	
35	1	IF 1, DO NOT RECOMPUTE PERTURBATIONS FOR CORRECTOR. IF 2, DO.	
36		FLAG FOR PERTURBATION COMPUTATION (USED IF INTEG(35)=1).	
37	.001	LEAST SQUARES CONVERGENCE CRITERION (RELATIVE)	
38	0	LEAST SQUARES CONVERGENCE CRITERION (ABSOLUTE)	
39		(NOT USED)	
40	2820.1763	SPEED OF LIGHT (EARTH-RADII/MINUTE)	
41	1	IF NON-ZERO, ROUND POSITION AND VELOCITY VECTORS FOR ACCELERATION COMPUTATION. IF ZERO, TRUNCATE.	
42		RESERVED FOR LATER USE	

Figure A-1. Standard Entries to Integration Constants (INTEG) Region

C CONSTANTS.

			INPUT NAME
1	.0043752691	EARTH ROTATION RATE (RAD/MIN)	
2	.0055303935	GM, EARTH GRAVITATION CONSTANT (ER**3/MIN**2)	
3		OPTION FOR SPEED OF LIGHT CORRECTION	SLT
4		S=1-E, RELATIVE SEMI-MINOR AXIS OF ELLIPSOID (COMPUTED IN CSET)	
5		S**2/A**2 = (1-E)**2 (COMPUTED IN CSET)	
6	.5	FACTOR FOR DECREASING BOUNDS IN L.S. SOLUTION	
7		2*E-E**2 (COMPUTED IN CSET)	
8	.0043752691	ATMOSPHERE ROTATION RATE (RAD/MIN)	
9	1.	EARTH RADIUS	
10		INPUT N FOR N-SIGMA RESIDUAL EDITOR	CEDIT
11		INPUT SCALE FACTOR FOR N-SIGMA RESIDUAL EDITOR	
12		IF INPUT NON-ZERO, GO THRU 1ST ITERATION TWICE FOR SUMS	
13	3280.8399	FEET/KILOMETER	
14	57.2957795	ANGLE CONVERSION FACTOR	
15	20925738.	A, EARTH RADIUS IN FEET	
16	332951.3	RELATIVE MASS OF SUN	
17	.0122995	MOON	
18	.814979	VENUS	
19	.107821	MARS	
20	317.887	JUPITER	
21	95.129	SATURN	
22	23454.665	EARTH-RADII/ASTRONOMICAL UNIT	
23	3443.9336	NAUTICAL MILES (6076.1155 FT)/EARTH RADIUS	
24	20925738.	I/O DISTANCE CONVERSION FACTOR	
25	348762.3	I/O VELOCITY CONVERSION FACTOR	
26	32.174	GO (USED FOR CDA/W AND THRUST/W)	
27		INPUT PARAMETER DIFFERENCE FOR TRAJECTORY DIFFERENCING	PDIFF
28		INPUT THRESHOLD FOR PERCENT DIFF FOR TRAJ DIFFERENCING	
29		CONSTANT FOR DOPPLER RATE	
30	348762.3	FT/SEC PER ER/MIN	
31	1.5	FACTOR FOR INCREASING BOUNDS IN L.S.SOLUTION	
32	1.0471976	RAD/MIN PER DEG/SEC	
33	3.14159265	PI	
34	298.3	RECIPROCAL OF E = ELLIPTICITY	
35	300000.	CRITICAL ALTITUDE (FT)	
36	82505.922	DEG/DAY PER RAD/MIN	
37-39		DIRECTION COSINES OF BODY AXIS FOR LOOK ANGLE	

Figure A-2. Standard Entries to Physical Constants (C) Region

40		APPROX TIME STEP FOR RISE-SET PREDICT (=2,IF NOT INPUT)		
41		NO. OF REVS PREDICTED FOR (=1,IF NOT INPUT)		
42		NORMAL TIME INCREMENT AND FLAG FOR 2ND DIFFERENCE EDITOR IF INPUT = 0, NO 2ND DIFFERENCE EDITING DONE		
43		CLIM, INPUT MULTIPLICATIVE CONSTANT FOR 2ND DIFF EDITOR		
44-47		EDITING LIMITS FOR R,A,E,R DOT-USED WITH 2ND DIFF EDITOR		
48	4	N1 FOR EQUATIONS OF MOTION	(EARTH MODEL)	IOB
49	0	N2 FOR EQUATIONS OF MOTION	(EARTH MODEL)	(2)
50	4	N1 FOR V-MATRIX	(EARTH MODEL)	(3)
51	0	N2 FOR V-MATRIX	(EARTH MODEL)	(4)
52-60		J2-J10 ZONALS	(EARTH MODEL)	OBJZ(2-10)
61-80		J21-J66 TESSERALS *	(EARTH MODEL)	OBJT(1-20)
81-100		L21-L66 LONGITUDES *	(EARTH MODEL)	OBLT(1-20)

* ORDERED J21,J22,J31,J32,...J65,J66

Figure A-2. Standard Entries to Physical Contants (C) Region (Concluded)

```
                      TRACE-D BASIC RUNNING DECK

$EXECUTE        FMS
*         XEQ
                                                        (REM. BINARY DECK)
*       DATA
GINTEG      TRACE-D STANDARD INTEGRATION C-NSTANTS
I1      1            I2    2              I3    0                    STDINTEG 01
  4     1             5    1              11    1.00302516           STDINTEG 02
 23     .1         -9 26   1              30    1                    STDINTEG 03
 31     .015625       32   64             33    1          -7        STDINTEG 04
I34     4            I35   1              37    .001                 STDINTEG 05
 38     0.            40   2820.1763     I41    1                    STDINTEG 06
GC          TRACE-D STANDARD PHYSICAL CONSTANTS    10/ 1/65
  1     .43752691 -2 2     .55303935 -2   6     .5                   STDCONST 01
  8     .43752691 -2 9     1.             10    10                   STDCONST 02
 13     3280.8399     14   57.2937795     15    20925733             STDCONST 03
 16     332951.3      17   .0122999       18    .814979              STDCONST 04
 19     .107821       20   317.887        21    95.129               STDCONST 05
 22     23454.865     23   3443.9336      24    20925738             STDCONST 06
 25     348762.3      26   32.174         30    348762.3             STDCONST 07
 31     1.5           32   1.0471976      33    3.14159265           STDCONST 08
 34     298.3         35   300000.        36    82505.922            STDCONST 09
IIOB    4            I2    0              I3    4                    STDCONST 10
I4      0            IOBJZ                2     .10823     -2        STDCONST 11
  3     -.23       -5 4    -.18        -5ITMATX4                     STDCONST 12
 NUMB                 42   3443.9336      DRAG                       STDCONST 13
I2      1             3    6.83           4     -15.684              STDCONST 14
 RREFC1.                                                             STDCONST 15
GC      STANDARD DECK FOR 6,TH ORDER GRAVITY MODEL.
  1     .043752695-1 2     .55304505 -2 15      20925741.            STAND 66 01
 24     20925741.    OBJT                 2     1.723      -6        STAND 66 02
  3     1.856      -6 4    .4482       -6 5     .1643      -6        STAND 66 03
  6     .7278      -6 7    .1536       -6 8     .05719     -6        STAND 66 04
  9     .004781    -6 10   .1509       -6 11    .05329     -6        STAND 66 05
 12     .005842    -6 13   .004093     -6 14    .0008462   -6        STAND 66 06
 15     .06611     -6 16   .03746      -6 17    .01166     -6        STAND 66 07
 18     .002256    -6 19   .0003328    -6 20    .00005153  -6        STAND 66 08
 OBLT                  2   -13.18         3     5.31                 STAND 66 09
```

Figure A-3. Standard FINP Entries

4	-13.33		5	16.00		6	-136.68	STAND 66 10
7	27.47		8	-1.10		9	36.01	STAND 66 11
10	-81.86		11	-2.60		12	-3.45	STAND 66 12
13	58.25		14	-15.32		15	157.21	STAND 66 13
16	113.15		17	-1.75		18	60.74	STAND 66 14
19	-17.75		20	-14.34		OBJZ		STAND 66 15
2	1082.76	-6	3	-2.693	-6	4	-1.56 -6	STAND 66 16
5	-.006	-6	6	.39	-6	7	-.633 -6	STAND 66 17
8	0.		9	.210		-61108	9	STAND 66 18
12	6		13	9		14	6	STAND 66 19

Figure A-3. Standard FINP Entries (Concluded)

APPENDIX B

DERIVATION OF V MATRIX

APPENDIX B

DERIVATION OF V MATRIX

In the variational equations associated with TRACE-D program procedure, the dependence of the gravitational force upon vehicle position is represented by the matrix

$$V = \frac{\partial}{\partial X}\left(-\frac{\mu X}{r^3}\right) + \frac{\partial F_1}{\partial X}$$

which for v-matrix derivation purposes may be written

$$V = \frac{\partial F}{\partial X}$$

where

$X = (x, y, z)^T$ = vector position of vehicle

F = gravitational force vector

The components of F in an equatorial coordinate system with the principal axis in the direction of the Greenwich meridian are

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \begin{pmatrix} \cos\phi\cos\lambda & -\sin\lambda & -\sin\phi\cos\lambda \\ \cos\phi\sin\lambda & \cos\lambda & -\sin\phi\sin\lambda \\ \sin\phi & 0 & \cos\phi \end{pmatrix} \begin{pmatrix} g_U \\ g_E \\ g_N \end{pmatrix}$$

where

ϕ = geocentric latitude of vehicle position

λ = longitude of vehicle position

$g_{U,E,N}$ - force vector components in the local horizontal coordinate system, wherein the coordinate axes are directed Up (along the radius vector), East, and North

The foregoing equation also may be written in vector notation, or

$$F = RG$$

The components in an equatorial inertial coordinate system with the principal axis in the direction of λ (vernal equinox) are obtained by a similar transformation in which α replaces λ.

Using subscripts to denote differentiation, and expanding in spherical coordinates (r, ϕ, λ),

$$F_X = F_r r_X + F_\phi \phi_X + F_\lambda \lambda_X$$

$$= RG_r r_X + R_r G r_X + RG_\phi \phi_X + R_\phi G \phi_X + RG_\lambda \lambda_X + R_\lambda G \lambda_X$$

$$= R(G_R r_X + G_\phi \phi_X + G_\lambda \lambda_X) + R_\phi G \phi_X + R_\lambda G \lambda_X$$

Noting that

$$R_\phi = R \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = RE_1$$

$$R_\lambda = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} R = E_2 R$$

then

$$F_X = R(G_r r_X + (G_\phi + E_1 G)\phi_X + G_\lambda \lambda_X) + E_2 RG\lambda_X$$

$$= R(\frac{1}{r} G_r X^T + (G_\phi + E_1 G)\phi_X + G_\lambda \lambda_X) + E_2 F\lambda_X$$

$$= R(\frac{1}{r} G_r X^T + (\cos\phi\, G_\phi + \cos\phi\, E_1 G)\frac{\phi_X}{\cos\phi} + G_\lambda \lambda_X) + E_2 F\lambda_X$$

The latter expression permits the desired matrix $V = F_X$ to be computed after $\phi_X/\cos\phi$, λ_X and G_r, G_ϕ, G_λ, respectively, have been calculated from geometrical formulas and from applicable equations for G (see Section 3.6.1).

In deriving expressions for G_r, G_ϕ, and G_λ it is convenient to make use of the relations $\sin\phi = z/r$ and $\lambda = \tan^{-1} y/x$, wherefrom

$$\frac{\phi_x}{\cos\phi} = \frac{-xz}{r^3\cos^2\phi} = \frac{-xz}{r(x^2+y^2)}$$

$$\frac{\phi_y}{\cos\phi} = \frac{-yz}{r(x^2+y^2)}$$

$$\frac{\phi_z}{\cos\phi} = \frac{-z^2}{r(x^2+y^2)} + \frac{r}{(x^2+y^2)} = \frac{r^2-z^2}{r(x^2+y^2)}$$

$$\lambda_x = \frac{-y}{x^2+y^2}$$

$$\lambda_y = \frac{x}{x^2+y^2}$$

$$\lambda_z = 0$$

In the foregoing expressions it should be noted that $\alpha_x = \lambda_x$, inasmuch as α and λ differ only by a time-dependent term.

The vector G has previously been given (Section 3.6.1) as

$$g_U = -\frac{\mu}{r^2}\left[1 - \sum_{n=2}^{n_1} (n+1) J_n \left(\frac{a_e}{r}\right)^n P_n(\sin\phi) + \sum_{n=2}^{n_2}\sum_{m=1}^{n} (n+1) J_{nm} \left(\frac{a_e}{r}\right)^n P_n^m(\sin\phi)\cos m(\lambda - \lambda_{nm})\right]$$

$$g_E = -\frac{\mu}{r^2\cos\phi}\sum_{n=2}^{n_2}\sum_{m=1}^{n} m J_{nm}\left(\frac{a_e}{r}\right)^n P_n^m(\sin\phi)\sin m(\lambda - \lambda_{nm})$$

$$g_N = \frac{\mu}{r^2}\left[-\sum_{n=2}^{n_1} J_n\left(\frac{a_e}{r}\right)^n P_n'(\sin\phi)\cos\phi + \sum_{n=2}^{n_2}\sum_{m=1}^{n} J_{nm}\left(\frac{a_e}{r}\right)^n P_n^{m\prime}(\sin\phi)\cos\phi\cos m(\lambda - \lambda_{nm})\right]$$

where

$g_{U,E,N}$ = Up, East, and North components of vector G

μ = product GM of Newtonian gravitational constant and the earth mass

r, ϕ, λ = geocentric distance, geocentric latitude, and east longitude, respectively, of a point

a_e = mean equatorial radius of the earth

J_n, J_{nm} = numerical coefficients

P_n - Legendre polynomial of the first kind of degree n

P_n^m - Associated Legendre function of the first kind

λ_{nm} - longitudes associated with the J_{nm}

Differentiating the foregoing expressions for g_U, g_E, and g_N, then

$$\frac{\partial g_U}{\partial r} = \frac{\mu}{r^3}\left[2 - \sum_{n=2}^{n_1}(n+1)(n+2)J_n\left(\frac{a_e}{r}\right)^n P_n(\sin\phi)\right.$$

$$\left. + \sum_{n=2}^{n_2}\sum_{m=1}^{n}(n+1)(n+2)J_{nm}\left(\frac{a_e}{r}\right)^n P_n^m(\sin\phi)\cos m(\lambda - \lambda_{nm})\right]$$

$$\frac{\partial g_E}{\partial r} = \frac{\mu}{r^3\cos\phi}\left[\sum_{n=2}^{n_2}\sum_{m=1}^{n} m(n+2)J_{nm}\left(\frac{a_e}{r}\right)^n P_n^m(\sin\phi)\sin m(\lambda - \lambda_{nm})\right]$$

$$\frac{\partial g_N}{\partial r} = \frac{\mu}{r^3}\left[\sum_{n=2}^{n_1}(n+2)J_n\left(\frac{a_e}{r}\right)^n P_n'(\sin\phi)\cos\phi\right.$$

$$\left. - \sum_{n=2}^{n_2}\sum_{m=1}^{n}(n+2)J_{nm}\left(\frac{a_e}{r}\right)^n P_n^{m\prime}(\sin\phi)\cos\phi\cos m(\lambda - \lambda_{nm})\right]$$

$$\frac{\partial g_U}{\partial\phi}\cos\phi = \frac{\mu\cos\phi}{r^2}\left[\sum_{n=2}^{n_1}(n+1)J_n\left(\frac{a_e}{r}\right)^n P_n'(\sin\phi)\cos\phi\right.$$

$$\left. - \sum_{n=2}^{n_2}\sum_{m=1}^{n}(n+1)J_{nm}\left(\frac{a_e}{r}\right)^n P_n^{m\prime}(\sin\phi)\cos\phi\cos m(\lambda - \lambda_{nm})\right]$$

$$\frac{\partial g_E}{\partial \phi} \cos \phi = \frac{-\mu \cos \phi}{r^2} \left[\sum_{n=2}^{n2} \sum_{m=1}^{n} mJ_{nm} \left(\frac{a_e}{r}\right)^n P_n^{m'} (\sin \phi) \sin m(\lambda - \lambda_{nm}) \right] + g_E \sin \phi$$

$$\frac{\partial g_N}{\partial \phi} \cos \phi = \frac{-\mu \cos \phi}{r^2} \left[- \sum_{n=2}^{n1} J_n \left(\frac{a_e}{r}\right)^n \{ P_n'' (\sin \phi) \sin \phi \} \right.$$

$$+ \sum_{n=2}^{n2} \sum_{m=1}^{n} J_{nm} \left(\frac{a_e}{r}\right)^n \left\{ P_n^{m''} (\sin \phi) \cos^2 \phi \right.$$

$$\left. \left. - P_n^{m'} (\sin \phi) \sin \phi \right\} \cos m(\lambda - \lambda_{nm}) \right]$$

$$\frac{\partial g_U}{\partial \lambda} = \frac{\mu}{r^2} \left[\sum_{n=2}^{n2} \sum_{m=1}^{n} m(n+1) J_{nm} \left(\frac{a_e}{r}\right)^n P_n^m (\sin \phi) \sin m(\lambda - \lambda_{nm}) \right]$$

$$\frac{\partial g_E}{\partial \lambda} = \frac{-\mu}{r^2 \cos \phi} \left[\sum_{n=2}^{n2} \sum_{m=1}^{n} m^2 J_{nm} \left(\frac{a_e}{r}\right)^n P_n^m (\sin \phi) \cos m(\lambda - \lambda_{nm}) \right]$$

and

$$\frac{\partial g_N}{\partial \lambda} = \frac{-\mu}{r^2} \left[\sum_{n=2}^{n2} \sum_{m=1}^{n} mJ_{nm} \left(\frac{a_e}{r}\right)^n P_n^{m'} (\sin \phi) \cos \phi \sin m(\lambda - \lambda_{nm}) \right]$$

The terms containing second derivatives of Legendre polynomials in the preceding equation for $\partial g_N / \partial \phi \cos \phi$ are calculated using the recursion formulas

$$\cos^2 \phi \, P_n''(\sin \phi) = 2 \sin \phi \, P_n'(\sin \phi) - n(n+1) P_n(\sin \phi)$$

and

$$\cos^2\phi\, P_n^{m''}(\sin\phi) = 2\sin\phi\, P_n^{m'}(\sin\phi) - \left\{n(n+1) - \frac{m^2}{\cos^2\phi}\right\} P_n^m(\sin\phi)$$

which may be rearranged in the forms

$$\cos^3\phi\, P_n''(\sin\phi) - \cos\phi\sin\phi\, P_n'(\sin\phi) = \sin\phi\cos\phi\, P_n'(\sin\phi) - n(n+1)\cos\phi\, P_n(\sin\phi)$$

$$\cos^3\phi\, P_n^{m''}(\sin\phi) - \sin\phi\cos\phi\, P_n^{m'}(\sin\phi) = \sin\phi\cos\phi\, P_n^{m'}(\sin\phi) - \{n(n+1)\cos^2\phi - m^2\}\frac{P_n^m(\sin\phi)}{\cos\phi}$$

The equation containing the second derivatives of the Legendre polynomials consequently becomes

$$\frac{\partial g_N}{\partial\phi}\cos\phi = \frac{\mu}{r^4}\left[-\sum_{n=1}^{n1} J_n\left(\frac{a_e}{r}\right)^n \cos\phi(\sin\phi\, P_n'(\sin\phi) - n(n+1)P_n(\sin\phi)) + \sum_{n=2}^{n2}\sum_{m=1}^{n} J_{nm}\left(\frac{a_e}{r}\right)^n \left(\sin\phi\cos\phi\, P_n^{m'}(\sin\phi) - \{n(n+1)\cos^2\phi - m^2\}\frac{P_n^m(\sin\phi)}{\cos\phi}\right)\cos m(\lambda - \lambda_{nm})\right]$$

Using the GRAV subroutine notation (see Appendix F), wherein

$$A_n = -(n+1)J_n\left(\frac{a_e}{r}\right)^n P_n(\sin\phi)$$

$$B_{nm} = (n+1)J_{nm}\left(\frac{a_e}{r}\right)^n P_n^m(\sin\phi)$$

$$C_{nm} = \frac{m}{\cos\phi} J_{nm}\left(\frac{a_e}{r}\right)^n P_n^m(\sin\phi)$$

$$D_n = J_n\left(\frac{a_e}{r}\right)^n P_n'(\sin\phi)\cos\phi$$

$$E_{nm} = -J_{nm}\left(\frac{a_e}{r}\right)^n P_n^{m'}(\sin\phi)\cos\phi$$

$$F_{nm} = \sin m(\lambda - \lambda_{nm})$$

$$G_{nm} = \cos m(\lambda - \lambda_{nm})$$

the expressions for G_r, G_ϕ, G_λ then become

(for G_r)

$$\frac{\partial g_U}{\partial r} = \frac{\mu}{r^3}\left[2 + \sum_{n=2}^{n1}(n+2)A_n + \sum_{n=2}^{n2}\sum_{m=1}^{n}(n+2)B_{nm}G_{nm}\right]$$

$$\frac{\partial g_E}{\partial r} = \frac{\mu}{r^3}\left[\sum_{n=2}^{n2}\sum_{m=1}^{n}(n+2)C_{nm}F_{nm}\right]$$

$$\frac{\partial g_N}{\partial r} = \frac{\mu}{r^3}\left[\sum_{n=2}^{n1}(n+2)D_n + \sum_{n=2}^{n2}\sum_{m=1}^{n}(n+2)E_{nm}G_{nm}\right]$$

(for G_ϕ)

$$\frac{\partial g_U}{\partial \phi} \cos\phi = \frac{\mu \cos\phi}{r^2} \left[\sum_{n=2}^{n1} (n+1) D_n + \sum_{n=2}^{n2} \sum_{m=1}^{n} (n+1) E_{nm} G_{nm} \right]$$

$$\frac{\partial g_E}{\partial \phi} \cos\phi = \frac{\mu}{r^2} \left[\sum_{n=2}^{n2} \sum_{m=1}^{n} m E_{nm} F_{nm} \right] + g_E \sin\phi$$

$$\frac{\partial g_N}{\partial \phi} \cos\phi = \frac{\mu}{r^2} \left[\sum_{n=2}^{n1} - (\sin\phi\, D_n + n \cos\phi\, A_n) \right.$$

$$\left. - \sum_{n=2}^{n2} \sum_{m=1}^{n} \left(\sin\phi\, E_{nm} + \left| \frac{n(n+1)}{m} \cos^2\phi - m \right| C_{nm}\right) G_{nm} \right]$$

and (for G_λ)

$$\frac{\partial g_U}{\partial \lambda} = \frac{-\mu}{r^2} \left[\sum_{n=2}^{n2} \sum_{m=1}^{n} m B_{nm} G_{nm} \right]$$

$$\frac{\partial g_E}{\partial \lambda} = \frac{-\mu}{r^2} \left[\sum_{n=2}^{n2} \sum_{m=1}^{n} m C_{nm} G_{nm} \right]$$

$$\frac{\partial g_N}{\partial \lambda} = \frac{\mu}{r^2} \left[\sum_{n=2}^{n2} \sum_{m=1}^{n} m E_{nm} F_{nm} \right]$$

The three vectors G_r, $\cos\phi\, G_\phi$, and G_λ thus obtained are then applied in the previously indicated computation for $V = F_X$.

APPENDIX C

DERIVATION OF T MATRIX

APPENDIX C

DERIVATION OF T MATRIX

In the variational equations associated with TRACE-D program procedure, the dependence of the drag force upon vehicle position is represented by the matrix

$$T = \frac{\partial F_3}{\partial X}$$

$$= -\frac{1}{2}\left(\frac{C_D A}{W}\right)\frac{\partial}{\partial X}(\rho V_A \dot{X}_A)$$

$$= -\frac{1}{2}\left(\frac{C_D A}{W}\right)\left(V_A \dot{X}_A \frac{\partial \rho}{\partial X} + \rho \dot{X}_A \frac{\partial V_A}{\partial X} + \rho V_A \frac{\partial \dot{X}_A}{\partial X}\right)$$

The derivatives of the factors ρ, V_A, and $\dot{X}_A$ appearing in the latter expression then are

$$\frac{\partial \rho}{\partial X} = \frac{\partial \rho}{\partial h}\frac{\partial h}{\partial X}$$

$$\frac{\partial V_A}{\partial X} = \frac{\partial}{\partial X}\left[(\dot{x} + \omega_e y)^2 + (\dot{y} - \omega_e x)^2 + \dot{z}^2\right]^{1/2}$$

$$= \frac{\omega_e}{V_A}(-\dot{y}_A, \dot{x}_A, 0)$$

$$\frac{\partial \dot{X}_A}{\partial X} = \begin{bmatrix} 0 & \omega_e & 0 \\ -\omega_e & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

In connection with the first of the above relationships it should be noted that

$$\frac{\partial h}{\partial X} = \left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}, \frac{\partial h}{\partial z}\right)$$

where

$$\frac{\partial h}{\partial x} = \frac{x}{r}\left\{1 - \frac{a_e \epsilon (2 - 3\epsilon + \epsilon^2) z^2}{[r^2 - (2\epsilon - \epsilon^2)(x^2 + y^2)]^{3/2}}\right\}$$

$$\frac{\partial h}{\partial y} = \frac{y}{r}\left\{1 - \frac{a_e \epsilon (2 - 3\epsilon + \epsilon^2) z^2}{[r^2 - (2\epsilon - \epsilon^2)(x^2 + y^2)]^{3/2}}\right\}$$

$$\frac{\partial h}{\partial z} = \frac{z}{r}\left\{1 - \frac{a_e \epsilon (2 - 3\epsilon + \epsilon^2)(x^2 + y^2)}{[r^2 - (2\epsilon - \epsilon^2)(x^2 + y^2)]^{3/2}}\right\}$$

$$h = r - \frac{a_e(1 - \epsilon)}{\left[1 - (2\epsilon - \epsilon^2)\frac{x^2 + y^2}{r^2}\right]^{1/2}}$$

ϵ = ellipticity of reference ellipsoid

and also that $\partial \rho / \partial h$, the rate of change of density with altitude, depends both upon the model atmosphere and its parameters and upon h.

The term $\partial \rho / \partial h$ either may be approximated by using

$$\frac{\partial \rho}{\partial h} = \rho' \frac{\rho}{h}$$

with values of p' specified respectively within the altitude intervals $0 \le h < 108$ and $108 \le h < 378$ nautical miles or may be calculated from expressions for density.

In the latter case, for $76 \leq h < 108$ nautical miles ($\rho' = \rho_1$),

$$\rho_1 = 5.606 \times 10^{-12}\left(\frac{76}{h}\right)^{d_1}\left[\frac{108 - h}{32} + 0.85\left(\frac{h - 76}{32}\right)^{4/3} F_{10.7}\right]\left[1 + \frac{h - 76}{153}\left(\frac{1 + \cos\psi'}{2}\right)^3\right]$$

and for $108 \leq h < 378$ nautical miles ($\rho' = \rho_2$),

$$\rho_2 = \rho_o(h)(0.85F_{10.7})\left\{1 + 0.19[\exp(0.012h) - 1.9]\left(\frac{1 + \cos\psi'}{2}\right)^3\right\}$$

where

$$\log_{10}\rho_o(h) = d_2 - 0.00368h + 6.363\exp[-0.0048h]$$

d_1, d_2 = numbers input to the program at execution time

$F_{10.7}$, ψ' = Lockheed-Jacchia model-atmosphere parameters (Ref. 12)

Differentiating each of these expressions with respect to h then yields

$$\frac{\partial\rho_1}{\partial h} = \frac{-d_1\rho}{h} - 5.606 \times 10^{-12}\left(\frac{76}{h}\right)^{d_1}\left[\frac{1}{32} - \left(\frac{1 + \cos\psi'}{2}\right)^3\left(\frac{184 - 2h}{4896}\right)\right]$$

$$+ 5.606 \times 10^{-2}\left(\frac{76}{h}\right)^{d_1}(0.85)F_{10.7}\left(\frac{h - 76}{32}\right)^{1/3}\left[\frac{1}{24} - \left(\frac{1 + \cos\psi'}{2}\right)^3\left(\frac{532 - 7h}{14688}\right)\right]$$

and

$$\frac{\partial\rho_2}{\partial h} = -\rho\,\frac{[0.00368 + 0.0305424\exp(-0.0048h)]}{0.4342944819} + (0.85)F_{10.7}$$

$$\times \exp\{2.302585[d_2 - 0.00368h + 6.363\exp(-0.0048h)]\}$$

$$\times\left[0.001938\left(\frac{1 + \cos\psi'}{2}\right)^3 \exp(0.0102h)\right]$$

Combining the foregoing results, the elements T_{ij} of the T matrix then are

$$T_{11} = -\frac{C_D A}{2W}\left(V_A \dot{x}_A \frac{\partial\rho}{\partial h}\frac{\partial h}{\partial x} - \frac{\rho\omega_e \dot{x}_A \dot{y}_A}{V_A}\right)$$

$$T_{12} = -\frac{C_D A}{2W}\left(V_A \dot{x}_A \frac{\partial\rho}{\partial h}\frac{\partial h}{\partial y} + \frac{\rho\omega_e \dot{x}_A^2}{V_A} + \rho V_A \omega_e\right)$$

$$T_{13} = -\frac{C_D A}{2W}\left(V_A \dot{x}_A \frac{\partial\rho}{\partial h}\frac{\partial h}{\partial z}\right)$$

$$T_{21} = -\frac{C_D A}{2W}\left(V_A \dot{y}_A \frac{\partial\rho}{\partial h}\frac{\partial h}{\partial x} - \frac{\partial\omega_e \dot{y}_A^2}{V_A} - \rho V_A \omega_e\right)$$

$$T_{22} = -\frac{C_D A}{2W}\left(V_A \dot{y}_A \frac{\partial\rho}{\partial h}\frac{\partial h}{\partial y} + \frac{\partial\omega_e \dot{x}_A \dot{y}_A}{V_A}\right)$$

$$T_{23} = -\frac{C_D A}{2W}\left(V_A \dot{y}_A \frac{\partial\rho}{\partial h}\frac{\partial h}{\partial z}\right)$$

$$T_{31} = -\frac{C_D A}{2W}\left(V_A \dot{z}_A \frac{\partial\rho}{\partial h}\frac{\partial h}{\partial x} - \frac{\partial\omega_e \dot{y}_A \dot{z}_A}{V_A}\right)$$

$$T_{32} = -\frac{C_D A}{2W}\left(V_A \dot{z}_A \frac{\partial\rho}{\partial h}\frac{\partial h}{\partial y} + \frac{\partial\omega_e \dot{x}_A \dot{z}_A}{V_A}\right)$$

$$T_{33} = -\frac{C_D A}{2W}\left(V_A \dot{z}_A \frac{\partial\rho}{\partial h}\frac{\partial h}{\partial z}\right)$$

APPENDIX D

ANALYTIC SOLUTION OF CERTAIN VARIATIONAL EQUATIONS

APPENDIX D

ANALYTIC SOLUTION OF CERTAIN VARIATIONAL EQUATIONS

When the forces acting on a vehicle are symmetric about the polar axis of the earth, the variational equations for the two initial-condition parameters α (right ascension) and Ω (right ascension of ascending node) have the analytic solutions

$$\frac{\partial x}{\partial \alpha} = -y \qquad \frac{\partial \dot{x}}{\partial \alpha} = -\dot{y}$$

$$\frac{\partial y}{\partial \alpha} = x \qquad \frac{\partial \dot{y}}{\partial \alpha} = \dot{x}$$

$$\frac{\partial z}{\partial \alpha} = 0 \qquad \frac{\partial \dot{z}}{\partial \alpha} = 0$$

and

$$\frac{\partial x}{\partial \Omega} = -y \qquad \frac{\partial \dot{x}}{\partial \Omega} = -\dot{y}$$

$$\frac{\partial y}{\partial \Omega} = x \qquad \frac{\partial \dot{y}}{\partial \Omega} = \dot{x}$$

$$\frac{\partial z}{\partial \Omega} = 0 \qquad \frac{\partial \dot{z}}{\partial \Omega} = 0$$

These solutions may be derived from consideration of the vector equation

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r \cos \delta \cos \alpha \\ r \cos \delta \cos \alpha \\ r \sin \delta \end{pmatrix}$$

wherein r, δ, and α represent instantaneous values. Differentiating with respect to α_o, the initial value of right ascension, then leads to

$$\frac{\partial X}{\partial \alpha_o} = \frac{\partial X}{\partial r} \frac{\partial r}{\partial \alpha_o} + \frac{\partial X}{\partial \delta} \frac{\partial \delta}{\partial \alpha_o} + \frac{\partial X}{\partial \alpha} \frac{\partial \alpha}{\partial \alpha_o}$$

Noting that

$$\frac{\partial X}{\partial \alpha} = \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$$

it thus is necessary only to show that the relations

$$\frac{\partial r}{\partial \alpha_o} = \frac{\partial \delta}{\partial \alpha_o} = 0 \quad , \quad \frac{\partial \alpha}{\partial \alpha_o} = 1$$

which are intuitively plausible in the case of axially symmetric forces, hold in order to complete the derivation of the analytic solution

$$\frac{\partial X}{\partial \alpha_o} = \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$$

Letting

$$R = \begin{pmatrix} r \\ \delta \\ \alpha \end{pmatrix}$$

a differential equation for R under the symmetry assumption would be of the form

$$\ddot{R} = G(r, \ \delta, \ \dot{r}, \ \dot{\delta}, \ \dot{\alpha})$$

or

$$\ddot{R} = G(R, \ \dot{R})$$

with initial conditions

$$R(t_o) = \begin{pmatrix} r_o \\ \delta_o \\ \alpha_o \end{pmatrix}$$

and

$$\dot{R}(t_o) = \begin{pmatrix} \dot{r}_o \\ \dot{\delta}_o \\ \dot{\alpha}_o \end{pmatrix}$$

but with no dependence upon α.

Differentiating with respect to α_o and interchanging orders of differentiation then yields

$$\ddot{R}_{\alpha_o} = \frac{\partial G}{\partial R} R_{\alpha_o} + \frac{\partial G}{\partial \dot{R}} \dot{R}_{\alpha_o}$$

with initial conditions

$$R_{\alpha_o}(t_o) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

and

$$\dot{R}_{\alpha_o}(t_o) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

Inasmuch as the third column of the matrix $\partial G/\partial R$ is $\partial G/\partial \alpha$, which is always zero by the symmetry assumption, then

$$R_{\alpha_o}(t) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

necessarily is a unique solution which, when expressed in component form, gives the required relations

$$\frac{\partial r}{\partial \alpha_o} = 0 \quad ; \quad \frac{\partial \delta}{\partial \alpha_o} = 0 \quad ; \quad \frac{\partial \alpha}{\partial \alpha_o} = 1$$

APPENDIX E

BINARY EPHEMERIS (B7) TAPE FORMATS

APPENDIX E

BINARY EPHEMERIS (B7) TAPE FORMATS

The format and characteristics of the TRACE-D program binary ephemeris (B7) tape are illustrated in Figures E-1 through E-4. Setup information for generation of this tape in conjunction with a TRACE-D program run has been fully described in the basic document (see Section 5.1.2.2).

1. EVEN-MINUTE FORMAT

The RESIDUE link may be used to difference two ephemeris tapes and to resolve resulting differences into orbit-plane coordinates. Tapes applicable to this procedure are typically contractor-produced, but may also, for example, involve a combination of one contractor tape and one TRACE-D generated tape. These tapes must be compatible with the IBM 7094 computer and IBM 729-VI tape units and must be in the binary mode with 36 bits per word and a density of 800 bits per inch.

It should be noted that the rectangular coordinates involved in this connection are in the usual inertial system, with the principal axis along the mean equinox at midnight of epoch day and the Z axis along the axis of rotation.

2. EXPANDED FORMAT

Future modifications to the TRACE-D program will result in addition of a number of words to the data records of the B7 tape. The logic for specifying the time points will also be generalized. The expanded format, as well as revised usage instructions, will be made available as such programming changes are introduced.

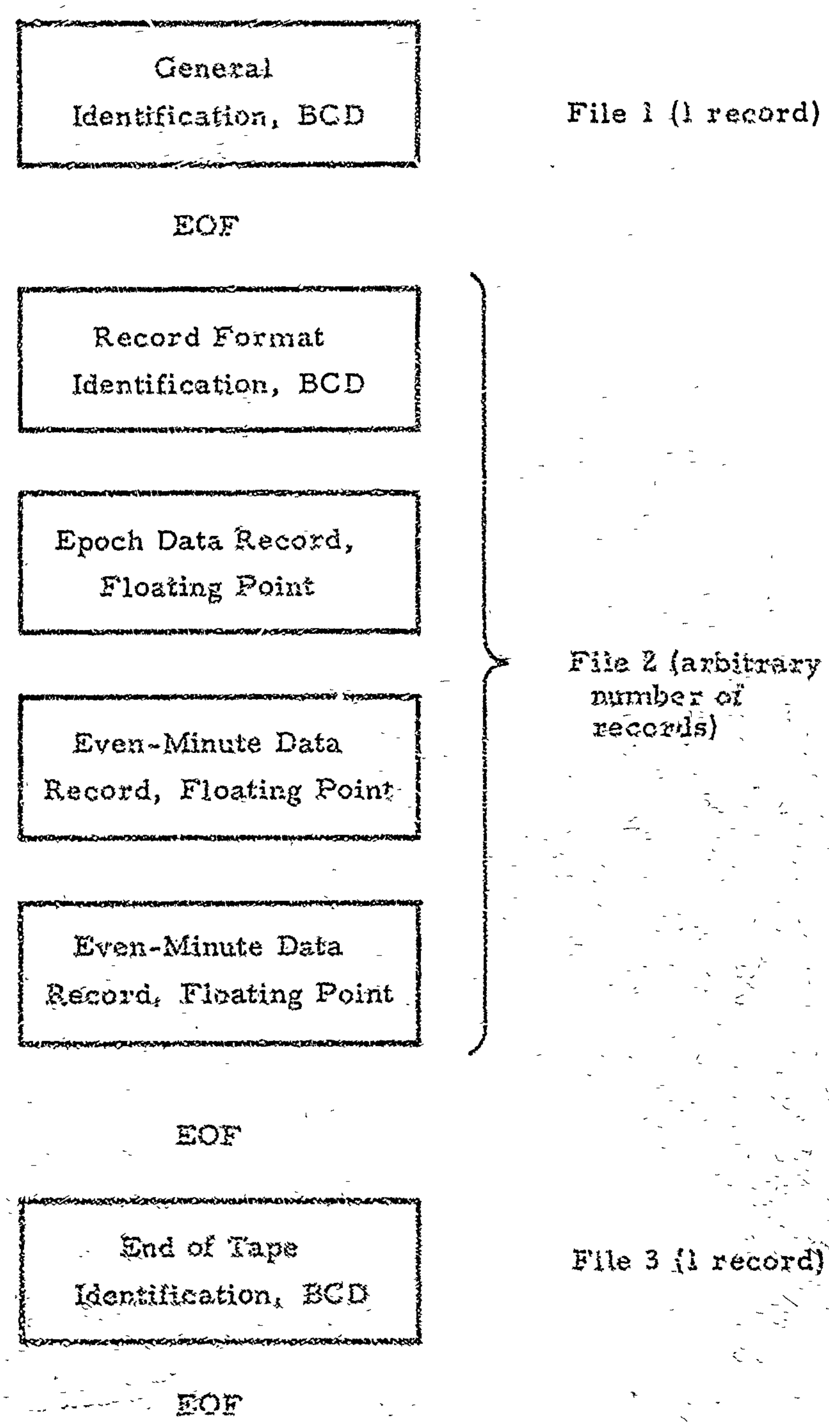


Figure E-1. Binary Ephemeris (B7) Tape Schematic Format

FILE 1

Word	Content
1	(Contractor Designation)
2	(Year) | (Month) | (Day)
3	(System time, integral part) | Dec. Pt.
4	(System time, fractional part)
5	(Starting Rev. No. | (Ending Rev. No.)
6	
7	
8	
9	(Description)
10	
11	
12	
13	S | D | T | O | O | 1

General ID file containing BCD characters

Notes:

1. Quantities enclosed in parentheses describe entry content. Characters not enclosed in parentheses indicate literal content (i. e., the BCD representation of a decimal point, or 73, should occupy the last six bits of Word No. 3.

2. System time is in seconds from midnight of current day.

3. Words No. 2 through 4 designate epoch time.

4. Words No. 6 through 12 are used to describe a particular ephemeris run, and should include earth and atmosphere model designations, any unusual parameters associated with the fit which determined the vector, and any other information distinguishing the ephemeris.

Figure E-2. File 1 Detailed Format

FILE 2

1	(Description)						ID record, BCD characters
2	(Description)						
3	0	0	0	0	1	0	
4	M	J	D				
5	S	Y	S	T	I	M	
6	X						
7	Y						
8	Z						
9	X	D	O	T			
10	Y	D	O	T			
11	Z	D	O	T			
12	L	A	T				
13	L	O	N	G			

1	(Modified Julian date)	Epoch time and position expressed in floating point numbers
2	(System time, epoch)	
3	(X)	
4	(Y)	
5	(Z)	
6	($\dot{X}$)	
7	($\dot{Y}$)	
8	($\dot{Z}$)	
9	(Vehicle geodetic latitude)	
10	(Longitude east from Greenwich)	

1	(Modified Julian date)	Time and position at each even minute after epoch expressed in floating point numbers
2	(System time, even minute)	
3	(X)	
4	(Y)	
5	(Z)	
6	($\dot{X}$)	
7	($\dot{Y}$)	
8	($\dot{Z}$)	
9	(Vehicle geodetic latitude)	
10	(Longitude east from Greenwich)	

1	(Modified Julian date)	Time and position at last even minute expressed in floating point numbers
2	(System time, even minute)	
3	(X)	
4	(Y)	
5	(Z)	
6	($\dot{X}$)	
7	($\dot{Y}$)	
8	($\dot{Z}$)	
9	(Vehicle geodetic latitude)	
10	(Longitude east from Greenwich)	

END OF FILE MARK

<u>Notes:</u>

1. Quantities enclosed in parentheses describe entry content. Characters not enclosed in parentheses indicate literal content.
2. System time is in seconds from midnight of current day.
3. Record 1, File 2 defines the numbers and positions of the various quantities in the following data records.
4. System time of the first data record (Record 2, File 2) is time of epoch. System time of the second data record is the first even minute after epoch. Subsequent times are at each even minute.
5. Modified Julian date is defined as Julian date minus 2,400,000.5
6. Vehicle geodetic latitude is defined as the angle from the equatorial plane to a line normal to the surface of the WGS-1960 ellipsoid and intersecting the satellite position.
7. X and $\dot{X}$ quantities are in units of feet and feet per second, latitude in units of degrees positive north and negative south, and longitude in units of degrees positive east in the range 0 to 360 referenced to Greenwich.
8. Intermediate even-minute records have format identical to first even-minute record. An arbitrary number of even-minute records is allowed.

Figure E-3. File 2 Detailed Format

FILE 3

E	N	D	O	F	T
A	P	E	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

End of tape file containing BCD characters

END OF FILE MARK

Figure E-4. Special File Indicating End of Tape

APPENDIX F

CHARACTERISTICS OF COW, FINP, AND GRAV SUBROUTINES

Appendix F

CHARACTERISTICS OF COW, FINP, AND GRAV SUBROUTINES

The material contained in Appendix F consists of information relating to subroutines COW, FINP, and GRAV, and is included to satisfy interest in the specific methods employed by these routines or in details regarding their usage in connection with the TRACE-D program. The following pages have been extracted from a Programming Handbook used by members of the Aerospace Corporation Computation and Data Processing Center.

It should be noted that since the following subroutine writeups are reproductions of instructions appearing in the Handbook, none of the contained symbols, notation, or references should be associated with other portions of this document. Thus, within the context of these extracted pages, reference to Appendices A or B would refer to the material appended to a particular subroutine writeup rather than to Appendices A or B of this primary report.

Appendix F

CHARACTERISTICS OF COW, FINP, AND GRAV SUBROUTINES

The material contained in Appendix F consists of information relating to subroutines COW, FINP, and GRAV, and is included to satisfy interest in the specific methods employed by these routines or in details regarding their usage in connection with the TRACE-D program. The following pages have been extracted from a Programming Handbook used by members of the Aerospace Corporation Computation and Data Processing Center.

It should be noted that since the following subroutine writeups are reproductions of instructions appearing in the Handbook, none of the contained symbols, notation, or references should be associated with other portions of this document. Thus, within the context of these extracted pages, reference to Appendices A or B would refer to the material appended to a particular subroutine writeup rather than to Appendices A or B of this primary report.

I2 $\geqslant$ +0; Variable step-size mode of operation is used.
I2 $<$ 0; Fixed step-size mode of operation is used.

IB; the first IB($\leqslant$ N) equations are tested to determine whether it is necessary to halve, or possible to double, the step-size or to proceed with a Cowell integration step.

IR; for a given step-size H, the Cowell integration step is H and the Runge-Kutta integration step is H/IR. If IR = 0 in the calling sequence, it will be set to 4.

ER= 1.0E-S, where S is the number of significant figures desired. If ER = 0 in the call sequence, it will be set to 1E-9.

HMIN is the minimum step size.

HMAX is the maximum step size, set to 1E18 if HMAX = 0 in the calling sequence.

YMI is the minimum y_i value allowed for testing. (See Appendix A, RW DE6F for further details.) (Page F-14) If y_{min} = 0 in the calling sequence, it will be set to 1.

DAUX is the location of the entry point of a subroutine which evaluates the second-order derivatives. This must be defined by use of an F card in the main program and must use COMMON for input and output.

Region T contains the following information prior to set-up entry:

T(2)= x , initial value of independent variable.
T(3)= h , value of step-size.

$$\left.\begin{array}{l} T(4) = y_1 \\ \cdot \\ \cdot \\ T(3+N) = y_N \end{array}\right\} \text{values of dependent variables } y_i$$

$$\left.\begin{array}{l} T(4+N) = y'_1 \\ \cdot \\ \cdot \\ T(3+2N) = y''_N \end{array}\right\} \text{values of the first derivatives } y'_i$$

$$\left.\begin{array}{l} T(4+2N) = y''_1 \\ \cdot \\ \cdot \\ T(3+3N) = y''_N \end{array}\right\} \begin{array}{l}\text{values of the second derivatives } y''_i \text{ to be supplied} \\ \text{by the auxiliary DAUX}\end{array}$$

Note: This region and the parameter N should be placed in CØMMØN since it is necessarily referred to in the main program and in the auxiliary. The cell T(1) is set up by the subprogram RW CØW and will contain N scaled at 35.

b. Calling sequence for integrating one step:

CALL ELL (TEST)

Upon return from ELL, TEST will be plus if the integration was a Runge-Kutta step and minus if the integration was a Cowell step.

c. Calling Sequence for Special Usage (See Appendices A and B, RW DE6F) (Pages F-14 and F-18)

CALL BULL (K,H)

K = 1. Running change of coordinates. After any Cowell integration step (TEST = -), this entry will initialize the beginning of a change of coordinates. Starting with the present values, one begins to save eight consecutive sets of y_i starting at T(11N+4) (and y_i' starting at T(3N+4) if they appear in the y_i''). He continues to use the integration entry above. The next seven integrations will be Cowell steps and all testing will be discontinued during this period. After the eighth set of $y_i(y_{i7})$, and $y_i'(y_{i7}')$ if necessary, have been stored, the user may change the second derivative evaluation routine DAUX and the units of x and h. The units of the eight sets of y_i and y_i' may be changed while storing each set or after all eight sets have been stored. When another integration step is asked for, the routine will perform the change of coordinates and proceed to a Cowell integration step. The routine will be ready for another change of coordinates and will operate under standard conditions.

K = 2. Change of step-size, not prior to a change of coordinates. During the integration procedure, the user may wish to output for a specific value of x without interrupting the Cowell/Runge-Kutta integration procedure. Or, he may wish to change the value of the step-size h prior to a running change of coordinates. He can do this after any integration entry with the following procedure:

With the new value of H, CALL BULL (2,H).

Thus, the integration step will be h/R. Continue with the regular integration entry to integrate further.

K = 3. This is simply a direct transfer to the Runge-Kutta integration subroutine and should be used to end exactly at a specific value of x. The integration step will be H. This procedure could be used in the middle of the integration procedure if 2. above is used immediately afterwards to restart in the Cowell/Runge-Kutta system.

If a change of step-size prior to a change of coordinates is desired, the user must prevent the routine from doubling again until after the change of coordinates. The following sequence will accomplish this:

1) CALL BULL (4,H), value of H is ignored.

2) CALL BULL (2,H) - see option K = 2.

   After the change of coordinates:

3) CALL BULL (5,H), value of H is ignored.

Number of Pages	
Write-up	4
Listing	21
Total	25

Identification

RWDE5F: Floating Point Runge-Kutta Integration of Second-Order Equations
704 - SAP
J. F. Holt, June 29, 1959
Space Technology Laboratories, Inc.

Purpose

To solve a set of N simultaneous second-order, ordinary differential equations, in which first derivatives may or may not appear.

Restrictions

No internal checks are made for overflow or underflow. The user must provide an auxiliary subroutine which evaluates the second order derivatives. Initial conditions must be set up prior to the first entry.

Method

A fourth order Runge-Kutta method* is used for second-order equations of the general type, $y''=f(x,y,y')$. However, the subroutine can also be used for special second-order equations, $y''=f(x,y)$. (See Reference and Appendix A.) (Page F-8)

While input and output to the routine are single precision, double precision is used internally in the calculation of the dependent variables and the first derivatives in order to control round-off errors. Truncation error can be controlled by choosing an appropriate step-size.

Before returning to the main program, both entries utilize the auxiliary subroutine to compute the second derivatives. Thus, the values of the variables and derivatives are consistent at all times.

Usage

Calling sequence to set up a problem:

A. With 1st Derivatives

Loc.	Instruction
α	TSX DE5F,4
α+1	PZE T,0,V
α+2	Return

B. Without 1st Derivatives

Loc.	Instruction
α	TSX DE5F,4
α+1	MZE T,0,V
α+2	Return

Calling sequence to integrate all variables one step:

Loc.	Instruction	Comments
α	TSX DE5F+1,4	Integration entry. The step-size
α+1	Return	h may be varied with each entry.

The address T is the first of 9N+3 cells arranged as follows:

Loc.	Contents	Comment.
T	PZE N,0,0	N is the number of equations. Fixed point.

---

* J. B. Scarborough, Numerical Mathematical Analysis. 3rd Ed., Johns Hopkins Press, Baltimore, 1955. (pp.300-301)

Usage - continued

Loc.	Contents	Comment
T+1	x	Value of independent variable. Floating point.
T+2	h	Value of step-size. Floating point.
T+3 to	y_1 thru y_N	Values of dependent variables y_i. Floating point.
T+N+3 to	y_1' thru y_N'	Values of first derivatives y_i'. Floating point.
T+2N+3 to	y_1'' thru y_N''	Locations where the user's auxiliary subroutine must store the second derivatives y_i''.
T+3N+3	etc.	6N cells of temporary storage.

T+4N+3 through T+5N+2 and T+8N+3 through T+9N+2 of the 6N cells of temporary storage contain the least significant parts of y_i' and y_i respectively. These 2N cells must be preserved throughout the integration procedure. The other 4N cells of temporary storage may be utilized between integration steps. T through T+3N+2 (except for varying the step-size) must also be preserved between integrations.

The address V is the entry point of the auxiliary subroutine which evaluates and stores the second-derivatives y_i'' and is entered by the calling sequence:

Loc.	Instruction	Comments
α	TSX V,4	Index registers need not be saved by the auxiliary subroutine.
α+1	Return	

User must return via TRA 1,4 from V.

Space Requirements

212 cells of program and constants.
3 cells of COMMON through COMMON + 2.

Timing

Set-up Time:

.012 (12N + 152) ms. + time for 1 entry to the auxiliary subroutine.

To integrate one step:

1. With 1st Derivatives

   .012 (474N + 182) ms. + time for 4 entries to the auxiliary subroutine

2. Without 1st Derivatives

   .012 (381N + 182) ms. + time for 3 entries to the auxiliary subroutine

Number of Pages

Writeup
Appendix
Listing

APPENDIX A

Method

Reference: J. B. Scarborough, Numerical Analysis, 3rd Ed., Johns Hopkins Press, Baltimore 1955.

Let the system of equations to be solved be given in the form:

1) $$y_i'' = f_i\,(x,\, y_1, \ldots, y_n,\, y_1' \ldots y_n') \qquad (i = 1,2,\ldots,N)$$
$$y_i(x_o) = y_{io},\; y_i'(x_o) = y_{io}'$$

Let y_{in} and y_{in}' be the values of y_i and y_i' at $x = x_n$; f_{in} be the second derivative of y_i at $x = x_n$; and h be the increment (step-size) of the independent variable x. The Runge-Kutta formulas (Ref. (1), pp.300-301) used in this subroutine are as follows:

2)
$$k_{i1} = h\, f_i\,(x_n,\, y_{in},\, y_{in}')$$
$$k_{i2} = h\, f_i\,(x_n + \tfrac{h}{2},\, y_{in} + \tfrac{h}{2} y_{in}' + \tfrac{h}{8} k_{i1},\, y_{in}' + \tfrac{k_{i1}}{2})$$
$$k_{i3} = h\, f_i\,(x_n + \tfrac{h}{2},\, y_{in} + \tfrac{h}{2} y_{in}' + \tfrac{h}{8} k_{i1},\, y_{in}' + \tfrac{k_{i2}}{2})$$
$$k_{i4} = h\, f_i\,(x_n + h,\, y_{in} + h\, y_{in}' + \tfrac{h}{2} k_{i3},\, y_{in}' + k_{i3})$$
$$\Delta y_{in} = h\left[y_{in}' + \tfrac{1}{6}(k_{i1} + k_{i2} + k_{i3})\right]$$
$$\Delta' y_{in} = \tfrac{1}{6}\left[k_{i1} + 2k_{i2} + 2k_{i3} + k_{i4}\right]$$
$$y_{i,n+1} = y_{in} + \Delta y_{in}$$
$$y_{i,n+1}' = y_{in}' + \Delta' y_{in}$$

For the special second-order equation,

3) $$y_i'' = f_i\,(x,\, y_1, \ldots, y_n) \qquad \text{(1st derivatives missing)}$$

it should be noted that $k_{i2} = k_{i3}$, so that the above formulas reduce to the following Runge-Kutta formulas:

4)
$$k_{i1} = h\, f_i\,(x_n,\, y_{in})$$
$$k_{i2} = h\, f_i\,(x_n + \tfrac{h}{2},\, y_{in} + \tfrac{h}{2} y_{in}' + \tfrac{h}{8} k_{i1})$$
$$k_{i3} = h\, f_i\,(x_n + h,\, y_{in} + h\, y_{in}' + \tfrac{h}{2} k_{i2})$$
$$\Delta y_{in} = h\left[y_{in}' + \tfrac{1}{6}(k_{i1} + 2k_{i2})\right]$$

4) - continued

$$\Delta' y_{in} = \frac{1}{6}(k_{i1} + 4k_{i2} + k_{i3})$$

$$y_{i,n+1} = y_{in} + \Delta y_{in}$$

$$y'_{i,n+1} = y'_{in} + \Delta' y_{in}$$

where k_{i4} of (2) is now k_{i3} of (4).

The subroutine can be made to take advantage of this fact by a simple change in the calling sequence--thus speeding up the integration process.

The user must provide a starting value for h. Since each integration step is independent, the value of h may be changed at will between integration steps. Negative values of h may be supplied for backward integration.

Except for the initial starting conditions, the values of y_i and y'_i are kept in double precision. All intermediate values, which are supplied for the user's auxiliary program, are done in single precision. The equations of Δy_i and $\Delta' y_i$ have been modified as follows:

5) $$\Delta y_{in} = h \left[y'_{in} + \frac{h}{6}(k^*_{i1} + k^*_{i2} + k^*_{i3}) \right],$$

6) $$\Delta' y_{in} = \frac{h}{6}(k^*_{i1} + 2k^*_{i2} + 2k^*_{i3} + k^*_{i4}),$$

where the $k^* = f(x,y,y')$ (i.e., not multiplied by h).

The values of k^* in these equations are accumulated in single precision. These sums are multiplied by h/6, and the most and least significant parts of this product are used to complete the formation of $y_{i,n+1}$ and $y'_{i,n+1}$ in double precision.

These double precision values of $y_{i,n+1}$ and $y'_{i,n+1}$ are saved for the next integration step.

The values of the variables and derivatives (i.e., x,y,y', and y") are consistent at the end of each integration step.

Number of Pages

Writeup	4
Listing	4
Total	8

Identification

RW DE6F Floating Point Cowell (Second Sum), Runge-Kutta Integration of Second-Order Equations
* 709/7090 SCAT
J. F. Holt and W. G. Strang, August 10, 1959
Space Technology Laboratories, Inc.

Purpose

To solve a set of N simultaneous second-order ordinary differential equations, in which first derivatives may or may not appear.

Restrictions

No internal checks are made for overflow or underflow. The user must provide an auxiliary subroutine which evaluates the second-order derivatives. The initial conditions must be set up prior to the first entry.

Method

A fourth-order Runge-Kutta method** (RW DE5F) is used to start the integration and to change the step-size during integration. A Cowell "second-sum" method based on sixth differences is used to continue the integration. While input to this routine is single precision, double precision is used internally to control round-off errors. Truncation error can be controlled by choosing an appropriate step-size, or by using the variable step-size mode of operation. The set-up entry uses the auxiliary subroutine to evaluate the second-order derivatives. The values of the variables and derivatives are consistent at all times. A detailed description of the method used is available in Appendix A. (Page F-14.)

Usage

Calling sequence to set up a problem:

Loc.	Instruction		Comments
α	TSX	DE6F,4	Set up entry
α+1	PO	T,0,V	Option, addresses of storage and auxiliary subroutines.
α+2	P1	B,0,R	Option and Parameters
α+3	DEC	1E-S	S is the number of significant figures desired.

** J. B. Scarborough Numerical Mathematical Analysis, Third Edition, Johns Hopkins Press, Baltimore, 1955 (pp. 301-302)

* Modified by Jim Holt, Aerospace Corporation, April 1, 1963.

RW DE6F
Page 2
August 10, 1959

Usage (continued)

Loc.	Instruction	Comments
$\alpha+4$	DEC h_{min}	Minimum step-size. Floating point.
$\alpha+5$	DEC h_{max}	Maximum step-size. Floating point.
$\alpha+6$	DEC y_{min}	Minimum y_i value allowed for testing. Floating point. (See Appendix A for details of y_{min}.) (Page F-14.)

Calling sequence to integrate all variables one step:

α	TSX DE6F+1,4	Integration entry.
$\alpha+1$	Return	Sign of AC will be plus if the integration was Runge-Kutta and minus if Cowell.

The address T is the first of 30N+3 cells arranged as follows:

T	PZE N,0,0	N is the number of equations. Fixed point.
T+1	x	Value of independent variable. Floating point.
T+2	h	Value of step-size. Floating point.
T+3 thru T+N+2	y_i thru y_N	Values of the dependent variables y_i. Floating point.
T+N+3 thru T+2N+2	y_i' thru y_N'	Values of the first derivatives y_i'. Floating point.
T+2N+3 thru T+3N+2	y_i'' thru y_N''	Locations where the user's auxiliary subroutine must store the second derivatives y_i''.
T+3N+3 thru	T+30N+2	27N cells of temporary storage.

T+3N+3 thru T+9N+2 (6N) are used by the Runge-Kutta subroutine (RW DE5F). T+4N+3 thru T+5N+2 and T+8N+3 thru T+9N+2 contain the least significant parts (except when a change of coordinates is in progress) of y_i' and y_i respectively, and must be preserved throughout the entire integration procedure. The final 21N cells of the T storage are used by the Cowell subroutine and must also be preserved. (See Appendix B for a detailed description of these 27N cells.) (Page F-18.)

The address V is the entry point of the auxiliary subroutine which evaluates the derivatives y_i'' and is entered by the calling sequence:

Loc.	Instruction	Comments
α	TSX V,4	Index registers need not be saved in V.
α+1	Return	Return must be made via a TRA 1,4.

The first B ($\leq$ N) equations are tested to determine whether it is necessary to halve or possible to double the step-size or to proceed with a Cowell integration step.

For a given step-size h, the Cowell integration step is h and the Runge-Kutta integration step is h/R.

Options

PO = PZE 1st derivatives are present in the evaluation of the second derivatives.
= MZE 1st derivatives are missing in the evaluation of the second derivatives.
P1 = PZE Variable step-size mode of operation is used.
= MZE Fixed step-size mode of operation is used.

If 1E-S, h_{min}, h_{max}, and y_{min} are not specified (0 in first calling sequence), the subroutine will set them to 1E-9, 0, 1E18, and 1., respectively.

Special Usage (See Appendices A and B for complete details.)(Pages F-14, F-18)

The following special usages are possible:

1. Running change of coordinates.
2. Running start.
3. Change of step-size by use in the Cowell/Runge-Kutta system.
4. Change of step-size for a final integration or at some prescribed value of x.

Space Required (In addition to T and V).

955 cells of program and constants. (Includes DE5F)

44 cells of COMMON thru COMMON + 43.

Timing

Set-up time. (V=time for 1 entry to the auxiliary subroutine.)

$$.00218 \left[12N + 512\right] \text{ ms. } + iV.$$

Timing (continued)

To integrate one step:

1. Runge-Kutta (AC=+ after an integration return.)

   a) With 1st derivatives.

   * $.00218 \left[476N + (8N+18)/R + 240\right]$ ms. + 4V.

   b) Without 1st derivatives.

   * $.00218 \left[383N + (8N + 18)/R + 240\right]$ ms. + 3V.

2. Cowell (AC= - after an integration return.)

   a) With 1st derivatives.

   * $.00218 \left[2901N + 92B + 194\right]$ ms. + 2V.

   * $+.00218 \left[2124N + 88B + 34\right]$ ms. if previous integration was Runge-Kutta.

   b) Without 1st derivatives.

   * $.00218 \left[2334N + 92B + 194\right]$ ms. + 2V.

   * $+.00218 \left[2124N + 88B + 34\right]$ ms. if previous integration was Runge-Kutta

3. * Change of Coordinates. (In addition to first part of 2A or 2B)

   $.00218 \left[2286N\right]$ ms. + 8V.

Number of Pages

Writeup	4
Appendix A	4
Appendix B	7

APPENDIX A - METHOD

This routine is prepared to solve the following system:

$$1)\quad \left.\begin{array}{l} y_i'' = f_i\ \ (x, y_1, \ldots, y_N, y_1', \ldots, y_N') \\ y_i(x_0) = y_{i0}, \quad y_i'(x_0) = y_{i0}' \end{array}\right\} \quad i = 1, 2, \ldots, N$$

In case none of the f_i involve the first derivatives y_i', time is saved by indicating this in the set-up. The Runge-Kutta routine RWDE5F is used to start the integration and also to change the step-size h. The user must ask for each integration step, and the routine will follow this sequence:

1. R Runge-Kutta steps of size $\frac{h\ \text{start}}{R}$ are taken to obtain y_{i1}, y_{i1}', y_{i1}''. This is continued until we reach y_{i6}, y_{i6}', y_{i6}'', after a total of 6R Runge-Kutta steps. The integer R (=4 if unspecified) simply allows Runge-Kutta to operate at a smaller step than the main program.

2. For each of the N equations, that part of the difference table above the diagonal line is constructed in three steps:

		y_{i0}''						
			Δ^{I}_{i0}					
		y_{i1}''		Δ^{II}_{i0}				
			Δ^{I}_{i1}		Δ^{III}_{i0}			
		y_{i2}''		Δ^{II}_{i1}		Δ^{IV}_{iC}		
			Δ^{I}_{i2}		Δ^{III}_{i1}		Δ^{V}_{i0}	
$\underline{''F_{i4}}$		$\underline{y_{i3}''}$		$\underline{\Delta^{II}_{i2}}$		$\underline{\Delta^{IV}_{i1}}$		$\underline{\Delta^{VI}_{i0}}$
	$\underline{'F_{i4}}$		$\underline{\Delta^{I}_{i3}}$		$\underline{\Delta^{III}_{i2}}$		$\underline{\Delta^{V}_{i1}}$	
$''F_{i5}$		y_{i4}''		Δ^{II}_{i3}		Δ^{IV}_{i2}		Δ^{VI}_{i1}
	$'F_{i5}$		Δ^{I}_{i4}		Δ^{III}_{i3}		Δ^{V}_{i2}	
$''F_{i6}$		y_{i5}''		Δ^{II}_{i4}		Δ^{IV}_{i3}		
	$'F_{i6}$		Δ^{I}_{i5}		Δ^{III}_{i4}			
$''F_{i7}$		y_{i6}''		Δ^{II}_{i5}				
	$'F_{i7}$		Δ^{I}_{i6}					
$''F_{i8}$		y_{i7}''						
	$'F_{i8}$							
$''F_{i9}$								

APPENDIX A (Continued)

First the known y''_{i0} through y''_{i6} are differenced to give the right half of the table. Next are calculated in extra precision:

$$2)\quad "F_{i4} = \frac{y_{i3}}{h^2} - C_0\, y''_{i3} - C_2\, \Delta^{II}_{i2} - C_4\, \Delta^{IV}_{i1} - C_6\, \Delta^{VI}_{i0}$$

$$3)\quad 'F_{i4} = \frac{y'_{i3}}{h} - D_0\, y''_{i3} - D_1\, \Delta^{I}_{i3} - D_2\, \Delta^{II}_{i2} - D_3\, \Delta^{III}_{i2}$$

$$- D_4\, \Delta^{IV}_{i1} - D_5\, \Delta^{V}_{i1} - D_6\, \Delta^{VI}_{i0}$$

The table is then completed down to the diagonal line, by requiring the difference between any entry and the entry above to equal the entry to the right. The constants used in equations (2) - (7) are given in the description of the Livermore Cowell routine.

3. Before going to a Cowell step, the step-size h is tested. The tests are omitted, however, if the user so indicates in the initial calling sequence, in which case h is fixed. Only the first B equations are used to test, where $1 \leqslant B \leqslant N$ and B = N if unspecified. We determine--

$$V = \max_{1 \leqslant i \leqslant B} \left| \frac{\Delta^{V}_{i1}}{\max(y_{i6}, y_{min})} \right| . \quad \text{If} \quad V \geqslant \frac{10^{3-S}}{h^2}, \quad \text{then the}$$

ratio of 5th difference to function is too large--if S decimal places are to be retained at that step. Therefore, h is reduced to h/2 and Runge-Kutta re-entered for another sequence of 6R steps. These begin with the latest calculated values (y_{i6}, y'_{i6}) and no ground is retraced. The constant y_{min} (=1. if unspecified) prevents division by y near a zero; for example, in sine calculation y_{min}=.1 avoids difficulty near 180°. The integer S, taken as 9 if unspecified, allows a larger h if chosen smaller, say S = 7.

If $\frac{10^{-1-S}}{h^2} < V < \frac{10^{3-S}}{h^2}$, we proceed to a Cowell step.

If $V \leqslant \frac{10^{-1-S}}{h^2}$, we may be able to double h. We test further to see that

$$W = \max_{1 \leqslant i \leqslant B} \left| \frac{\Delta^{VI}_{i0}}{\max(y_{i6}, y_{min})} \right| \leqslant \frac{10^{-1-S}}{h^2},$$

and if so, we re-enter Runge-Kutta after replacing h by 2h, since the step-size h has led to needlessly small difference to function ratios. Of course, h is not halved or doubled if this would violate h_{min} or h_{max}, which are 0 and 10^{18} if unspecified.

APPENDIX A (Continued)

4. The Cowell integration begins with predictions--

$$4) \quad y_{17} = h^2 \left({}''F_{18} + N_0 y''_{16} + N_1 \Delta^{I}_{15} + \cdots + N_6 \Delta^{VI}_{10} \right)$$

$$5) \quad y'_{17} = h \left({}'F_{17} + \dot{N}_0 y''_{16} + \dot{N}_1 \Delta^{I}_{15} + \cdots + \dot{N}_6 \Delta^{VI}_{10} \right)$$

These use the row of the difference table above the diagonal line; only this row is needed for a Cowell step and is kept up to date as the integration proceeds. (We mention that the above prediction for y'_{17} is omitted in the missing first derivative option.) Now from y_{17} and y'_{17}, we obtain y''_{17} and then complete the row of differences out to Δ^{VI}_{11} under the diagonal line in the table. With this row, we calculate final corrected values--

$$6) \quad y_{17} = h^2 \left({}''F_{18} + B_0 y''_{17} + \cdots + B_6 \Delta^{VI}_{11} \right)$$

$$7) \quad y'_{17} = h \left({}'F_{17} + \dot{B}_0 y''_{17} + \cdots + \dot{B}_6 \Delta^{VI}_{11} \right)$$

From these we get corrected values for y''_{17}, and recalculate the entire row under the diagonal line. This completes the integration step. Using the new row of differences, the next step begins by testing the step-size (i.e., at 3.).

Further Properties Of The Program

In some problems, information about the first derivative (velocity) may be less reliable than information about the function (position). The user may then choose a "running change of coordinates" or a "running start;" these depend on the fact that with 8 consecutive values of the y_i (and the y'_i in case first derivatives are present in the f_i) the Cowell part of the program can be self-starting. The mathematics is simple: step No. 1 is omitted, and No. 2 modified to calculate $''F_{14}$ and $''F_{15}$ from Eq. (2) (instead of $''F_{14}$ and $'F_{14}$). The difference table may again be completed, and Cowell integration begins. The user, having tested the AC to establish that the previous step was a Cowell step, begins a running change of coordinates by setting cell DE6F + 500 to non-zero. He then sets up a counter and begins immediately to store 8 consecutive values of the y_i starting at T+3+11N (and y'_i starting at T+3+3N, if they appear in the f_i). After changing the coordinates the 8th time, the user may change the second derivative evaluation routines; if x and h are to be in new units this should also be done. When another step is asked for, the routine will form a difference table in the new coordinates and proceed to a Cowell step.

APPENDIX A (Continued)

The mechanics of a running start are similar; after going through the set-up routine, the user loads his values of y_{i0} through y_{i7} (and the y_i' if needed in the f_i) into the same locations as above and makes the required transfer.

There may also be occasions on which the user will wish to modify h himself; e.g., if he wishes to produce the numerical solution at some prescribed value of x, or if he wishes to approach a running change of coordinates at a step-size smaller than that being used by the routine. The technique of modifying h is described in Appendix B. (Page F-18.)

APPENDIX B

USAGE AND CODING INFORMATION

There are essentially two entries to the subroutine. The first entry must be made once at the beginning to set up the addresses, options, parameters, etc. of the routine for integration of N simultaneous second-order, ordinary differential equations, in which first derivatives may or may not appear. The first entry utilizes the auxiliary subroutine to evaluate the second-order derivatives at the initial conditions. Thus, the initial conditions must be set up prior to the first entry. The second entry may be used any number of times after the first to integrate all y_i from x to x+h by a Cowell step; or x to x+h/R by a Runge-Kutta step.

The first entry has the following calling sequence:

Loc.	Instruction	Comments
α	TSX DE6F,4	Setup entry.
α+1	PO T,O,V	Option, addresses of storage and auxiliary subroutine.
α+2	P1 B,O,R	Option and parameters.
α+3	DEC 1E-S	S is the number of significant figures desired.
α+4	DEC h_{min}	Minimum step-size. Floating point.
α+5	DEC h_{max}	Maximum step-size. Floating point.
α+6	DEC y_{min}	Minimum y_i value allowed for testing. Floating point.
α+7	Return	

(α+1): T is the address of the first word of a block of 30N+3 cells of temporary storage arranged as follows:

Loc.	Contents	Comments
T	PZE N,0,0	N is the number of equations. Fixed point.
T+1	x	Value of independent variable. Floating point.
T+2	h	Value of step-size. Floating point.
T+3 thru T+N+2	y_1 thru y_N	Values of the dependent variables y_i. Floating point.
T+N+3 thru T+2N+2	y_1' thru y_N'	Values of the first derivatives y_i'. Floating point.
T+2N+3 thru T+3N+2	y_1'' thru y_N''	Locations where the user's auxiliary subroutine must store the second derivatives y_i''. Floating point.
T+3N+3 thru	T+30N+2	27N cells of temporary storage.

APPENDIX B (Continued)

The next 27N storages of T are temporary storages. The Runge-Kutta subroutine uses the first 6N cells (T+3N+2 thru T+9N+2) and the Cowell routine uses the final 21N cells (T+9N+3 thru T+30N+2). However, if a change of coordinates (see SPECIAL USAGE) is made, the Cowell routine will also use the first 6N cells. The attached T STORAGE CHART shows the setup of the entire T region. The N cells starting at T+5N+3 and the N cells starting at T+6N+3 are used by the Runge-Kutta subroutine to compute $\Delta' y_{1N}$ and Δy_{1N}. However, $\Delta' y_{1N}$ and Δy_{1N} are destroyed before final exit, and these cells contain intermediate values of no significance to the user. The left side of the chart shows the storage for the normal case where 6R Runge-Kutta steps are taken (using the first 9N cells of T for each integration) before an attempt is made to proceed to a Cowell step. At the beginning of each set of R steps, the Cowell subroutine saves the values of the second derivatives (7 sets starting at T+11N+3). In addition, the values of y_{13} and y'_{13} are saved, starting at T+9N+3 and T+10N+3 respectively, for use in the central difference equations where $''F_{14}$ and $'F_{14}$ are calculated. Care must be exercised in using certain values in the temporary region. For instance, after a Runge-Kutta integration step, the most significant values of y_i and y'_i, starting at T+7N+3 and T+3N+3 respectively, will be the values of the previous integration; while the least significant values of y_i and y'_i, starting at T+8N+3 and T+4N+3 respectively, will be the values of the present integration. The Cowell routine also saves the least significant values of y_i and y'_i (unless a change of coordinates is in progress) in these same storages at the end of each integration. The 11N storages starting at T+19N+3 contain the right half of the N difference tables, an example of which is shown in Appendix A. (p F-14) The right side of the chart shows other values which are stored in the T region during a change of coordinates and will be explained later under SPECIAL USAGE. Even though only one symbol is given (y'_{10}, etc.), it should be understood that N values are stored as in the left side of the chart. Thus, y'_{10} signifies $y'_{10}, y'_{20}, y'_{30}, \ldots, y'_{N0}$.

The address V is the entry point of an auxiliary subroutine which the user must provide to evaluate the second derivatives y''_i. This subroutine must store y''_i in T+2N+3 through T+3N+2 as shown above. The subroutine is entered by the calling sequence:

Loc.	Instruction	Comments
α	TSX V,4	Index registers need not be saved.
α+1	Return	Return via a TRA 1,4.

The derivatives y''_i are evaluated during the setup and at the end of each integration step. Thus, the values of the variables and the derivatives are consistent at all times. Extra precision is recommended for the evaluation of the second derivatives y''_i.

PO should be set to PZE if the first derivatives are present in the evaluation of the second derivatives. If first derivatives are not present, PO should be set to MZE.

APPENDIX B (Continued)

(α+2): P1 should be set to PZE if a variable step-size is wanted. For a fixed step-size, P1 should be set to MZE. The former allows doubling and halving while the latter restricts the routine to a fixed h. The user may change the mode of operation externally at any time by setting cell DE6F+501 to plus for a variable step-size and minus for a fixed step-size.

Only the first B ($1 \leqslant B \leqslant N$) equations are tested to determine doubling or halving of h. Thus, the user should arrange the N equations in descending order of importance, and specify B accordingly. If B = 0 in the calling sequence, it will be set to N.

R is the ratio of the Cowell step-size to the Runge-Kutta step-size. Thus, smaller integration steps can be taken in the Runge-Kutta subroutine by setting R greater than 1. If R = 0 in the calling sequence, it will be set to 4. R is saved in the decrement of cell DE6F+516 and in floating point in cell DE6F+517. After any Cowell integration step (AC=-), the user could change R by changing BOTH of these cells.

(α+3): 1E-S is a floating point number where S is the number of significant figures of accuracy desired at each step. The user should experiment with S to fit his own particular problem. The 1 of 1E-S may also be varied from 1 to 9 (1E-S thru 9E-S) for a finer degree of control over the accuracy testing. If 1E-S=0 in the calling sequence, it will be set to 1E-9.

(α+4): h_{min} is a floating point number giving a lower bound for h. h_{min} is saved in cell DE6F+509 and can be changed at any time.

(α+5): h_{max} is a floating point number giving an upper bound for h. If h_{max}=0 in the calling sequence, the upper bound will be set to 1E18. h_{max} is saved in cell DE6F+510 and can be changed at any time.

(α+6): y_{min} is a positive floating point number which is used in testing the step-size. If y_{min}=0 in the calling sequence, it will be set to 1. y_{min} is saved in cell DE6F+511 and can be changed at any time.

If the fixed step-size mode of operation is selected (P1=MZE), then B, 1E-S, h_{min}, h_{max}, and y_{min} are all ignored by the subroutine. (If P1=MZE, set B = 1 for maximum efficiency.)

The integration entry has the following calling sequence:

Loc.	Instruction	Comments
α	TSX DE6F+1,4	Integrates all variables one step.
α+1	Return	

APPENDIX B (Continued)

Upon return from the integration entry, the accumulator will be plus if the integration was a Runge-Kutta step and minus if the integration was a Cowell step. Ordinarily, x will have been advanced to $x + h/R$ for a Runge-Kutta step and to $x + h$ for a Cowell step. However, in the variable h mode, it is possible that the value of h in T+2 prior to the integration entry has been changed to $h/2$ or $2h$. In this case, the integration step will be a Runge-Kutta step, and the value of x will be either $x + h/2R$ or $x + 2h/R$. All values of y_i, y_i', and y_i'' will be consistent with the new value of x. The user must never change the value of the step-size h except as described under SPECIAL USAGE.

Special Usage (See Appendix A. Further Properties Of The Program.) (Page F-16.)

1. Running change of coordinates. (Normal Entries.)

After any Cowell integration step (AC=-), the user may initialize the beginning of a change of coordinates by setting cell DE6F+500 to non-zero. Starting with the present values, he begins to save eight consecutive sets of y_i starting at T+11N+3 (and y_i' starting at T+3N+3 if they appear in the y_i''). He continues to use the integration entry above. The routine will detect the non-zero value stored in cell DE6F+500 and will begin a count-down in cell DE6F+502 from 8 (-1) 0. The next seven integrations will be Cowell steps and all testing will be discontinued during this period. After the eighth set of y_i (y_{i7}), and y_i' (y_{i7}') if necessary, have been stored (DE6F+502 will have a fixed point 1 in the address), the user may change the second derivative evaluation routine V and the units of x and h. The units of the eight sets of y_i and y_i' may be changed while storing each set, or after all eight sets have been stored. When another integration step is asked for, the routine will perform the change of coordinates and proceed to a Cowell integration step. Cell DE6F+500 will be restored to zero, and an 8 will be restored to the address of cell DE6F+502. Thus, the routine will be ready for another change of coordinates and will operate under standard conditions.

2. Running start. (Special Entry.)

A running start is similar to a running change of coordinates except that the user must supply all eight sets of y_i (and y_i' if necessary) at one time. The following sequence of operations must be followed:

A. Set up the initial conditions in the T storage. Only N, x, and h are needed, although the eight sets of y_i (but not y_i') can also be stored at this time. x must correspond to y_{i7} and h must be the interval at which the y_i have been obtained. Thus, $x = x_0 + 7h$ where x_0 corresponds to y_0 (and y_0').

B. Use the first entry calling sequence to set up all parameters and options. The V subroutine will be used but will have no effect on the problem. Also, cells T+4N+3 thru T+5N+2 and T+8N+3 thru T+9N+2 will be set to zero. Thus, the eight sets of y_i' must be stored AFTER the setup entry.

APPENDIX B (Continued)

C. Store eight consecutive sets (equal intervals) of y_i starting at cell T+11N+3 (and eight consecutive sets of y_i' starting at cell T+3N+3, if needed).

D. Execute the following calling sequence one time:

Loc.	Instruction	Comments
α	TSX DE6F+16,4	Enter only once.
α+1	Return	Integrates 1 step.

E. Continue with the regular integration entry (TSX DE6F+1,4) to integrate further. Step D integrates all variables one Cowell step, and $x(x_0 + 7h)$ is advanced to $x + h(x_0 + 8h)$. From this point, the routine will operate under normal conditions.

3. Change of step-size in the Cowell/Runge-Kutta System. (Special Entry.)

During the integration procedure, the user may wish to output for a specific value of x without interrupting the Cowell/Runge-Kutta integration procedure. Or, he may wish to change the value of the step-size h prior to a running change of coordinates. He can do this after any integration entry with the following procedure:

With the new value of h in the AC,

Loc.	Instruction	Comments
α	TSX DE6F+22,4	Changes h and starts new series of 6R
α+1	Return	Runge-Kutta steps.

Thus, the integration step will be h/R. Continue with the regular integration entry (TSX DE6F+1,4) to integrate further.

If the above procedure is being used to reduce the step-size prior to a change of coordinates, the user must prevent the routine from doubling again until after the change of coordinates. Doubling can be prevented either by storing zero in cell DE6F+510 (h_{max}), or by setting cell DE6F+501 (fixed step-size) negative prior to the above entry. After the change of coordinates, the user may restore the above cells.

4. Change of step-size for a final integration. (Special Entry.)

This is simply a direct transfer to the Runge-Kutta integration subroutine and should be used to end exactly at a specific value of x.

After changing the value of h in T+2,

Loc.	Instruction	Comment
α	TSX DE6F+588,4	Integrates one step with the Runge-Kutta
α+1	Return	subroutine.

APPENDIX B (Continued)

The integration step will be the value of h in T+2. This procedure could be used in the middle of the integration procedure if 3. above (TSX DE6F+22,4) is used immediately afterwards to restart in the Cowell/Runge-Kutta system.

In addition to the user's auxiliary subroutine and the 30N+3 cells for the T storage, the storage requirements are 955 words for RWDE6F plus 44 words of COMMON.

The value of the independent variable x is accumulated in double precision when incremented by h. The least significant part of x is saved in cell DE6F+718.

APPENDIX B - T STORAGE CHART

T												
T	N	x	h									
T+3	y_1	y_2	·	·	y_N	Initial conditions which user must supply.						
T+N+3	y'_1	y'_2	·	·	y'_N							
T+2N+3	y''_1	y''_2	·	·	y''_N	2nd derivatives stored by V subroutine.						
T+3N+3	y'_1	y'_2	·	·	y'_N	Most significant				y'_{10}	y''_{10}	Coordinate Change
T+4N+3	y'_1	y'_2	·	·	y'_N	Least significant				y'_{11}	y''_{11}	8 sets of consecu-
T+5N+3	$\Delta y'_1$	$\Delta y'_2$	·	·	$\Delta y'_N$	Destroyed by R.K.				y'_{12}	y''_{12}	tive y'_i saved by
T+6N+3	Δy_1	Δy_2	·	·	$\Delta y'_N$	Destroyed by R.K.				y'_{13}	y''_{13}	user if $y''=f(x,y,$
T+7N+3	y_1	y_2	·	·	y_N	Most significant				y'_{14}	y''_{14}	y'). Cowell
T+8N+3	y_1	y_2	·	·	y_N	Least significant				y'_{15}	y''_{15}	stores y''_i over
T+9N+3	y_{13}	y_{23}	·	·	y_{N3}	Saved for central				y'_{16}	y''_{16}	y'_i.
T+10N+3	y'_{13}	y'_{23}	·	·	y'_{N3}	difference equations.				y'_{17}	y''_{17}	
T+11N+3	y''_{10}	y''_{20}	·	·	y''_{N0}	Normal Case.				y_{10}		Coordinate Change
T+12N+3	y''_{11}	y''_{21}	·	·	y''_{N1}	Saved from 6R R-K				y_{11}		8 sets of consecu-
T+13N+3	y''_{12}	y''_{22}	·	·	y''_{N2}	steps. Used to form				y_{12}		tive y_i saved by
T+14N+3	y''_{13}	y''_{23}	·	·	y''_{N3}	difference tables in				y_{13}		user. Cowell uses
T+15N+3	y''_{14}	y''_{24}	·	·	y''_{N4}	an attempt to proceed				y_{14}		(with V sub) to
T+16N+3	y''_{15}	y''_{25}	·	·	y''_{N5}	to a Cowell step.				y_{15}		form y''_i above.
T+17N+3	y''_{16}	y''_{26}	·	·	y''_{N6}					y_{16}		
T+18N+3	Used only if coordinates changed.									y_{17}		
T+19N+3	Δ^{VI}_{10}	Δ^{V}_{11}	Δ^{IV}_{12}	Δ^{III}_{13}	Δ^{II}_{14}	Δ^{I}_{15}	y''_{16}	$''F_{18}$	$''F_{18}$	$'F_{17}$	$'F_{17}$	11N storages.
T+19N+14	Δ^{VI}_{20}	Δ^{V}_{21}	Δ^{IV}_{22}	Δ^{III}_{23}	Δ^{II}_{24}	Δ^{I}_{25}	y''_{26}	$''F_{28}$	$''F_{28}$	$'F_{27}$	$'F_{27}$	N sets of
T+19N+25	Δ^{VI}_{30}	Δ^{V}_{32}	Δ^{IV}_{32}	Δ^{III}_{33}	Δ^{II}_{34}	Δ^{I}_{35}	y''_{36}	$''F_{38}$	$''F_{38}$	$'F_{37}$	$'F_{37}$	differences
·	·	·	·	·	·	·	·	↑	↑	↑	↑	saved for
·	·	·	·	·	·	·	·	Most Sig.	Lst. Sig.	Most Sig.	Lst. Sig.	next Cowell step.
·	·	·	·	·	·	·	·	↓	↓	↓	↓	(See App. A)
T+30N-8	Δ^{VI}_{N0}	Δ^{V}_{N1}	Δ^{IV}_{N2}	Δ^{III}_{N3}	Δ^{II}_{N4}	Δ^{I}_{N5}	y''_{N6}	$''F_{N8}$	$''F_{N8}$	$'F_{N7}$	$'F_{N7}$	(Page F-14)

FORTRAN Subroutine

Identification

RW FINP - Decimal, Octal, BCD, Variable Data Input
7090 FAP Subroutine
W. J. Stoner, August 24, 1961
Aerospace Corporation

Purpose

To read a set of Hollerith punched data and/or header cards into core with one FORTRAN CALL statement.

To convert the data fields to binary and store in core according to their associated conversion codes.

Restrictions

This routine uses (CSH)S to accomplish the BCD card image read. Tape troubles or other errors from this routine are indicated by the print-out of HPR 1,4.

This routine uses (EXE) to print HPR 2,4 in case of errors such as non-Hollerith characters, data out of range, illegal format, subscripts too large for the array previously defined, etc. Upon detection of any error, control is immediately sent to (EXE) and no more cards are processed.

Method

Decimal numbers are converted to binary integers and then scaled to the indicated power of ten.

Octal numbers are converted to binary integers.

Hollerith words are stored directly.

Range: Decimal to floating binary conversion $10^{\pm 38}$
Decimal to fixed binary; 1 to 9 digits*
Decimal integer to binary integer; 1 to 5 digits
Octal integer to binary integer; 0 to $2^{35} - 1$

*the magnitude of the number depends upon the location of the decimal point.

Usage

Format:

1. The data card format, available on keypunch form M-1, consists of four subfields containing the conversion code, location, number, and exponent respectively.

FORTRAN Subroutine
RW FINP
Page 2
August 24, 1961

	Data Field	Data Field	Data Field	Data Field
Sub field	1	2	3	4
Conversion code	1	19	37	55
Location	2-6	20-24	38-42	56-60
Value	7-16	25-34	43-52	61-70
Exponent	17-18	35-36	53-54	71-72

where conversion code is one of the alphabetic characters defined below which specifies the type of conversion to be used on the value field, the location specifies the cell into which the converted value field is to be stored, the value subfield contains the data to be converted, and the exponent contains the power of ten by which floating data is to be scaled, or the location of the binary point of fixed point data.

2. The header card format consists of a conversion code in column 1, a sequence number in columns 2-6 and any Hollerith information in columns 7-72.

Decimal Points:

Decimal points may be placed anywhere in the value field except that they may not occur in the same column as a minus sign (11 punch) since this results in a non-Hollerith character. If the decimal point would normally appear at the right of the number punched in the value field, then it is optional.

Minus Signs:

Minus signs are 11 punches over any digit of the field. If all of the available columns of the field are not used, minus signs may be punched as the left character of the field.

Values:

Values must always be written to the extreme left of a field. It is not necessary that the entire field be filled as the first blank denotes the end of value. Superfluous low order zeros should be omitted as they increase conversion error.

The only exception to partial fields is BCI data where the entire field, including blanks, is stored.

Location:

~~The location may be specified by either absolute octal, a variable or~~ array name, or the element subscripts in a one or two dimensional array. If the locations contains five digits, it is interpreted as octal. All five columns must be punched for octal locations.

If the location contains at least one (1) non-numeric character, it is interpreted as a variable or array name which must appear exactly as given in the CALL statement (see Calling Sequence below). The contents of the number and exponent fields, if they are numeric data, are stored in the cell for the variable or the first cell for the array. This location then becomes the origin for all subscript locations following until another variable or array name is encountered. Caution must be taken to load an array name prior to subscript locations.

If the location contains four or fewer digits, it is interpreted as a subscript except for conversion code H explained below. Single dimension array subscripts must be left-justified with leading zeros optional. Two dimension array subscripts must be denoted by two subfields of two columns each containing i and j respectively. The i and j subfields must be separated by a comma and must contain two-digit integers.

If the location is left blank, then the location counter within the routine is decreased by 1 and the associated number is stored in the cell immediately preceding the cell where the last number was stored. Thus, an entire array may be read in by specifying the initial location only.

Conversion Codes:

Blank: Floating decimal

The number in the value field times the power of ten in the exponent field is converted to floating binary. Checks are made for overflow and format errors.

F: Fixed decimal

The number in the value field is converted to fixed point binary and stored with the binary point located at the position specified by the number in the exponent field. An overflow error check is made.

I: Decimal integer

The number in the value field is converted to a fixed point binary integer with the binary point following position 17. The exponent field is ignored. A decimal point is considered an error.

B: Octal

The value plus exponent fields are converted as a logical octal word.

It is not necessary to include leading zeros but the first octal digit must always occupy the leftmost position of the field.

D: BCI Data

The contents of the value plus exponent fields are interpreted as two BCI words and stored in two consecutive cells in descending order beginning at the location specified by the location field.

H: Heading Card

A card with an H in column 1 is considered a BCI heading card. If the location field is blank, the card is ignored. If the location field contains a left-justified one to four digit positive decimal integer V (octal, negative, or variable locations are not permitted), columns 7-72 of the card are stored directly in 11 consecutive words in descending order. The location of the first of these words is calculated by the routine as HEAD (1+11*(V-1)) where HEAD is defined as the last variable or array named in the CALL statement. Each card may be used as one record of output using FORMAT option A with column 7 of the card providing the code for printer spacing on output.

A: Variable names as data

The value plus exponent fields are interpreted in a pseudo FAP instruction format AAAAA T DDDDD P where the fields to replace are address, tag, decrement and prefix respectively. The address and decrement fields are defined normally to be 5 characters and the tag and prefix as one octal numeric character each. Any field containing less than the normal number of characters must end with a comma while fields of normal length must not. Any address or decrement field containing less then 5 numeric characters is converted as decimal while those of all 5 numeric characters are converted as octal. Any address or decrement field containing at least one non-numeric character is interpreted as a variable or array name. Variable addresses cause the entire word from the compiler generated calling sequence to be loaded into the location word (i.e., the TSX X is stored in the location specified if X is the variable appearing in the address field). Variable decrements cause the right-most 18 bits from the compiler generated calling sequence to be loaded into the location word's prefix and decrement. Numeric tags and prefixes are loaded directly into the corresponding parts of the location words. Null fields are not loaded. Since the first blank indicates the end of the loading of a word, address only, address-tag, address-tag-decrement, or entire word may be loaded as desired.

G: Temporary Origin

The value in the first location field on the card is used as a temporary origin for tables. The location is saved and if data cards follow with blank location fields the corresponding data is stored consecutively in descending order beginning with the cell specified in the location in the G card. Columns 7-72 are ignored and may be used to identify the table.

The first non-blank location starts a new origin. If this non-blank location is a subscript, it references the last variable or array named, which may or may not have been on the G card.

J: Transfer

The location specified with this prefix must be an octal address and is the only part of the data field that is interpreted. The subroutine causes a transfer to the octal location specified and does not interpret the remaining fields on the card.

L: Two dimension array i_{max}, j_{max} definition

The location field contains the name of the array to be loaded. The value field is defined to consist of 2 subfields, separated by a comma, of 2 columns each containing the two-digit decimal integers for i_{max} and j_{max} respectively where i_{max} and j_{max} generally appear in a DIMENSIØN statement. The i_{max} and j_{max} values are retained to compute the successive subscripted locations until redefined. Blank address fields may follow this array definition if successive elements of the array are to be loaded.

M: Two dimension array i_{max}, j_{max} definition

Conversion is identical to L except the entire array is preset to zero.

E: End Case

This defines an end-of-case and control is returned to the FORTRAN object program. The rest of this field and the remaining fields on the card are ignored.

Calling Sequence:

The following two types of CALL statements may be used:

I. CALL FINP (n,X,Y,ZETA,...,mHX5Y5ZETA2...) where

A. n is the number of variables and/or arrays in the list, excluding n itself.

B. X, Y, ZETA,... are the names of variables and/or arrays restricted to at most 5 characters each, one character of which is non-numeric.

C. m is 6 times n. Hence, mH allows for 6n Hollerith characters to follow.

D. X(5)Y(5)ZETA(2)... is a list of the items previously named in exactly the same order with (i) indicating the number, i, of blanks necessary to provide six Hollerith characters for each item. Since each item name is restricted to 5 characters, the minimum value of (i) is (1).

II. CALL FINP (0) where the number of items is given as zero. This CALL statement must be used only after a CALL statement of type I has been executed. When the subroutine encounters a zero for the number of items, it immediately refers to the last executed CALL FINP with a non-zero number of items for the names of the items to be loaded.

Space Requirements

613 cells.

Number of Pages

Writeup	6
Listing	12
Total	18

Identification

ASC GRAV - Gravitational Force Components
709/7090 FAP Language Subroutine*
R. J. Mercer, 30 August 1963
Aerospace Corporation

Purpose

Compute the components of the gravitational force, as derived from a general earth potential function, in a local horizontal coordinate system, and, optionally, obtain by rotation the components in an equatorial system with either the Greenwich meridian or an inertial direction as principal axis.

Restrictions

Only 18 cells are currently provided for the storage of sin λ_{nm} and cos λ_{nm}. This is sufficient for $n_2 = 4$. For larger values of n_2, the BSS instructions at the end of the subroutine must be appropriately modified.

GRAV refers to, but does not include, the subroutines SIN and CØS.

Mathematical Method

The potential function is

$$U = \frac{\mu}{r}\left[1 - \sum_{n=2}^{n_1} J_n \left(\frac{a_e}{r}\right)^n P_n(\sin\varphi) + \sum_{n=2}^{n_2} \sum_{m=1}^{n} J_{nm}\left(\frac{a_e}{r}\right)^n P_n^m(\sin\varphi)\cos m(\lambda - \lambda_{nm})\right]$$

where

μ	is the product GM of the Newtonian gravitational constant and the mass of the earth,
r, φ, λ	are the geocentric distance, geocentric latitude and (east) longitude of a point,
a_e	is the mean equatorial radius of the earth,
J_n, J_{nm}	are numerical coefficients,
P_n	is the Legendre polynomial of the first kind of degree n,
P_n^m	is the Legendre associated function of the first kind,
λ_{nm}	are longitudes associated with the J_{nm}.

* Easily converted to SCAT; see Usage.

In the local horizontal coordinate system, in which the coordinate axes are directed Up (along the radius vector), East and North (see figure), the force components are

$$g_U = \frac{\partial U}{\partial r}$$

$$= -\frac{\mu}{r^2}\left[1 - \sum_{n=2}^{n_1} (n+1)\, J_n \left(\frac{a_e}{r}\right)^n P_n (\sin\varphi)\right.$$

$$\left. + \sum_{n=2}^{n_2} \sum_{m=1}^{n} (n+1)\, J_{nm} \left(\frac{a_e}{r}\right)^n P_n^m (\sin\varphi) \cos m (\lambda - \lambda_{nm})\right]$$

$$g_E = \frac{1}{r\cos\varphi}\ \frac{\partial U}{\partial\lambda}$$

$$= -\frac{\mu}{r^2 \cos\varphi} \sum_{n=2}^{n_2} \sum_{m=1}^{n} m\, J_{nm} \left(\frac{a_e}{r}\right)^n P_n^m (\sin\varphi) \sin m (\lambda - \lambda_{nm})$$

$$g_N = \frac{1}{r}\ \frac{\partial U}{\partial\varphi}$$

$$= \frac{\mu}{r^2}\left[- \sum_{n=2}^{n_1} J_n \left(\frac{a_e}{r}\right)^n P_n' (\sin\varphi) \cos\varphi\right.$$

$$\left. + \sum_{n=2}^{n_2} \sum_{m=1}^{n} J_{nm} \left(\frac{a_e}{r}\right)^n P_n^{m'} (\sin\varphi) \cos\varphi \cos m (\lambda - \lambda_{nm})\right]$$

The Legendre functions and their derivatives are computed from the recursion formulas

$$P_n (\sin\varphi) = \frac{-(n-1)\, P_{n-2} (\sin\varphi) + (2n-1) \sin\varphi\, P_{n-1} (\sin\varphi)}{n}$$

$$P_n' (\sin\varphi) = \sin\varphi\, P_{n-1}' (\sin\varphi) + n\, P_{n-1} (\sin\varphi)$$

$$\frac{P_n^m (\sin\varphi)}{\cos\varphi} = \frac{-(n+m-1)\dfrac{P_{n-2}^m (\sin\varphi)}{\cos\varphi} + (2n-1)\sin\varphi\dfrac{P_{n-1}^m (\sin\varphi)}{\cos\varphi}}{n-m}$$

$$\frac{P_m^m (\sin\varphi)}{\cos\varphi} = 1 \cdot 3 \cdot \ldots (2m-1) (\cos\varphi)^{m-1}$$

$$P_n^{m'}(\sin\varphi)\cos\varphi = (n+1)\sin\varphi\,\frac{P_n^m(\sin\varphi)}{\cos\varphi} - (n-m+1)\,\frac{P_{n+1}^m(\sin\varphi)}{\cos\varphi}$$

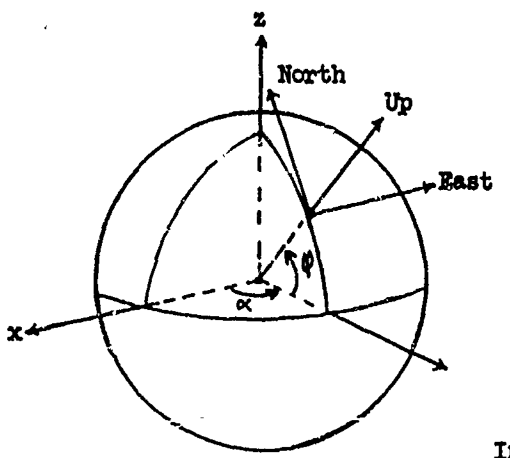
The use of the quotient $\dfrac{P_n^m(\sin\varphi)}{\cos\varphi}$ avoids numerical difficulties at high latitudes in the equation for g_E.

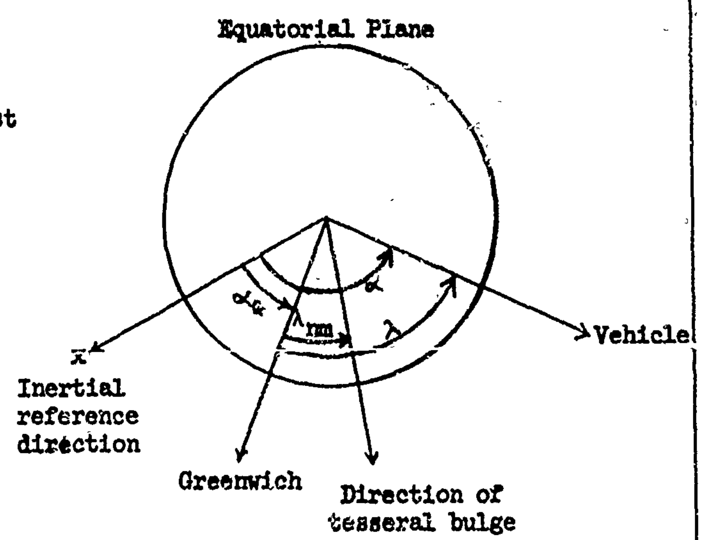
The components of the force vector in an equatorial coordinate system with x in the direction of the Greenwich meridian are

$$\begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix} = \begin{pmatrix} \cos\varphi\cos\lambda & -\sin\lambda & -\sin\varphi\cos\lambda \\ \cos\varphi\sin\lambda & \cos\lambda & -\sin\varphi\sin\lambda \\ +\sin\varphi & 0 & \cos\varphi \end{pmatrix} \begin{pmatrix} g_U \\ g_E \\ g_N \end{pmatrix}$$

The components in an equatorial inertial coordinate system with x in the direction of γ (vernal equinox) are obtained by a similar transformation in which α replaces λ.

Optionally, either longitude λ or both α and α_G (inertial right ascensions of the vehicle and Greenwich - see figure) may be input.





Usage

The subroutine is written in the FAP language for use with FORTRAN II programs, in which arrays are stored backwards. With three simple changes, which are carefully described in the comments of the symbolic cards, the subroutine can be assembled for use with machine language programs in which arrays are stored forwards.

The subroutine has an initial, a regular and a third entry point. In the initial entrance, all addresses are set up from the calling sequence, and certain preliminary computations are performed. At the regular entrance (for which no calling sequence is used), the force components are evaluated using the options and the locations of the constants and variables (input and output) specified in the initial entrance calling sequence.

The initial entry to the FAP version must be with the statement:

```
CALL GRAV1 (GM, AE, FN1, FJN, FN2, FJNM, FLNM, ØP1, ØP2, ARG,
            GLH, GX, ØP3, A, B, C, D, E, F, G, R, U)
```

Only address set ups and preliminary computations are performed; the acceleration components are not evaluated. The regular entry is with the statement

```
CALL    GRAV
```

The machine language version is entered initially with

```
TSX     GRAV + 2, 4
PZE     GM
PZE     AE
  :  (etc.)
PZE     U
Normal return
```

and for the computation of acceleration components with

```
TSX     GRAV, 4
Normal return
```

In numerical integration, it is often unnecessary to recompute the perturbative accelerations for the corrector step. A third entry

```
        CALL    GRAV2
or      TSX     GRAV + 3, 4
```

provides this option. When so used, the subroutine uses the last-computed summations, but recomputes $\frac{\mu}{r^2}$ and the rotation matrix before forming g_U, g_E, g_N, g_x, g_y, g_z and U.

The subroutine arguments are the 22 locations

GM — GM = μ

AE — a_e

FN1 — n_1 (floating point)

FJN — Array containing $J_2, J_3, \ldots, J_{n_1}$

FN2 — n_2 (floating point)

FJNM — Array containing $J_{21}, J_{31}, \ldots, J_{n_2 1}, J_{22}, J_{32}, \ldots, J_{n_2 2}, J_{33}, \ldots, J_{n_2 n_2}$

FLNM — Array containing λ_{nm} (in degrees) in like order

ØP1 — Zero for longitude input; non-zero for right ascension input (see ARG).

ØP2 — Positive for rotation of output to inertial coordinates (right ascension input, ØP1 $\neq$ 0, must be used); negative for rotation of output to earth-fixed system; zero for no output rotation.

ARG — Array containing, in order, the input variables

$$r, \sin\varphi, \cos\varphi, \begin{cases} \sin\lambda, \cos\lambda & \text{if } ØP1 = 0 \\ \sin\alpha_G, \cos\alpha_G, \sin\alpha, \cos\alpha & \text{if } ØP1 \neq 0 \end{cases}$$

GLH — Output array containing accelerations g_U, g_E, g_N in basic local horizontal coordinate system.

GX — Output array containing (if ØP2 $\neq$ 0) accelerations in chosen (see ØP2) equatorial coordinate system.

ØP3 Non-zero if individual terms in the components of GLH are to be saved in regions A, B, ..., G; Zero if not. (Regions A, B, ..., G are not cleared by GRAV.)

A Array containing (if ØP3 ≠ 0) $A_n = -(n+1) J_n \left(\frac{a_e}{r}\right)^n P_n (\sin \varphi)$ for $n = 2, \ldots, n_1$.

B Array; $B_{nm} = (n+1) J_{nm} \left(\frac{a_e}{r}\right)^n P_n^m (\sin \varphi)$ in order as in FJNM.

C Array; $C_{nm} = \frac{m}{\cos \varphi} J_{nm} \left(\frac{a_e}{r}\right)^n P_n^m (\sin \varphi)$

D Array; $D_n = J_n \left(\frac{a_e}{r}\right)^n P_n' (\sin \varphi) \cos \varphi$

E Array; $E_{nm} = -J_{nm} \left(\frac{a_e}{r}\right)^n P_n^{m'} (\sin \varphi) \cos \varphi$

F Array; $\sin m(\lambda - \lambda_{nm})$ in order as in FJNM

G Array; $\cos m(\lambda - \lambda_{nm})$ in like order

R Rotation matrix, stored by columns, specified by ØP2.

U The value of the potential function.

Space Required

482 cells.

Checkout

All intermediate results were hand computed for a case with $n_1 = 4$, $n_2 = 2$. Spot checks were made on many other cases, in which the various options were tested.

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13 ABSTRACT

The TRACE-D computer program is designed for use on the IBM 7094 machine as the principal tool in design and analysis studies of all aspects of orbital operations. Its principal characteristics are completeness of the equations of motion, a comprehensive set of differential correction parameters, and the ability to simultaneously process observations of several satellites taken by a number of different types of sensors. The report includes objectives, equations, program structure, and complete instructions for input data preparation and program operation.

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